Hands-on Running Exercise:
Nonlinear simulations of peeling-ballooning modes

X. Q. Xu

Presented at

2011 BOUT++ Workshop

Lawrence Livermore National Laboratory, Livermore, CA

September 15, 2011

Work performed for U.S. DOE by U.C. LLNL under Contract DE-AC52-07NA27344, LLNL-PRES-499292

The basic set of equations is found for nonlinear simulations of non-ideal MHD peeling-ballooning modes

$$\frac{\partial \varpi}{\partial t} + v_E \cdot \nabla \varpi = B_0^2 \nabla_{||}\left(\frac{j_{||}}{B_0}\right) + 2b_0 \times \kappa \cdot \nabla P,$$

$$\frac{\partial P}{\partial t} + v_E \cdot \nabla P = 0,$$

$$\frac{\partial A_{||}}{\partial t} = -\nabla_{||}(\phi + \Phi_0) + \frac{\eta}{\mu_0}\nabla_\perp^2 A_{||} - \frac{\eta_H}{\mu_0}\nabla_\perp^4 A_{||},$$

$$\varpi = \frac{n_0 M_i}{B_0}\left(\nabla_\perp^2 \phi + \frac{1}{n_0 Z_i e}\nabla_\perp^2 P\right),$$

$$j_{||} = J_{||0} - \frac{1}{\mu_0}\nabla_\perp^2 A_{||}, v_E = \frac{1}{B_0} b_0 \times \nabla(\phi + \Phi_0)$$

Non-ideal physics

✓Using resistive MHD term, resistivity can renormalized as Lundquist Number $S=\mu_0 R v_A/\eta$

✓Using hyper-resistivity η_H $S_H=\mu_0 R^3 v_A/\eta_H=S/\alpha_H$

✓After gyroviscous cancellation, the diamagnetic drift modifies the vorticity and additional nonlinear terms

✓Using force balance and assuming no net rotation,

$E_{r0}=(1/N_i Z_i e)\nabla_\perp P_{i0}$

THE UNIVERSITY of York | Fusion Energy Program | GENERAL ATOMICS

The normalized basic set of equations

Define typical length scale $\overline{L}$, timescale $\overline{T}$ and magnetic field $\overline{B}$, $\overline{V_A} = \overline{B}/\sqrt{\mu_0\rho} = \overline{L}/\overline{T}$.

$$\hat{t} = \frac{t}{\overline{T}} \qquad \hat{B} = \frac{B}{\overline{B}} \qquad \hat{\nabla} = \overline{L}\nabla \qquad \hat{\kappa} = \overline{L}\kappa$$

$$\hat{U} = \overline{T}U \qquad \hat{\psi} = \frac{\psi}{\overline{L}} \qquad \hat{P} = \frac{2\mu_0 P}{\overline{B}^2} \qquad \hat{J}_{||} = -\frac{\mu_0\overline{L}}{B_0}J_{||} \qquad \hat{\phi} = \frac{\phi}{\overline{V_A}\overline{L}B_0}$$

$$\frac{\partial U}{\partial t} + v_E \cdot \nabla U = B_0^2 \nabla_{||} j_{||} + b_0 \times \kappa \cdot \nabla P,$$

$$\frac{\partial P}{\partial t} + v_E \cdot \nabla P = 0,$$

$$\frac{\partial \psi}{\partial t} = -\nabla_{||}(\phi + \Phi_0) + \frac{1}{S}\nabla_\perp^2 \psi - \frac{1}{S_H}\nabla_\perp^4 \psi,$$

$$U = \left(\nabla_\perp^2 \phi + \frac{1}{\omega_{ci} T}\nabla_\perp^2 P_i\right), A_{||} = -B_0\psi$$

$$j_{||} = \nabla_\perp^2 \psi, v_E = b_0 \times \nabla(\phi + \Phi_0)$$

Non-ideal physics

✓Using resistive MHD term, resistivity can renormalized as Lundquist Number $S=\mu_0 R v_A/\eta$

✓Using hyper-resistivity η_H

$S_H=\mu_0 R^3 v_A/\eta_H=S/\alpha_H$

✓After gyroviscous cancellation, the diamagnetic drift modifies the vorticity and additional nonlinear terms

✓Using force balance and assuming no net rotation,

$E_{r0}=(1/N_i Z_i e)\nabla_\perp P_{i0}$

Boundary conditions

On the inner (core) boundary, the conditions applied are

$$\omega = 0 \qquad \frac{\partial \phi}{\partial x} = 0, \nabla_\perp^2 \phi = 0 \qquad j_{||} = 0 \qquad \frac{\partial^2 A_{||}}{\partial x^2} = \frac{\partial A_{||}}{\partial x} = 0$$

and outer (vacuum) boundaries are

$$\omega = 0 \qquad \frac{\partial \phi}{\partial x} = 0, \nabla_\perp^2 \phi = 0 \qquad j_{||} = 0 \qquad \nabla_\perp^2 A_{||} = 0$$

Running a simulation

```
hyperion0:xu(xu)> git clone git://github.com/bendudson/BOUT-1.0.git

hyperion0:xu(xu)> cd BOUT-1.0
hyperion0:xu(BOUT-1.0)>source configure.hyperion.tcsh
hyperion0:xu(BOUT-1.0)>gmake
hyperion0:xu(BOUT-1.0)> cd examples/elm-pb
hyperion0:xu(elm-pb)> gmake

hyperion0:xu(elm-pb)> srun -n16 ./elm_pb
```

Running a simulation

```
hyperion0:xu(elm-pb)>idl
IDL> loadct, 39
IDL> device, decomposed=0
IDL> tek_color
IDL>g=file_import(“data/cbm18_dens8.grid_nx68ny64.nc”)
IDL>; to plot grid
IDL> plot, g.rxy, g.zxy, /iso, psym=3
IDL> contour2, g.bpxy, g.rxy, g.zxy, nlev=30,/fil,/iso, tit="poloidal B field"

IDL>  p = collect(path="data", var="P")
IDL> moment_xyzt, p, rms=rms
IDL> plot, deriv(alog(rms[40,32,*]))
IDL>
```

- **For low resolution mesh nx=68, data/cbm18_dens8.grid_nx68ny64.nc, the growth rate is 0.275002 *Alfven time.**
- **For high resolution mesh nx=516, data/cbm18_8_y064_x516_090309.nc, the growth rate is 0.186655 *Alfven time.**
- **The difference is 47%.**

Elm-pb running options

✓To advance the system state in time, BOUT++ uses the CVODE package with the Newton Krylov BDF implicit method

✓The spatial derivatives are discretized with finite differences. There is a range of finite difference schemes chosen at run time

- ➢4th order central difference
- ➢3rd order WENO.

✓Automatically handles details of parallel simulation with 2D domain decomposition

- ☞ NXPE
- ☞ NYPE

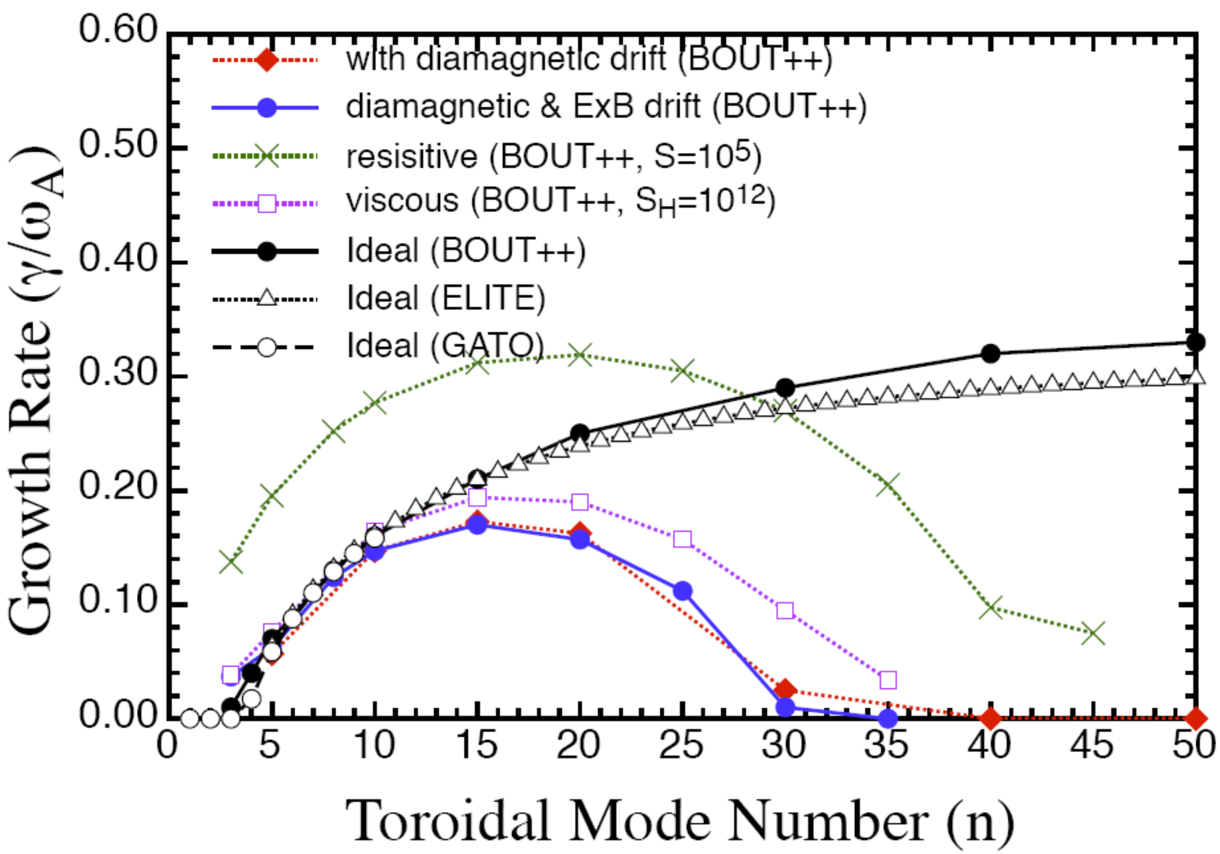
✓Toroidal segment, zperiod=n, the toroidal mode number

- ✓zperiod = 5

✓Filter option: keep only one mode for linear comparison

✓Hyper-resistivity for nonlinear ELM simulation

A good agreement for an ideal MHD model is shown between GATO, ELITE, and BOUT++ codes



$$\frac{D}{Dt} = \frac{\partial}{\partial t} + v_E \cdot \nabla,$$

$$v_E = \frac{1}{B_0} b_0 \times \nabla \phi - \frac{E_{r0}}{B_0},$$

$$\mathrm{E_{r0}} = (1/\mathrm{N_i Z_i e}) \nabla_r P_{i0}$$

$$\frac{\partial A_{||}}{\partial t} = -\nabla_{||}(\phi + \Phi_0)$$

$$+ \frac{\eta}{\mu_0} \nabla_\perp^2 A_{||}$$

$$- \frac{\eta_H}{\mu_0} \nabla_\perp^4 A_{||}$$

$$\eta_H = \eta \frac{\mu_e}{\nu_{ei}}$$

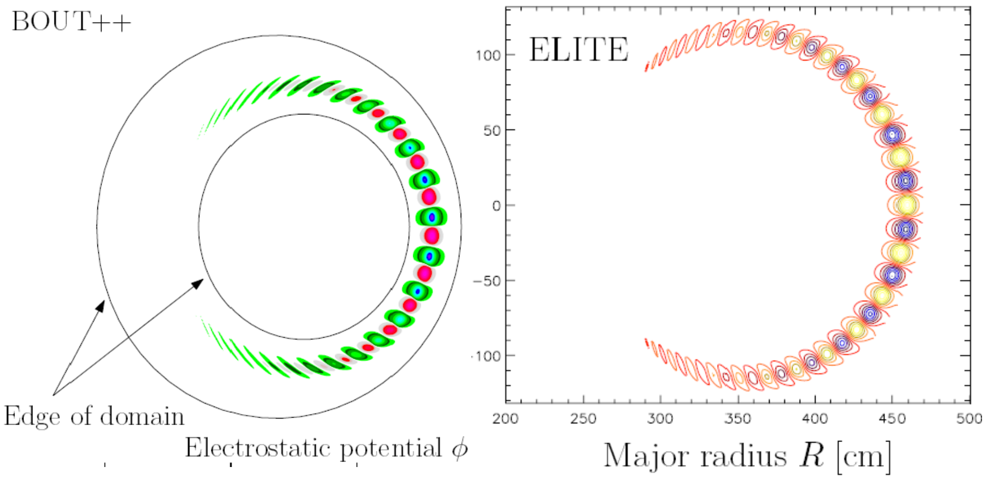
$$\mathrm{S} = \mu_0 R v_A / \eta$$

$$\mathrm{S_H} = \mu_0 R^3 v_A / \eta_H = S / \alpha_H$$

✓Resistivity & viscosity destabilizes the ideal modes

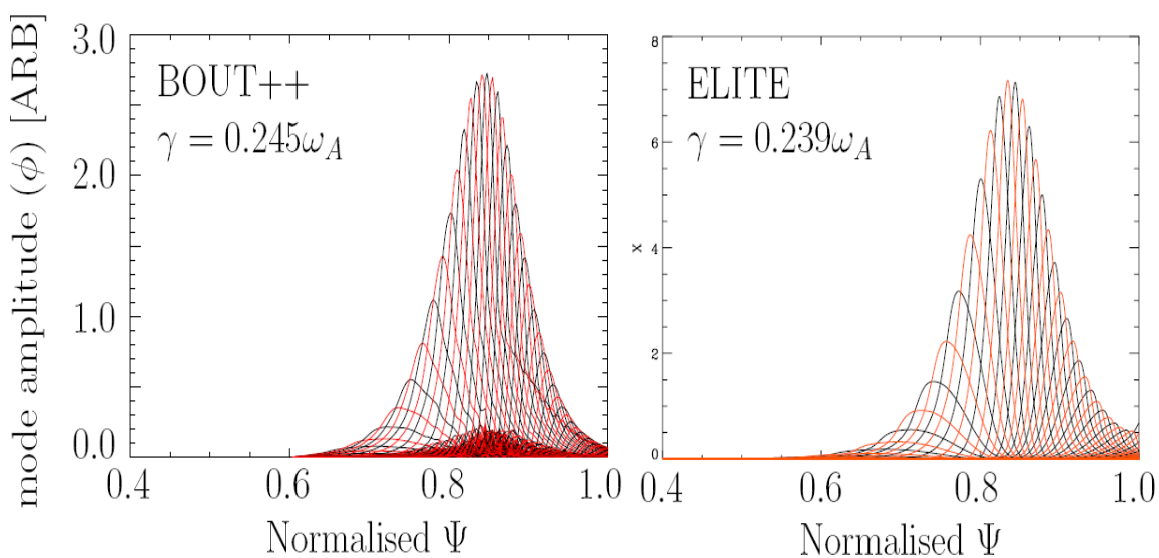
✓ion diamagnetic drift and ExB drift stabilize the ideal modes

Linear mode structures are similar in BOUT++ and ELITE



BOUT++ and ELITE show the same poloidal wavelength n=20

Radial Mode structures are similar in BOUT++ and ELITE



THE UNIVERSITY of York

Fusion Energy Program

GENERAL ATOMICS

Lundquist number S plays a critical role on nonlinear ideal ballooning mode

Time step collapses at high Lundquist number S w/o η_H