Hands-on Running Exercise:
3-field simulations of peeling-ballooning modes

X. Q. Xu

Presented at
2013 BOUT++ Workshop
Lawrence Livermore National Laboratory, Livermore, CA
September 3, 2013

Work performed for U.S. DOE by U.C. LLNL under Contract DE-AC52-07NA27344, LLNL-PRES-643037

The basic set of equations is found for nonlinear simulations of non-ideal MHD peeling-ballooning modes

$$\frac{\partial \varpi}{\partial t} + v_E \cdot \nabla \varpi = B_0^2 \nabla_\parallel \left(\frac{j_\parallel}{B_0} \right) + 2 b_0 \times \kappa \cdot \nabla P,$$

$$\frac{\partial P}{\partial t} + v_E \cdot \nabla P = 0,$$

$$\frac{\partial A_\parallel}{\partial t} = -\nabla_\parallel (\phi + \Phi_0) + \frac{\eta}{\mu_0} \nabla_\perp^2 A_\parallel - \frac{\eta_H}{\mu_0} \nabla_\perp^4 A_\parallel,$$

$$\varpi = \frac{n_0 M_i}{B_0} \left(\nabla_\perp^2 \phi + \frac{1}{n_0 Z_i e} \nabla_\perp^2 P \right),$$

$$j_\parallel = J_{\parallel 0} - \frac{1}{\mu_0} \nabla_\perp^2 A_\parallel, v_E = \frac{1}{B_0} b_0 \times \nabla (\phi + \Phi_0)$$

Non-ideal physics

✓Using resistive MHD term, resistivity can renormalized as Lundquist Number $S=\mu_0 R v_A/\eta$

✓Using hyper-resistivity η_H $S_H=\mu_0 R^3 v_A/\eta_H=S/\alpha_H$

✓After gyroviscous cancellation, the diamagnetic drift modifies the vorticity and additional nonlinear terms

✓Using force balance and assuming no net rotation,

$E_{r0}=(1/N_i Z_i e)\nabla_\perp P_{i0}$

The normalized basic set of equations

Define typical length scale $\overline{L}$, timescale $\overline{T}$ and magnetic field $\overline{B}$, $\overline{V_A} = \overline{B}/\sqrt{\mu_0\rho} = \overline{L}/\overline{T}$.

$$\hat{t} = \frac{t}{\overline{T}} \qquad \hat{B} = \frac{B}{\overline{B}} \qquad \hat{\nabla} = \overline{L}\nabla \qquad \hat{\kappa} = \overline{L}\kappa$$

$$\hat{U} = \overline{T}U \qquad \hat{\psi} = \frac{\psi}{\overline{L}} \qquad \hat{P} = \frac{2\mu_0 P}{\overline{B}^2} \qquad \hat{J}_{\|} = -\frac{\mu_0 \overline{L}}{B_0}J_{\|} \qquad \hat{\phi} = \frac{\phi}{\overline{V_A L B_0}}$$

$$\frac{\partial U}{\partial t} + v_E \cdot \nabla U = -B_0^2 \nabla_{\|} j_{\|} + b_0 \times \kappa \cdot \nabla P,$$

$$\frac{\partial P}{\partial t} + v_E \cdot \nabla P = 0,$$

$$\frac{\partial \psi}{\partial t} = -\nabla_{\|}(\phi + \Phi_0) + \frac{1}{S}\nabla_{\perp}^2 \psi - \frac{1}{S_H}\nabla_{\perp}^4 \psi,$$

$$U = \left(\nabla_{\perp}^2 \phi + \frac{1}{\omega_{ci} T}\nabla_{\perp}^2 P_i\right), A_{\|} = -B_0 \psi$$

$$j_{\|} = \nabla_{\perp}^2 \psi, v_E = b_0 \times \nabla(\phi + \Phi_0)$$

Non-ideal physics

✓Using resistive MHD term, resistivity can renormalized as Lundquist Number $S=\mu_0 R v_A/\eta$

✓Using hyper-resistivity η_H $S_H=\mu_0 R^3 v_A/\eta_H=S/\alpha_H$

✓After gyroviscous cancellation, the diamagnetic drift modifies the vorticity and additional nonlinear terms

✓Using force balance and assuming no net rotation,

$E_{r0}=(1/N_i Z_i e)\nabla_{\perp} P_{i0}$

Boundary conditions

On the inner (core) boundary, the conditions applied are

$$\omega = 0 \qquad \frac{\partial \phi}{\partial x} = 0, \nabla_\perp^2 \phi = 0 \qquad j_{||} = 0 \qquad \frac{\partial^2 A_{||}}{\partial x^2} = \frac{\partial A_{||}}{\partial x} = 0$$

and outer (vacuum) boundaries are

$$\omega = 0 \qquad \frac{\partial \phi}{\partial x} = 0, \nabla_\perp^2 \phi = 0 \qquad j_{||} = 0 \qquad \nabla_\perp^2 A_{||} = 0$$

Running a simulation

```
hopper04:xu(elm-pb)>cd bout/examples/elm-pb
hopper04:xu(elm-pb)>make
hopper04:xu(elm-pb)> qsub bout_hopper_debug_PE16.pbs
hopper04:xu(elm-pb)>qstat -uxu

IDL>  p = collect(path="data", var="P")
IDL> moment_xyzt, p, rms=rms
IDL> plot, deriv(alog(rms[317,32,*]))
IDL>
```

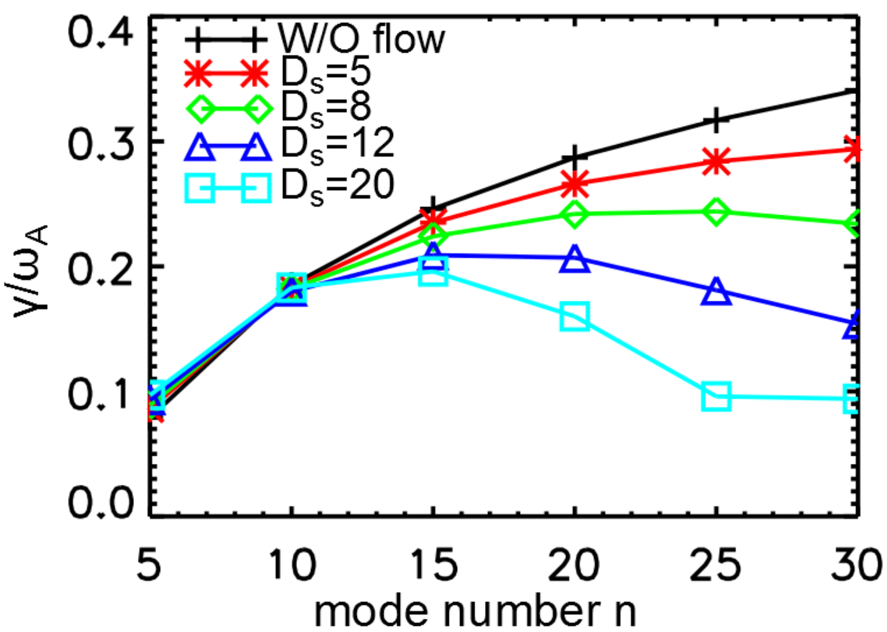
Elm-pb running options with shear flow

$$\mathbf{V} = K(\psi)\mathbf{B} + R^2\Omega(\psi)\nabla\zeta, \qquad \Omega(\psi) = \frac{d\Phi_{V0}}{d\psi}$$

$$\Phi_0 = \Phi_{dia0} + \Phi_{V0}, \qquad E_r = -\frac{d\Phi_0}{d\psi} RB_p$$

$$\begin{cases} \Phi_{dia0} = -\frac{1}{2Zen_0} P_0 = \frac{\overline{L}^2}{\overline{T}} B_0 * phi0 \\ \frac{d\Phi_{V0}}{d\psi} = D_0\left[1 - \tanh\left(D_s\left(x - x_0\right)\right)\right] + D_{\min} = \frac{1}{\overline{T}} * Dphi0 \end{cases}$$

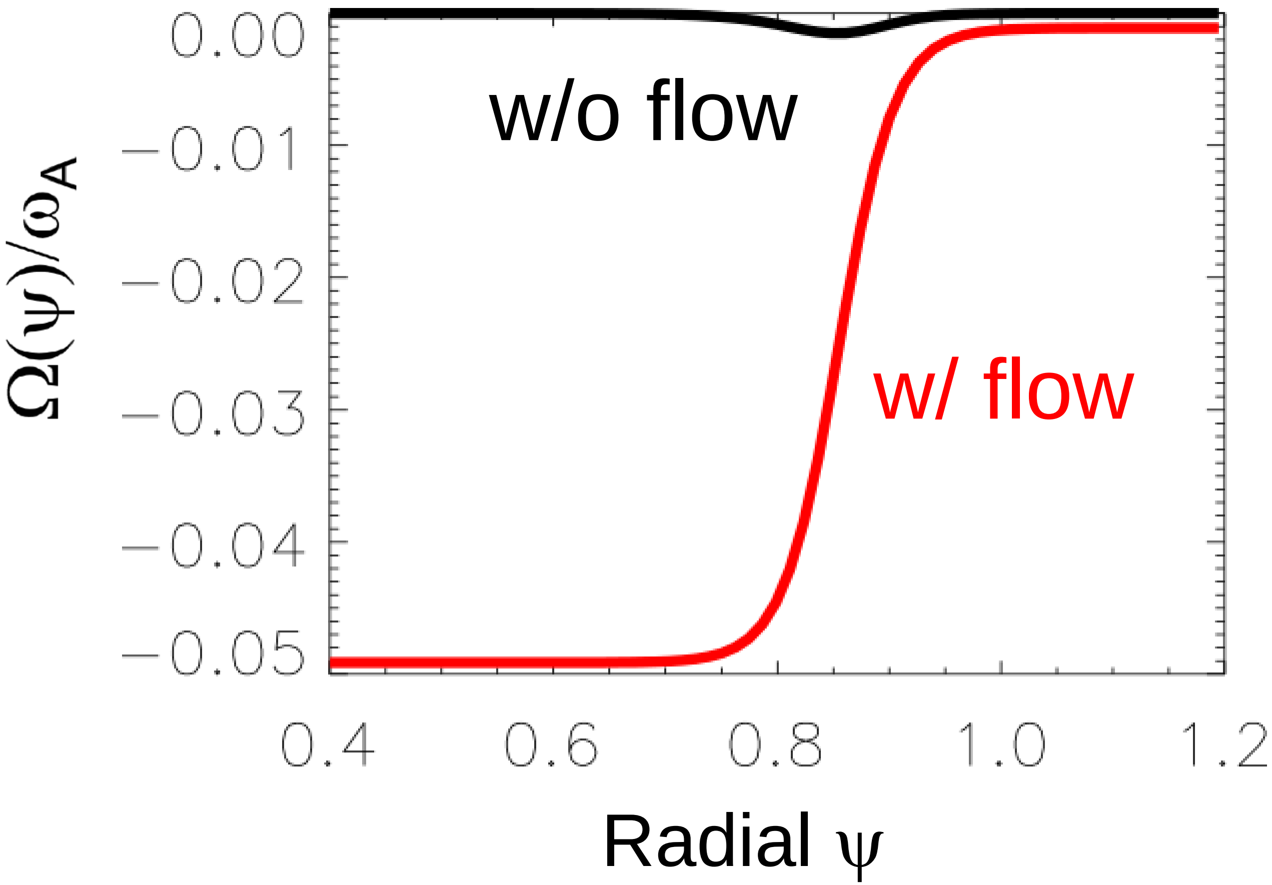
Read from the output file



Options	Values	Description
withflow	true	Net flow switch
D_0	130000	Flow amplitude
D_s	20.0	Flow shear
K_H_term	false	Kelvin-Helmholtz term
Sign	-1	Flow direction
x0	0.855	Flow location
D_min	3000	Flow amplitude at pedestal bottom

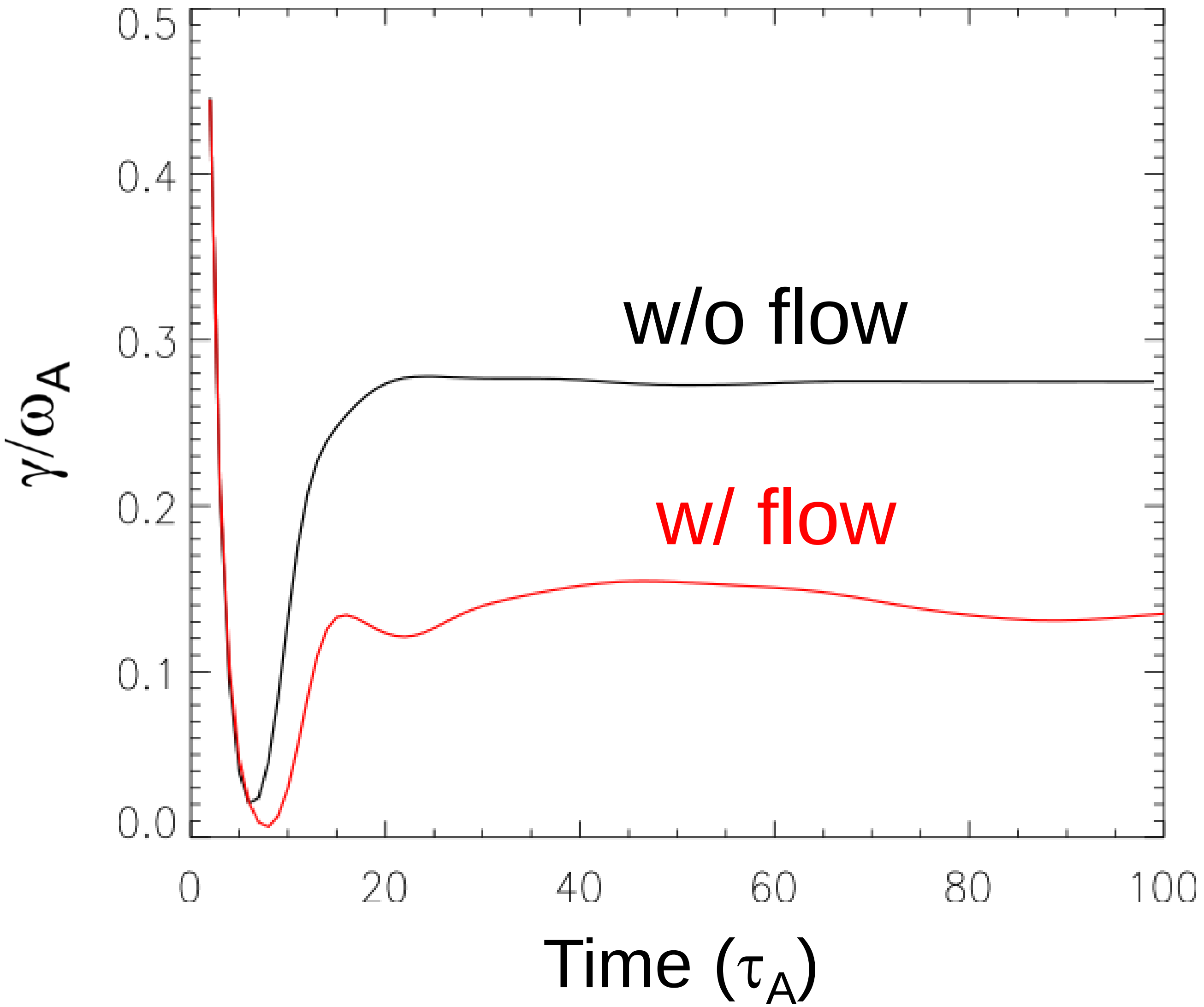
External shear flow profiles

```
IDL> psix=(g.psixy[*,32]-g.PSI_AXIS)/(g.PSI_BNDRY-g.PSI_AXIS)
IDL> Epsi=-deriv(psix,phi0[*,32])
IDL> plot, psix,-Dphi0[*,32],thick=2
IDL> oplot, psix,Epsi,col=2,thick=2
```

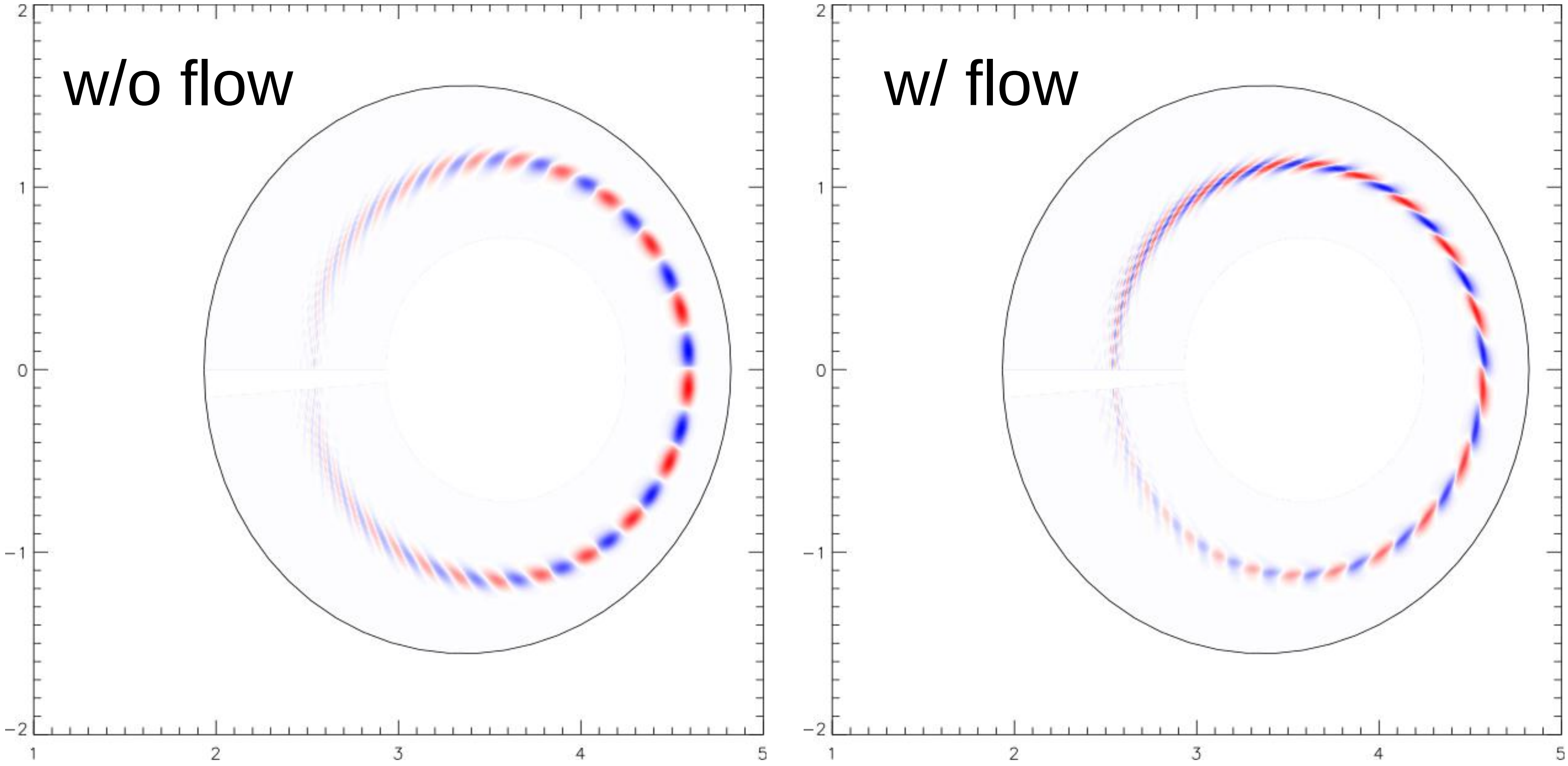


Linear growth rate is reduced with flow shear

```
IDL> plot, deriv(alog(rmsp_0f[34,32,*])),thick=3,chars=2
IDL> oplot, deriv(alog(rmsp_f[34,32,*])),col=2,thick=3
```

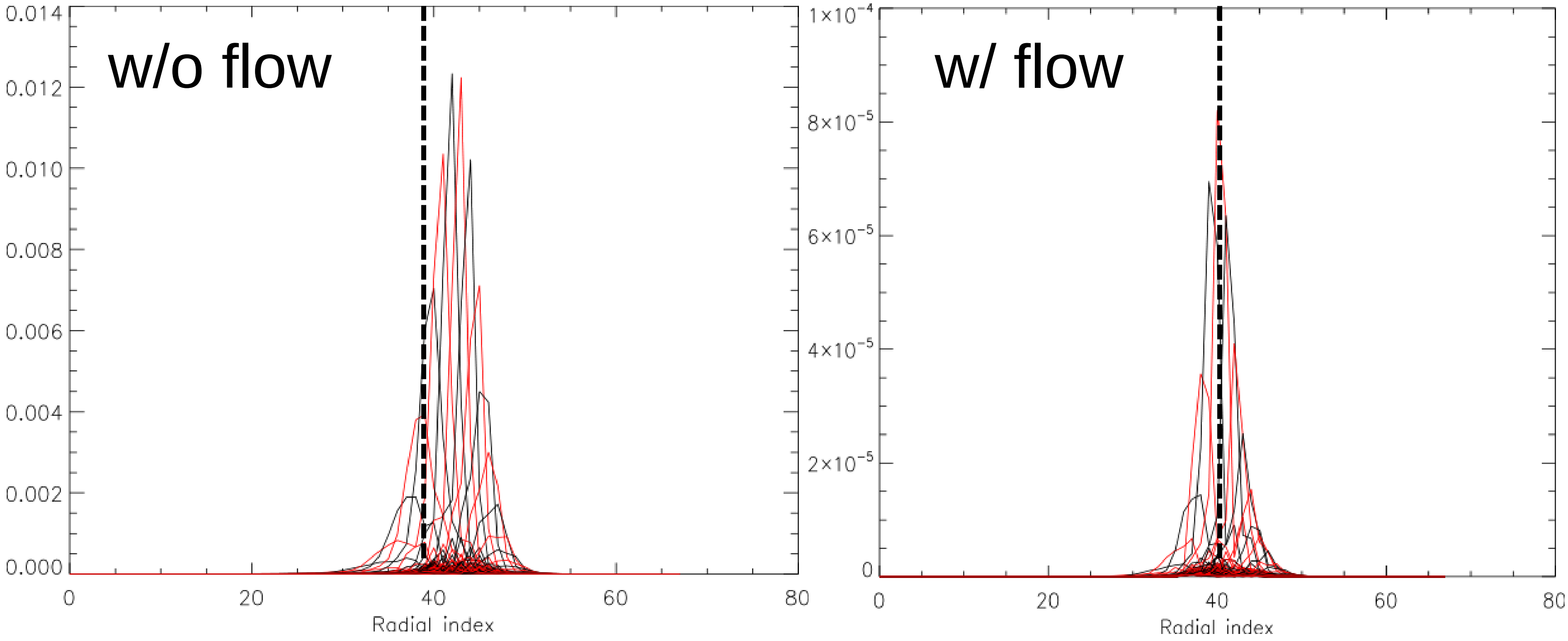


Poloidal slice shows the sheared filaments with flow




```
IDL> p_0f_t50=p_0f[*,*,*,50]
IDL> p_f_t50=p_f[*,*,*,50]
IDL> plotpolslice, p_f_t50, g, period=15
IDL> plotpolslice, p_f_t50, g, period=15
```

Radial mode structures are changed with flow shear




```
IDL> p_0f_t50=p_0f[*,*,*,50]
IDL> p_f_t50=p_f[*,*,*,50]
IDL> mode_structure, p_0f_t50, g, period=15
IDL> mode_structure, p_f_t50, g, period=15
```

Useful idl analysis commands

```
IDL> loadct, 39
IDL> device, decomposed=0

IDL> Dphi0 = collect(path="data", var="Dphi0") surface, Dphi0, chars=3
IDL> phi0 = collect(path="data", var="phi0") window, 1 surface, phi0,
IDL> chars=3
IDL> g=file_import("data/cbm18_dens8.grid_nx68ny64.nc")

IDL> help, g
IDL> psix=(g.psixy[*,32]-g.PSI_AXIS)/(g.PSI_BNDRY-g.PSI_AXIS)
IDL> Epsi=-deriv(psix,phi0[*,32])
IDL> plot, psix,-Dphi0[*,32],thick=2
IDL> oplot, psix,Epsi,col=2,thick=2

IDL> p_0f = collect(path="data.0flow", var="p")
IDL> p_f = collect(path="data.flow", var="p")

IDL>  moment_xyzt, p_f, rms=rmsp_f
IDL>  moment_xyzt, p_0f, rms=rmsp_0f

IDL> plot, deriv(alog(rmsp_0f[34,32,*])),thick=3,chars=2
IDL> oplot, deriv(alog(rmsp_f[34,32,*])),col=2,thick=3
IDL> showdata, p_0f[*,*,*,*], contour=contour, yr=yr, color=color, noscale=noscale, uedge=g, period=15,bmp=bmp

IDL> p_0f_t50=p_0f[*,*,*,50]
IDL> p_f_t50=p_f[*,*,*,50]
IDL> plotpolslice, p_f_t50, g, period=15
IDL> plotpolslice, p_f_t50, g, period=15
IDL> mode_structure, p_0f_t50, g, period=15
IDL> mode_structure, p_f_t50, g, period=15

IDL> showdata, p_0f[*,*,*,*], contour=contour, yr=yr, color=color, noscale=noscale, uedge=uedge, period=5,bmp=bmp ,output="Nonlinear_image/ptot0000"
```

Elm-pb running options

✓To advance the system state in time, BOUT++ uses the CVODE package with the Newton Krylov BDF implicit method

✓The spatial derivatives are discretized with finite differences. There is a range of finite difference schemes chosen at run time

- ➢4th order central difference
- ➢3rd order WENO.

✓Automatically handles details of parallel simulation with 2D domain decomposition

- ☞ NXPE
- ☞ NYPE

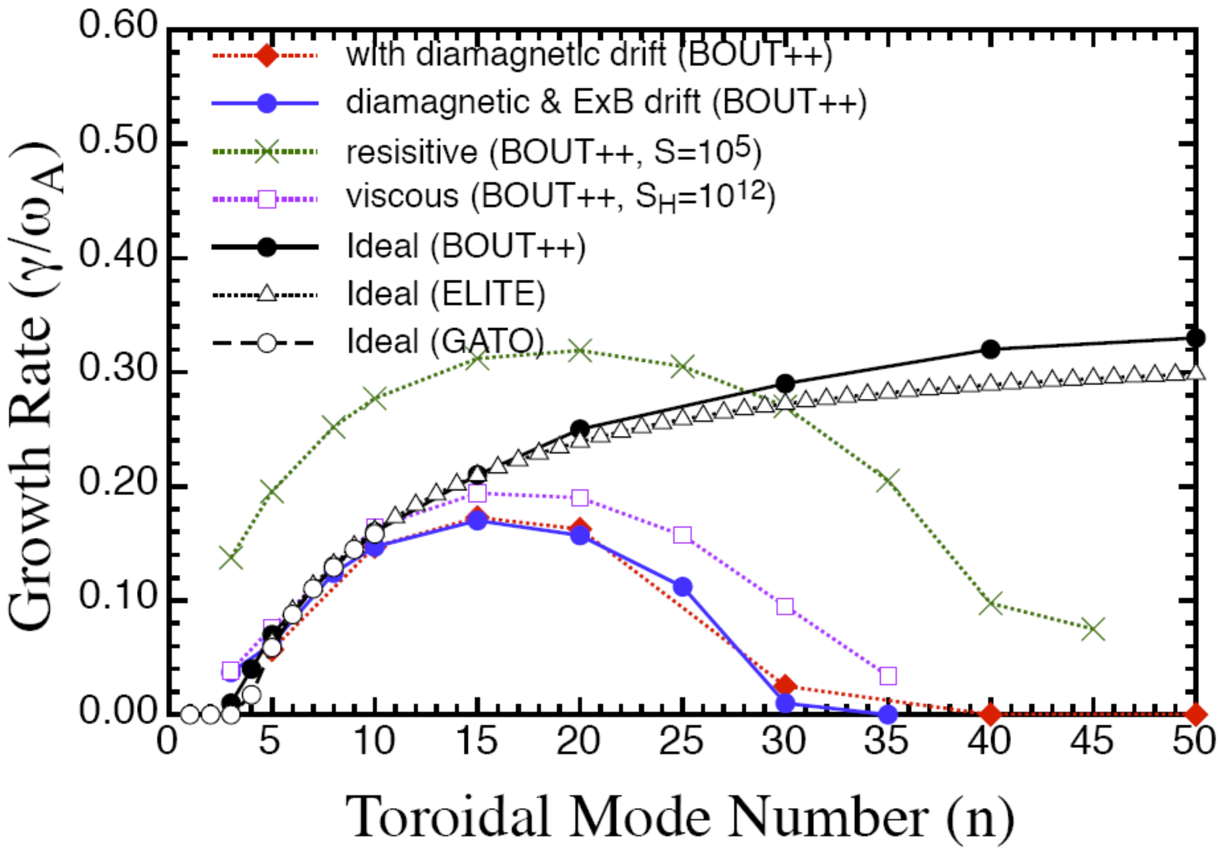
✓Toroidal segment, zperiod=n, the toroidal mode number

- ✓zperiod = 15

✓Filter option: keep only one mode for linear comparison

✓Hyper-resistivity for nonlinear ELM simulation

A good agreement for an ideal MHD model is shown between GATO, ELITE, and BOUT++ codes



$$\frac{D}{Dt} = \frac{\partial}{\partial t} + v_E \cdot \nabla,$$

$$v_E = \frac{1}{B_0} b_0 \times \nabla \phi - \frac{E_{r0}}{B_0},$$

$$E_{r0} = (1/N_i Z_i e) \nabla_r P_{i0}$$

$$\frac{\partial A_\parallel}{\partial t} = -\nabla_\parallel \left(\phi + \Phi_0\right) + \frac{\eta}{\mu_0} \nabla_\perp^2 A_\parallel - \frac{\eta_H}{\mu_0} \nabla_\perp^4 A_\parallel$$

$$\eta_H = \eta \frac{\mu_e}{\nu_{ei}}$$

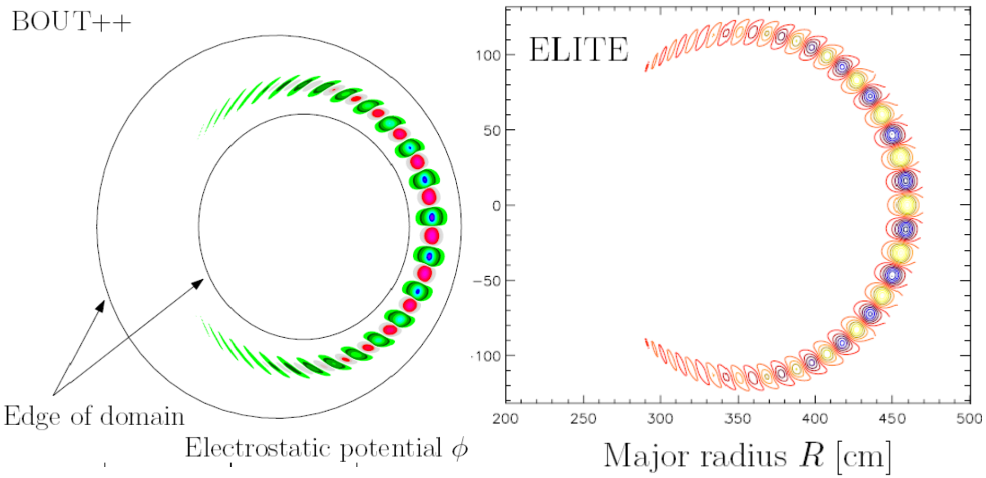
$$S = \mu_0 R v_A / \eta$$

$$S_H = \mu_0 R^3 v_A / \eta_H = S / \alpha_H$$

✓Resistivity & viscosity destabilizes the ideal modes

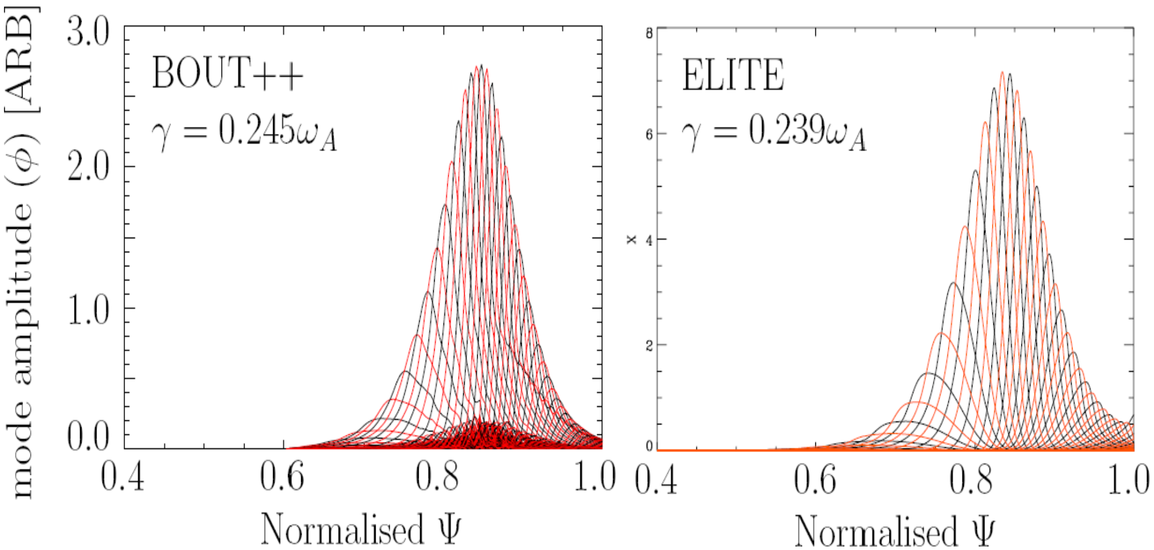
✓ion diamagnetic drift and ExB drift stabilize the ideal modes

Linear mode structures are similar in BOUT++ and ELITE



BOUT++ and ELITE show the same poloidal wavelength n=20

Radial Mode structures are similar in BOUT++ and ELITE



Lundquist number S plays a critical role on nonlinear ideal ballooning mode

Time step collapses at high Lundquist number S w/o η_H