Six-field two-fluid ELM simulations using BOUT++

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BOUT++
Boundary Plasma Turbulence Code

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Multi-field two-fluid model in BOUT++

Six-field two-fluid model is necessary to describe:

- pedestal energy loss
- density profile evolution through the ELM event,
- heat flux
- energy depositions on divertor target

Six-field (ϖ, n_i, T_i, T_e, $A_{||}$, $V_{||}$): based on Braginskii equations, the density, momentum and energy of ions and electrons are described in drift ordering[1,2].

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[1] X. Q. Xu et al., Commun. Comput. Phys. **4**, 949 (2008).

[2] T. Y. Xia et al., *Nucl. Fusion* **53**, 073009 (2013).

Simplified 6-field model in BOUT++

$$
\begin{aligned}
\frac{\partial}{\partial t}\varpi &= -\frac{1}{B_0}\boldsymbol{b}\times\boldsymbol{\nabla}_\perp\Phi\cdot\boldsymbol{\nabla}\varpi + B_0^2\nabla_\parallel\left(\frac{J_\parallel}{B_0}\right) + 2\boldsymbol{b}\times\boldsymbol{\kappa}\cdot\boldsymbol{\nabla}p_i \boxed{+\mu_{\parallel i}\nabla_{\parallel 0}^2\varpi,}\\
\frac{\partial}{\partial t}n_i &= -\frac{1}{B_0}\boldsymbol{b}\times\boldsymbol{\nabla}_\perp\Phi\cdot\nabla n_i \boxed{-n_iB_0\nabla_\parallel\left(\frac{V_{\parallel i}}{B_0}\right)},\\
&\boxed{\frac{\partial}{\partial t}V_{\parallel i} = -\frac{1}{B_0}\boldsymbol{b}\times\boldsymbol{\nabla}_\perp\Phi\cdot\boldsymbol{\nabla}V_{\parallel i} - \frac{1}{m_i n_{i0}}\boldsymbol{b}\cdot\boldsymbol{\nabla}P,}\\
\frac{\partial}{\partial t}A_\parallel &= -\nabla_\parallel\phi + \frac{\eta}{\mu_0}\nabla_\perp^2A_\parallel \boxed{-\frac{\eta_H}{\mu_0}\nabla_\perp^4A_\parallel}\\
\frac{\partial}{\partial t}T_i &= -\frac{1}{B_0}\boldsymbol{b}\times\boldsymbol{\nabla}_\perp\Phi\cdot\boldsymbol{\nabla}T_i \boxed{-\frac{2}{3}T_iB_0\nabla_\parallel\left(\frac{V_{\parallel i}}{B_0}\right)}\boxed{+\frac{2}{3n_{i0}k_B}\nabla_{\parallel 0}\left(\kappa_{\parallel i}\nabla_{\parallel 0}T_i\right)},\\
\frac{\partial}{\partial t}T_e &= -\frac{1}{B_0}\boldsymbol{b}\times\boldsymbol{\nabla}_\perp\Phi\cdot\boldsymbol{\nabla}T_e \boxed{-\frac{2}{3}T_eB_0\nabla_\parallel\left(\frac{V_{\parallel e}}{B_0}\right)}\boxed{+\frac{2}{3n_{e0}k_B}\nabla_{\parallel 0}\left(\kappa_{\parallel e}\nabla_{\parallel 0}T_e\right)}.
\end{aligned}
$$

Parallel velocity terms

Parallel viscosity

Hyper resistivity

Thermal conduction

Switch Name	Physics meanings
compress0	Parallel velocity
viscos_par	Parallel viscosity
spitzer_resist	Spitzer resistivity
hyperresist	Hyper resistivity
diffusion_par	Thermal conduction

6-field model in BOUT++ (cont.)

Definitions:

$$\varpi = n_{i0}\frac{m_i}{B_0}\left(\nabla_\perp^2\phi + \frac{1}{n_{i0}}\nabla_\perp\phi\cdot\nabla_\perp n_{i0} + \frac{1}{n_{i0}Z_i e}\nabla_\perp^2 p_{i1}\right),$$

$$J_\parallel = J_{\parallel 0} - \frac{1}{\mu_0}B_0\nabla_\perp^2\psi,$$

$$V_{\parallel e} = V_{\parallel i} + \frac{1}{\mu_0 Z_i e n_i}\nabla_\perp^2 A_\parallel.$$

$$\eta_{SP} = 0.51 \times 1.03 \times 10^{-4} Z_i \ln\Lambda T^{-3/2}\,\Omega\,\mathrm{m}^{-1}$$

Flux limited expression for parallel thermal conduction:

$$\kappa_{\parallel i} = 3.9 n_i v_{th,i}^2/\nu_i \qquad \kappa_{\parallel e} = 3.2 n_e v_{th,e}^2/\nu_e$$

$$\kappa_{fs,j} = n_j v_{th,j} q R_0$$

$$\kappa_{eff,j} = \frac{\kappa_{\parallel j}\kappa_{fs,j}}{\kappa_{\parallel j} + \kappa_{fs,j}}.$$

Boundary conditions and normalizations

Boundary conditions:

Inner boundary:

$$\partial n_i/\partial\Psi = 0,\ \partial T_j/\partial\Psi = 0, \varpi = 0,\ \nabla_\perp^2 A_\parallel = 0,\ \partial^2\phi/\partial^2\Psi = 0,\ \partial V_\parallel/\partial\Psi = 0$$

Outer boundary:

$$n_i = 0,\ T_j = 0,\ \varpi = 0,\ \nabla_\perp^2 A_\parallel = 0,\ \partial^2\phi/\partial^2\Psi = 0,\ V_\parallel = 0$$

Normalizations:

$$\hat{T}_j = \frac{T_j}{\bar{T}_j}, \qquad \hat{n} = \frac{n_i}{\bar{n}}, \qquad \hat{L} = \frac{L}{\bar{L}},$$

$$\hat{t} = \frac{t}{\bar{t}}, \qquad \hat{B} = \frac{B}{\bar{B}}, \qquad \hat{J} = \frac{\mu_0 \bar{L}}{B_0} J,$$

$$\hat{\psi} = \frac{\psi}{\bar{L}}, \qquad \hat{\phi} = \frac{\bar{t}}{\bar{L}^2 B_0}\phi, \qquad \hat{\varpi} = \frac{\bar{t}}{m_i \bar{n}}\varpi,$$

$$\tau = \frac{\bar{T}_i}{\bar{T}_e}, \qquad \hat{V} = \frac{V}{V_A}, \qquad V_A = \frac{\bar{L}}{\bar{t}} = \frac{\bar{B}}{\sqrt{\mu_0 m_i n_i}},$$

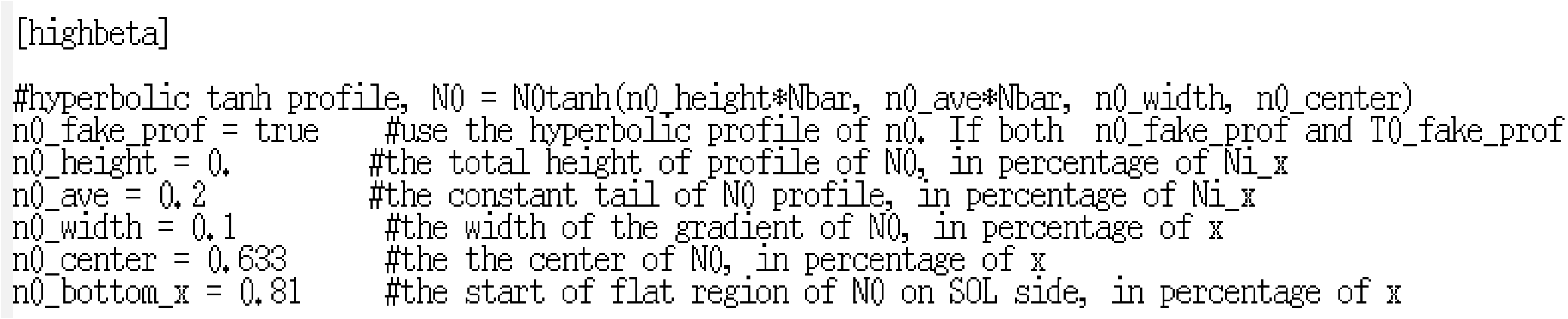
$$\hat{P}_j = \frac{P_j}{k_B \bar{n} \bar{T}_j} \qquad \hat{\kappa} = \bar{L}\kappa, \qquad \hat{\nabla} = \bar{L}\nabla$$

Density profile as the input

Density profile used in 6-field model:

$$n_{i0}(x) = \frac{(n_{\text{height}} \times n_{\text{ped}})}{2}\left[1 - \tanh\left(\frac{x - x_{\text{ped}}}{\Delta x_{\text{ped}}}\right)\right] + n_{\text{ave}} \times n_{\text{ped}},$$

The coefficients in BOUT.inp:



```
[highbeta]

#hyperbolic tanh profile, N0 = N0tanh(n0_height*Nbar, n0_ave*Nbar, n0_width, n0_center)
n0_fake_prof = true     #use the hyperbolic profile of n0. If both  n0_fake_prof and T0_fake_prof
n0_height = 0.         #the total height of profile of N0, in percentage of Ni_x
n0_ave = 0.2           #the constant tail of N0 profile, in percentage of Ni_x
n0_width = 0.1          #the width of the gradient of N0, in percentage of x
n0_center = 0.633       #the the center of N0, in percentage of x
n0_bottom_x = 0.81      #the start of flat region of N0 on SOL side, in percentage of x
```

Compiling and running of 6-field module

For the exercise, a simple linear test is prepared:

Compiling:

Set the environment first, then

```
> make
```

Go to the scratch directory to run the code:

```
> cd $SCRATCH
> cp –r $BOUT_TOP/examples/6field-simple/ .
> cp $BOUT_TOP/examples/6field-simple/
cbm18_dens8.grid_nx68ny64.nc .
> cd 6field-simple/
```

Edit the pbs file with:

```
#PBS –l advres=bout.10
```

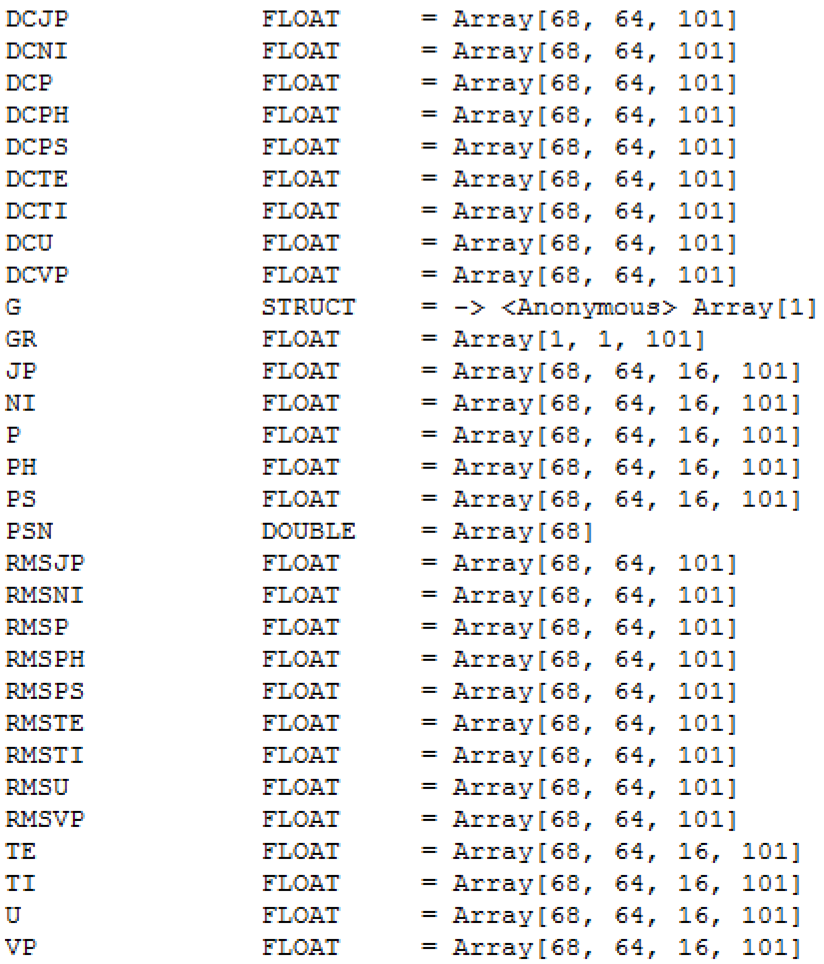
Submit job and run the job:

```
> qsub bout_hopper_debug.cmd
```

Data post-processing:

Add the idl library directory first

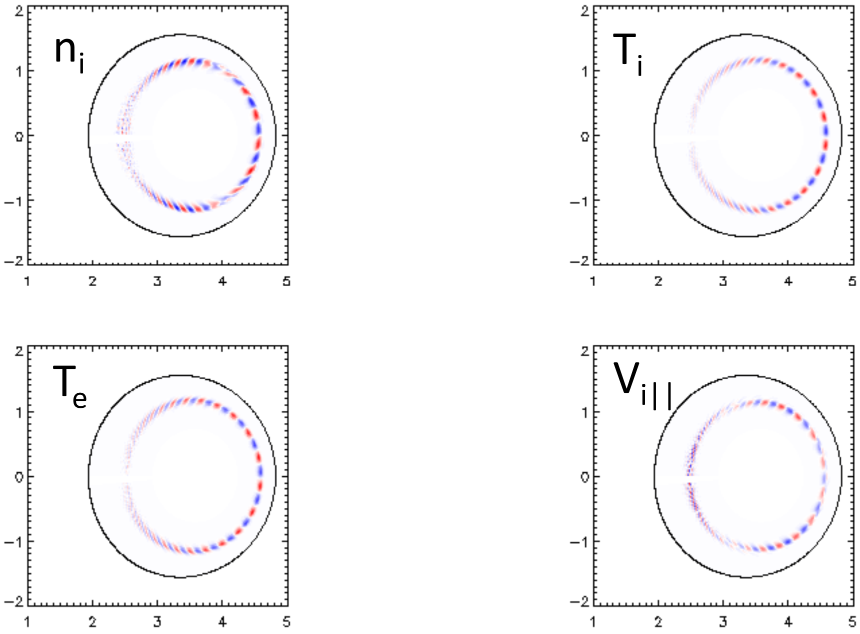
```
IDL> !path=!path+":$BOUT_TOP/tools/idllib”
IDL> @collect-all
```



```
DCJP            FLOAT     = Array[68, 64, 101]
DCNI            FLOAT     = Array[68, 64, 101]
DCP             FLOAT     = Array[68, 64, 101]
DCPH            FLOAT     = Array[68, 64, 101]
DCPS            FLOAT     = Array[68, 64, 101]
DCTE            FLOAT     = Array[68, 64, 101]
DCTI            FLOAT     = Array[68, 64, 101]
DCU             FLOAT     = Array[68, 64, 101]
DCVP            FLOAT     = Array[68, 64, 101]
G               STRUCT    = -> <Anonymous> Array[1]
GR              FLOAT     = Array[1, 1, 101]
JP              FLOAT     = Array[68, 64, 16, 101]
NI              FLOAT     = Array[68, 64, 16, 101]
P               FLOAT     = Array[68, 64, 16, 101]
PH              FLOAT     = Array[68, 64, 16, 101]
PS              FLOAT     = Array[68, 64, 16, 101]
PSN             DOUBLE    = Array[68]
RMSJP           FLOAT     = Array[68, 64, 101]
RMSNI           FLOAT     = Array[68, 64, 101]
RMSP            FLOAT     = Array[68, 64, 101]
RMSPH           FLOAT     = Array[68, 64, 101]
RMSPS           FLOAT     = Array[68, 64, 101]
RMSTE           FLOAT     = Array[68, 64, 101]
RMSTI           FLOAT     = Array[68, 64, 101]
RMSU            FLOAT     = Array[68, 64, 101]
RMSVP           FLOAT     = Array[68, 64, 101]
TE              FLOAT     = Array[68, 64, 16, 101]
TI              FLOAT     = Array[68, 64, 16, 101]
U               FLOAT     = Array[68, 64, 16, 101]
VP              FLOAT     = Array[68, 64, 16, 101]
```

Variables after the collecting

The output of the mode structure (1)

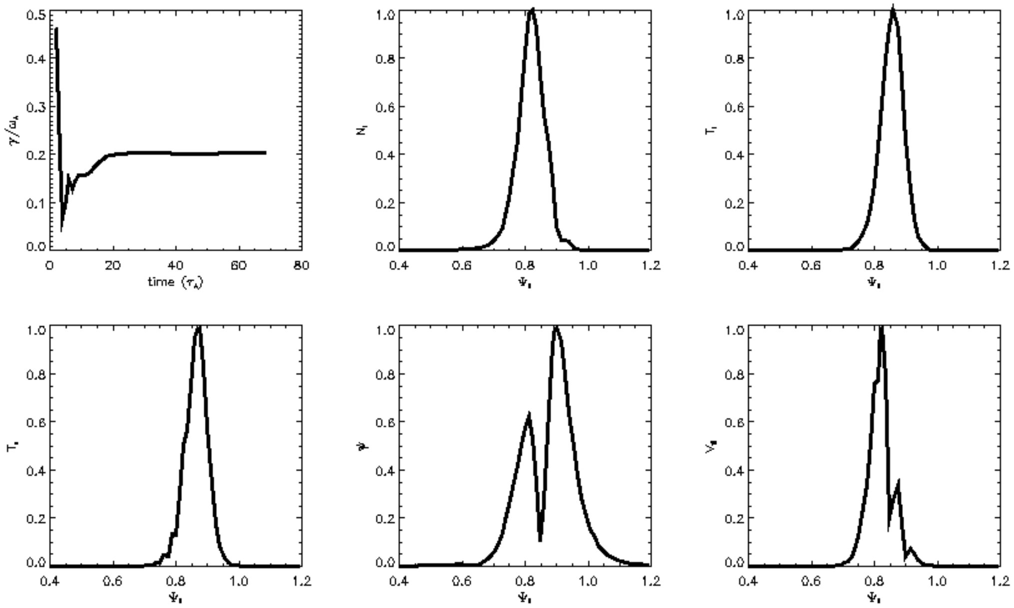


Poloidal mode structures

n0_height = 0.0
n0_ave = 0.2

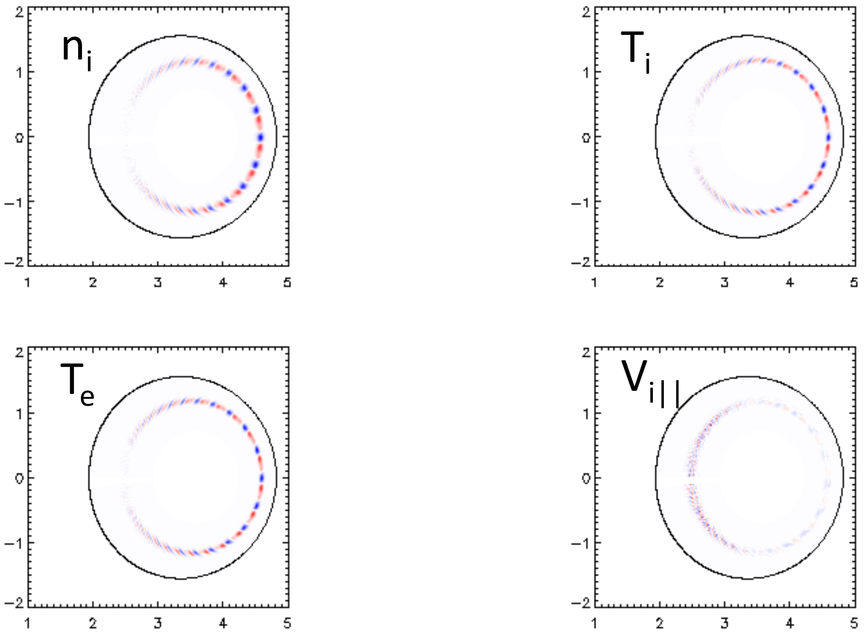
Linear growth rate for this test case:

```
IDL> print,gr[-1]
      0.202673
```



Linear growth rate and radial mode structures

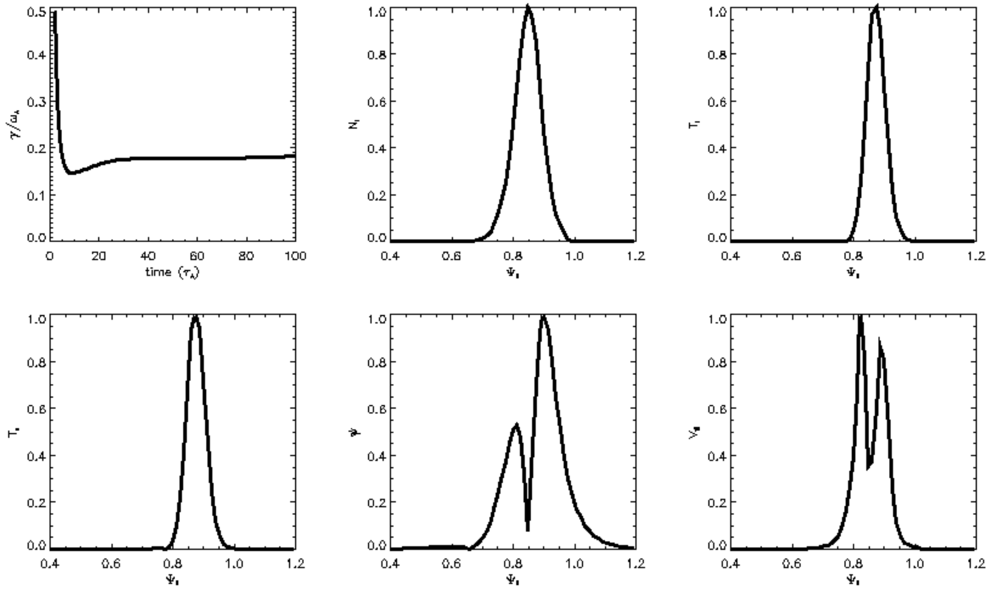
The output of the mode structure (2)



Poloidal mode structures

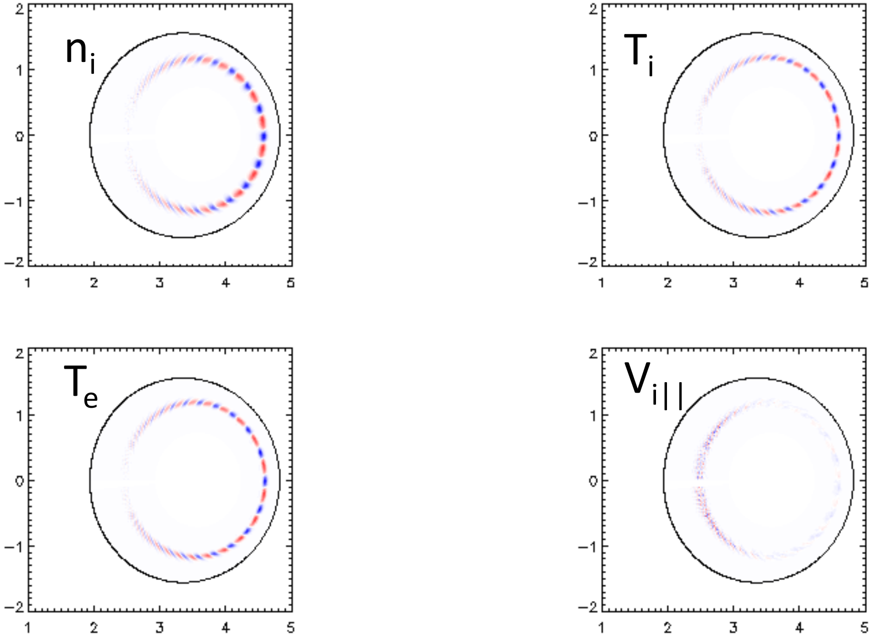
n0_height = 0.364
n0_ave = 0.2

Linear growth rate for this test case:
IDL> print,gr[-1]
      0.183418



Linear growth rate and radial mode structures

The output of the mode structure (3)

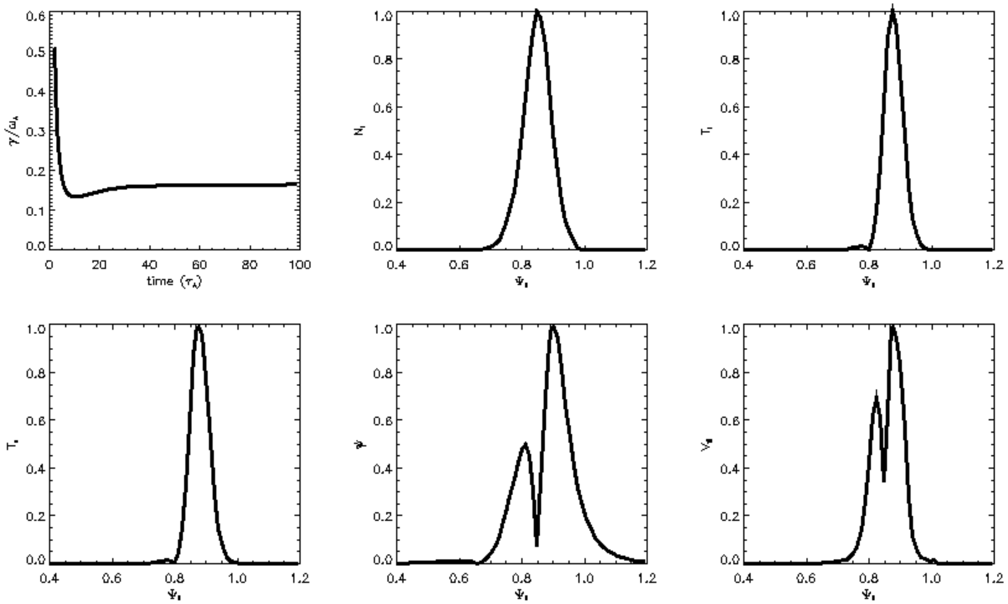


Poloidal mode structures

n0_height = 0.6
n0_ave = 0.2

Linear growth rate for this test case:

```
IDL> print,gr[-1]
       0.166440
```



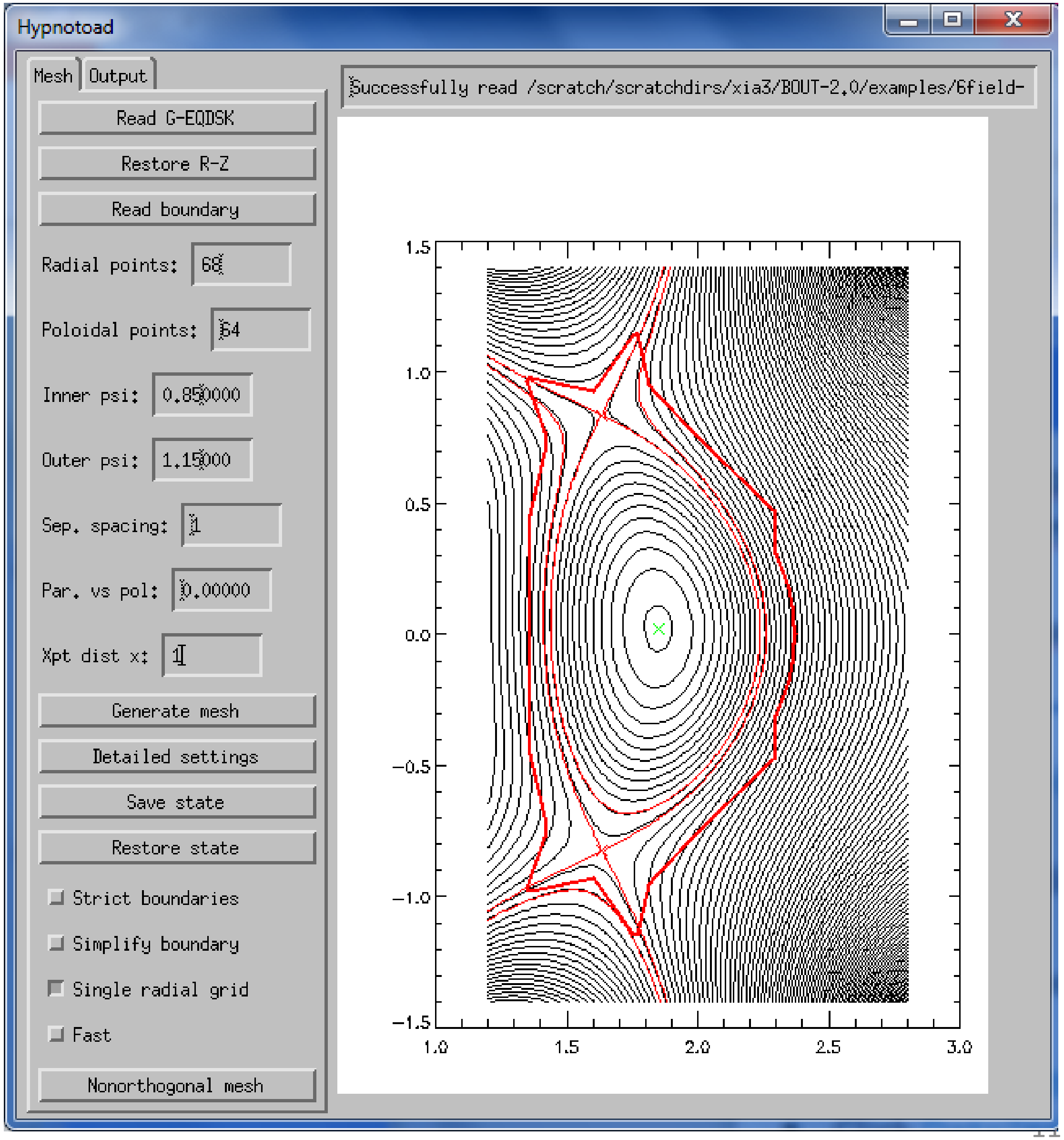
Linear growth rate and radial mode structures

Generate BOUT grid from g-file with hypnotoad

To start hypnotoad:

```
> cd tools/tokamak_grids/gridgen/
> idl
> IDL> hypnotoad
```

- Click Read G-EQDSK, choose a g-file
- Input the radial and poloidal points
- Input inner boundary and outer boudary
- Click Generate mesh
- Click Detailed settings and adjust the points for legs
- Click Generate mesh again to generate the modified grid
- Output, and name it, such as “EAST_033068.02900_x68y64_psi085to115.nc”



Successfully read /scratch/scratchdirs/xia3/BOUT-2.0/examples/6field-

Radial points: 68

Poloidal points: 64

Inner psi: 0.850000

Outer psi: 1.15000

Sep. spacing: 1

Par. vs pol: 0.00000

Xpt dist x: 1

☐ Strict boundaries

☐ Simplify boundary

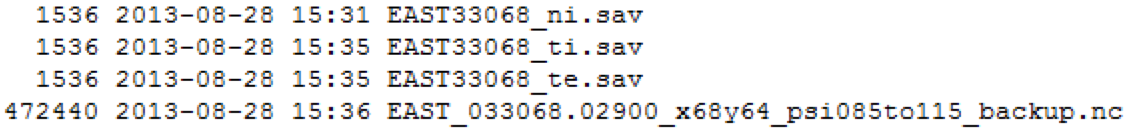
☑ Single radial grid

☐ Fast

Implement density and temperature profiles in to the grid file generated by hypnotoad

1. Backup the grid file.

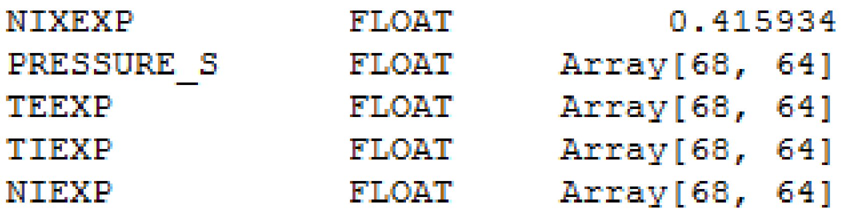
2. Get ready of the profiles of density, ion and electron temperatures which has already been interpolated with the radial coordinate in the grid file generated just now.



```
  1536 2013-08-28 15:31 EAST33068_ni.sav
  1536 2013-08-28 15:35 EAST33068_ti.sav
  1536 2013-08-28 15:35 EAST33068_te.sav
472440 2013-08-28 15:36 EAST_033068.02900_x68y64_psi085to115_backup.nc
```

3. Using idl routing Ni2Gridalls.pro to implement these profiles into the grid file:
   IDL> .compile Ni2Gridalls
   IDL> Ni2Gridalls, "EAST33068_ni.sav", "EAST33068_te.sav", "EAST33068_ti.sav", "EAST_033068.02900_x68y64_psi085to115.nc"

4. The new profiles in the grid file is renamed as:



Density at the inner boundary:	NIXEXP	FLOAT	0.415934
Pressure:	PRESSURE_S	FLOAT	Array[68, 64]
T_e:	TEEXP	FLOAT	Array[68, 64]
T_i:	TIEXP	FLOAT	Array[68, 64]
N_i:	NIEXP	FLOAT	Array[68, 64]

6-field model in BOUT++

$$
\begin{aligned}
\frac{\partial}{\partial t}\varpi = & -\frac{1}{B_0}\boldsymbol{b}\times\boldsymbol{\nabla}_\perp\Phi\cdot\boldsymbol{\nabla}\varpi + B_0^2\nabla_\parallel\left(\frac{J_\parallel}{B_0}\right) + 2\boldsymbol{b}\times\boldsymbol{\kappa}\cdot\boldsymbol{\nabla}p_i \\
& \boxed{-\frac{1}{2\Omega_i}\left[\frac{1}{B_0}\boldsymbol{b}\times\boldsymbol{\nabla}P_i\cdot\boldsymbol{\nabla}\left(\nabla_\perp^2\Phi\right) - Z_ieB_0\boldsymbol{b}\times\boldsymbol{\nabla}n_i\cdot\boldsymbol{\nabla}\left(\frac{\nabla_\perp\Phi}{B_0}\right)^2\right]} \\
& \boxed{+\frac{1}{2\Omega_i}\left[\frac{1}{B_0}\boldsymbol{b}\times\boldsymbol{\nabla}\Phi\cdot\boldsymbol{\nabla}\left(\nabla_\perp^2P_i\right) - \nabla_\perp^2\left(\frac{1}{B_0}\boldsymbol{b}\times\boldsymbol{\nabla}\Phi\cdot\boldsymbol{\nabla}P_i\right)\right]} + \mu_{\parallel i}\nabla_{\parallel 0}^2\varpi,
\end{aligned}
\tag{1}
$$

$$
\begin{aligned}
\frac{\partial}{\partial t}n_i = & -\frac{1}{B_0}\boldsymbol{b}\times\boldsymbol{\nabla}_\perp\Phi\cdot\nabla n_i \boxed{-\frac{2n_i}{B_0}\boldsymbol{b}\times\boldsymbol{\kappa}\cdot\boldsymbol{\nabla}\Phi} \\
& \boxed{-\frac{2}{Z_ieB_0}\boldsymbol{b}\times\boldsymbol{\kappa}\cdot\boldsymbol{\nabla}P_i}\boxed{-n_iB_0\nabla_\parallel\left(\frac{V_{\parallel i}}{B_0}\right)},
\end{aligned}
\tag{2}
$$

$$
\boxed{\frac{\partial}{\partial t}V_{\parallel i} = -\frac{1}{B_0}\boldsymbol{b}\times\boldsymbol{\nabla}_\perp\Phi\cdot\boldsymbol{\nabla}V_{\parallel i} - \frac{1}{m_in_{i0}}\boldsymbol{b}\cdot\boldsymbol{\nabla}P,}
\tag{3}
$$

$$
\frac{\partial}{\partial t}A_\parallel = -\nabla_\parallel\phi + \frac{\eta}{\mu_0}\nabla_\perp^2A_\parallel \boxed{+\frac{1}{en_{e0}B_0}\nabla_\parallel P_e}\boxed{+\frac{0.71k_B}{eB_0}\nabla_\parallel T_e} - \frac{\eta_H}{\mu_0}\nabla_\perp^4A_\parallel,
\tag{4}
$$

$$
\begin{aligned}
\frac{\partial}{\partial t}T_i = & -\frac{1}{B_0}\boldsymbol{b}\times\boldsymbol{\nabla}_\perp\Phi\cdot\boldsymbol{\nabla}T_i \\
& \boxed{-\frac{2}{3}T_i\biggl[\left(\frac{2}{B_0}\boldsymbol{b}\times\boldsymbol{\kappa}\right)\cdot\biggl(\boldsymbol{\nabla}\Phi + \frac{1}{Z_ien_{i0}}\boldsymbol{\nabla}P_i}\boxed{+\frac{5}{2}\frac{k_B}{Z_ie}\boldsymbol{\nabla}T_i}\biggr)\boxed{+B_0\nabla_\parallel\left(\frac{V_{\parallel i}}{B_0}\right)\biggr]} \\
& \boxed{+\frac{2}{3n_{i0}k_B}\nabla_{\parallel 0}\left(\kappa_{\parallel i}\nabla_{\parallel 0}T_i\right) + \frac{2}{3n_{i0}k_B}\boldsymbol{\nabla}_\perp\cdot\left(\kappa_{\perp i}\boldsymbol{\nabla}_\perp T_i\right)}\boxed{+\frac{2m_e}{m_i}\frac{Z_i}{\tau_e}\left(T_e-T_i\right)},
\end{aligned}
\tag{5}
$$

$$
\begin{aligned}
\frac{\partial}{\partial t}T_e = & -\frac{1}{B_0}\boldsymbol{b}\times\boldsymbol{\nabla}_\perp\Phi\cdot\boldsymbol{\nabla}T_e \\
& \boxed{-\frac{2}{3}T_e\biggl[\left(\frac{2}{B_0}\boldsymbol{b}\times\boldsymbol{\kappa}\right)\cdot\biggl(\boldsymbol{\nabla}\Phi - \frac{1}{en_{e0}}\boldsymbol{\nabla}P_e}\boxed{-\frac{5}{2}\frac{k_B}{e}\boldsymbol{\nabla}T_e}\biggr)\boxed{+B_0\nabla_\parallel\left(\frac{V_{\parallel e}}{B_0}\right)\biggr]} \\
& \boxed{+0.71\frac{2T_e}{3en_{e0}}B_0\nabla_\parallel\left(\frac{J_\parallel}{B_0}\right)}\boxed{+\frac{2}{3n_{e0}k_B}\nabla_{\parallel 0}\left(\kappa_{\parallel e}\nabla_{\parallel 0}T_e\right) + \frac{2}{3n_{e0}k_B}\boldsymbol{\nabla}_\perp\cdot\left(\kappa_{\perp e}\boldsymbol{\nabla}_\perp T_e\right)} \\
& \boxed{-\frac{2m_e}{m_i}\frac{1}{\tau_e}\left(T_e-T_i\right) + \frac{2}{3n_{e0}k_B}\eta_\parallel J_\parallel^2}
\end{aligned}
\tag{6}
$$

Compressible terms

Parallel velocity terms

Electron Hall

Thermal force

Gyro-viscosity

Energy exchange

Energy flux

Thermal conduction

The switches of the terms in 6-field model

Switch Name	Physics meanings
compress0	Parallel velocity
continuity	Compressible terms
eHall	Electron Hall effect term
energy_flux	Energy flux
energy_exch	Energy exchange between electrons and ions
thermal_force	Thermal force
gyroviscous	Gyro-viscosity
spitzer_resist	Spitzer resistivity
diffusion_par	Parallel thermal conduction
diffusion_perp	Perpendicular thermal conduction
gamma_i_BC, gamma_e_BC	Sheath boundary condition

Sheath Boundary conditions

Sheath boundary condition:

$$
\begin{aligned}
V_j &= c_{se} = \sqrt{\frac{k_B(T_i + T_j)}{M_j}} \\
J_{\|} &= n_i e \left[c_{se} - \frac{v_{Te}}{2\sqrt{\pi}} \exp\left(-\frac{e\phi}{k_B T_e} \right) \right] \\
q_{se} &= -\kappa_{\|e} \partial_{\|} T_e = \gamma_e n_e T_e c_{se} \\
q_{si} &= -\kappa_{\|i} \partial_{\|} T_i = \gamma_i n_i T_i c_{se} \\
\partial_{\|} \varpi &= 0 \\
\partial_{\|} n_i &= 0
\end{aligned}
$$