Gyrokinetic Simulations of Kinetic-MHD Processes in Fusion Plasmas

Z. Lin

University of California, Irvine
Fusion Simulation Center, Peking University


PPPL: W. Deng, G. Dong, S. Ethier

ORNL: S. Klasky; Nvidia: P. Wang

PKU: H. S. Zhang, P. Jiang, J. Bao, Y. Q. Liu

Zhejiang U.: Y. Xiao and students

IoP: W. L. Zhang and students

Xiamen U.: G. Sun; Sichuan U.: D. Liu

Tianhe-1a & Tianhe-2: X. Meng, Y. Zhao

Outlines

• GTC development strategy
  – Comprehensive simulation platform
  – Collaborative code development
  – Code capability driven by application requirement

• Capability for simulation of microturbulence
• Capability for simulation of kinetic-MHD
• Capability for simulation of RF heating & current drive

[Y. Xiao]
Gyrokinetic Simulation of Plasma Turbulence

- Nonlinear gyrokinetic equation (GKE) in 5D phase space \((R, v_{||}, \mu)\) for low frequency driftwave turbulence \([Lee, PoF1983]\)
  \[
  \frac{\partial F}{\partial t} + (v_{||} b + v_d) \cdot \frac{\partial F}{\partial R} + \dot{v}_{||} \frac{\partial F}{\partial v_{||}} = 0
  \]

- Poisson equation in long wavelength limit
  \[
  \frac{\rho_s^2}{\lambda_D^2} \nabla^2 \phi = 4\pi e (n_e - Zn_i)
  \]

- Ampere’s Law
  \[
  \nabla^2 A_{||} = \frac{4\pi e n_0}{c} (u_e - Zu_i)
  \]

- Particles (kinetic): electrons, thermal ions, fast ions etc

- Fields (fluid): density, temperature, electromagnetic fields etc
Magnetic Coordinates for Toroidal Geometry

- Magnetic coordinate \((\psi, \theta, \zeta)\)

- Flux surface: \(\mathbf{B} \cdot \nabla \psi = 0\)

- Straight field line: \(\frac{\mathbf{B} \cdot \nabla \zeta}{\mathbf{B} \cdot \nabla \theta} = q\)

  - Efficient for integrating particle orbits & discretizing field-aligned mode

- Boozer coordinates [Boozer, PoF1981]: \(J = (gq + l)/B^2 \sim \chi^2\)
Guiding Center Equation of Motion

• Gyrocenter Hamiltonian [White & Chance, PF1984]
  \[ H = \frac{1}{2} \rho^2 B^2 + \mu B + \phi \]

• Canonical variables in Boozer coordinates
  \[ P_\theta = I \rho + \psi \]
  \[ P_\zeta = \mu \rho - \psi \]

• Equation of motion
  \[ \frac{dP_\theta}{dt} = -\frac{\partial H}{\partial \theta}, \quad \frac{d\theta}{dt} = \frac{\partial H}{\partial P_\theta} \]
  \[ \frac{dP_\zeta}{dt} = -\frac{\partial H}{\partial \zeta}, \quad \frac{d\zeta}{dt} = \frac{\partial H}{\partial P_\zeta} \]

• Only scalar quantities needed, conserve phase space volume
Collisions: Monte-Carlo Method

- Electron-ion pitch angle $\xi = \nu_|| / \nu$ scattering in ion frame: Lorentz operator
  \[ C_{ei} (\delta f_e) = \nu_0 \frac{1}{2} \frac{\partial}{\partial \xi} (1 - \xi^2) \frac{\partial}{\partial \xi} \delta f_e \]
  \[ \xi = \xi_0 (1 - \nu \Delta t) + (r - 0.5) [12(1 - \xi_0^2) \nu \Delta t]^{1/2} \]

- Linear like-species guiding center collision operator [Xu & Rosenbluth, PFB1991]
  \[
  C(\delta f) = C(\delta f, F_0) + P(F_0, \delta f) = P(F_0, \delta f) + \frac{\partial}{\partial \nu_\parallel} (\nu_\parallel \delta f) + \frac{\partial}{\partial \nu_\perp} (\nu_\perp \delta f) \\
  + \frac{\partial^2}{\partial \nu_\parallel^2} (\nu_\parallel \delta f) + \frac{1}{2} \frac{\partial^2}{\partial \nu_\parallel^2} (\nu_\parallel \delta f) + \frac{1}{2} \frac{\partial^2}{\partial \nu_\perp^2} (\nu_\perp \delta f)
  \]

- Conserve momentum and energy [Dimits & Cohen, PRE1994], preserve Shifted Maxwellian [Lin et al, PoP1995]
  \[
  \Delta w = -3 \sqrt{\frac{\pi}{2}} \phi(x) \left( \frac{v_{th}}{\nu} \right)^3 v_\parallel \delta P - 3 \sqrt{\frac{\pi}{2}} \left[ \phi(x) - \frac{d \phi(x)}{dx} \right] \frac{v_{th}}{\nu} \delta E
  \]

Toroidal Perturbative Method

- Perturbative method: discrete particle noise reduced by \((\delta f/f)^2\)
  \[\text{[Dimits & Lee, PF1993; Parker & Lee, PF1993; Hu & Krommes, PoP1994]}\]

- ES GK equation: \(Lf(R,v_{||},\mu)=0\)
  \[
  L = \frac{\partial}{\partial t} + (v_{||}b + v_d + v_{E\times B}) \cdot \frac{\partial}{\partial R} - \mathbf{b} \cdot \nabla (\mu B + \Phi) \frac{\partial}{\partial v_{||}}
  \]

- Define \(f=f_0 + \delta f, L=L_0 + \delta L\), \(L_0 f_0=0\), then \(L \delta f = -\delta L f_0\)
  \[
  L_0 = \frac{\partial}{\partial t} + (v_{||}b + v_d) \cdot \frac{\partial}{\partial R} - \mathbf{b} \cdot \nabla (\mu B) \frac{\partial}{\partial v_{||}}
  \]

- \(F_0\): arbitrary function of constants of motion in collisionless limit.

  Canonical Maxwellian \([Idomura, PoP2003]\)

- Neoclassical \(\delta f\) simulation \([Lin et al, PoP1995]\)
  \(f_0 = f_M + f_{02}, L_0 = L_{01} + L_{02}, L_0 f_M = 0, L_0 f_{02} = -L_{02} f_M\)
  \[
  L_{01} = \frac{\partial}{\partial t} + v_{||} b \cdot \frac{\partial}{\partial R} - \mathbf{b} \cdot \nabla (\mu B) \frac{\partial}{\partial v_{||}} - C
  \]
Poisson Solver

- Gyrokinetic Poisson equation [Lee, JCP1987]

- Polarization density
  \[ \tilde{\phi}(x) = \langle \tilde{\phi}(R) F_M \rangle \]
  \[ \tilde{\phi}_k = \Gamma_0 \phi_k \]

- Solve in \( k \)-space: Pade approximation
  \[ \Gamma_0 \approx 1/[1 + (k_{\perp} \rho_i)^2] \]

- Solve in real space [Lin & Lee, PRE1995]

- Need to invert extremely large matrix
  \[ \sum_{m,n} c_{mnij} \phi_{mn} = (\delta n_i - \delta n_e)_{ij} \]

- Iterative method: good for electrostatic

- Electromagnetic: PETSc
Numerical Methods

• 2D spline function for equilibrium data from EFIT etc [Xiao, BOUT++ workshop]

• Gyroaveraging: performed on poloidal plane ($\zeta=constant$)
  › Assuming $k_\parallel << k_\perp$
  › Gyro-orbit elliptic
  › Linearized

• No fluctuation at radial boundary

• Numerical filter

\[ \bar{\phi}(R) = \phi(x + \rho) \]
\[ \rho = \frac{\nu}{\nu_{th}} \rho_{th} \]

DIII-D shot # 131997
Application: Turbulence Self-regulation by Zonal Flows

- GTC global gyrokinetic simulation finds self-regulation of turbulence by zonal flow (ZF) [Lin et al, Science 1998]
- GTC finds that ZF collisional damping induces oscillation of ZF and turbulence intensity with 90° phase shift [Lin et al, PRL 1999]
- Limit cycle oscillation (LCO) observed in TFTR RS to ERS transition [Mazzucato et al, PRL 1996]
- LCO with a phase shift observed in HL-2A L-I-H transition [Cheng et al, PRL 2013]
Global Field-aligned Mesh in GTC

- Discretization in \((\psi, \alpha, \zeta)\), rectangular mesh in \((\alpha, \zeta)\), \(\alpha = \theta - \zeta/q\)
  - Number of computation \(\sim (a/\rho)^2\), reduce computation by \(n\)
  - No approximation in geometry, loss of ignorable coordinate
  - Twisted in toroidal direction: periodicity preserved
- Decomposition in toroidal mode? \(\sim (a/\rho)^3\)
Multi-level Parallelization

- Simulation solves gyrokinetic-Poisson equations in 5D phase space: field data on 3D fixed grids; Particles moving in 5D.

- I. 1D MPI domain-decomposition for particle-field interactions

- II. Particle-decomposition with series/parallel field solvers

- III. Loop level parallelism using OpenMP

- Vectorization: work-vector for scatter operation

- Optimization for GPU & MIC
Application: Transition from Bohm to Gyro-Bohm

- Gradual transition from Bohm to gyro-Bohm \([Lin \ et \ al, \ PRL2002]\)
- Transport driven by local intensity
- Intensity driven nonlocally
- Turbulence spreading introduces nonlocality: breaking of Gyro-Bohm
- Time scales: key to understanding saturation vs. transport
Kinetic & Fluid Time Scales in ETG Turbulence

- Transport driven by local intensity: diffusive?
- Wave-particle decorrelation of parallel resonance $\delta(\omega-k_\parallel v_\parallel)$ dominates
- Quasilinear calculation of $\chi_e$ agrees well with simulation
- Saturation: wave-wave coupling determines fluctuation intensity
- Transport: wave-particle decorrelation determines transport level
- Streamers do not determine directly transport level

$\tau_{wp}$ \hline $\tau_\parallel$ \hline $\tau_\perp$ \hline $\tau_{rb}$ \hline $\tau_{eddy}$ \hline $\tau_{auto}$ \hline $\tau_s$ \hline $1/\gamma$
\hline 4.2 \hline 5.3 \hline 8.0 \hline 437 \hline 42 \hline 346 \hline $\infty$ \hline 33

[Lin et al, PRL 2007]
Time Scales in CTEM Turbulence

- CTEM instability drive is kinetic: toroidal precessional resonance of trapped electrons
- CTEM electron transport is from fluid eddy mixing, which regulated by zonal flows
- Leading to non-diffusive component of electron transport
- Ion transport is diffusive due to parallel decorrelation: ion transport follows local intensity of turbulence

[Xiao and Lin, PRL 2009]

|                  | $\tau_{wp}$ | $\tau_{||}$ | $\tau_{\perp}$ | $\tau_{rb}$ | $\tau_{edd}$ | $\tau_{au}$ | $\tau_{s}$ | $\frac{1}{\gamma_{max}}$ |
|------------------|-------------|-------------|-----------------|-------------|--------------|-------------|------------|---------------------|
| $CTEM_e$         | 0.61        | $\infty$   | $\infty$         | 5.1         | 1.6          | 11          | 0.66       | 4.0                 |
| $ITG_i$          | 1.7         | 1.8         | 2.0             | 21          | 4.9          | 7.2         | 1.4        | 9.1                 |
Energetic Particle Turbulent Transport: Energy Scaling

- GTC simulations find ITG diffusivity decreases drastically for high energy particles due to gyro- and orbit-averaging and wave-particle decorrelation
- GTC results are used to successfully to explain the transport of NBI fast ion transport in DIII-D tokamak

\[
D/D_1 = (E/T)^{-2}
\]

GTC diffusivity \( D \) as a function of particle energy for passing & trapped particles.


Measured neutron rate, beam-driven current, FIDA radiance from fast ions divided by (a) classical prediction & (b) simulation-based prediction vs. beam power in DIII-D

[W. Heidbrink et al, PRL2009]
Outlines

- GTC development strategy
- Capability for simulation of microturbulence
- Capability for simulation of kinetic-MHD
- Capability for simulation of RF heating & current drive

[Y. Xiao]
Multi-scale Gyrokinetic Simulation of Kinetic-MHD

• Macroscopic MHD modes limit burning plasma performance and threaten fusion device integrity

• Kinetic effects at microscopic scales and coupling of multiple processes play a crucial role in excitation and evolution MHD modes

• Neoclassical tearing modes (NTM): set principal performance limit in both ITER baseline and hybrid scenarios
  • Predictive NTM simulation needs to incorporate kinetic physics at multiple spatial and temporal scales: microturbulence, neoclassical bootstrap current, magnetic island dynamics

• Fully self-consistent simulation of Alfven eigenmode excited by energetic particle (EP) must incorporate three new physics elements
  • Linear and nonlinear kinetic effects of thermal particles
  • Nonlinear interaction of meso-scale shear Alfven waves with micro-scale kinetic effects and wave-particle resonance
  • Cross-scale couplings of meso-micro turbulence

http://phoenix.ps.uci.edu/GSEP
Kinetic-MHD via Gyrokinetic Particle Simulation

• Nonlinear gyrokinetic equation, Poisson equation, Ampere’s law

\[
\frac{\partial F}{\partial t} + (v_b + v_d) \cdot \frac{\partial F}{\partial R} + \dot{v}_i \frac{\partial F}{\partial v_i} = 0
\]

\[
\phi - \tilde{\phi} = -\sum_s 4\pi e Z_s \bar{n}_s
\]

\[
\nabla^2 A_i = -\sum_s \frac{4\pi e}{c} Z_s \bar{n}_s u_s
\]

• In fluid limit, gyrokinetic system recover MHD modes including Alfven wave, interchange mode, kink mode, KBM

\[
\frac{\omega (\omega - \omega_s P)}{v_A^2} \nabla^2 \delta \phi - iB_0 \cdot \nabla \left\{ \frac{\mathbf{b}_0 \cdot \nabla \times \left[ \nabla \times (k_{||} \delta \phi \mathbf{b}_0) \right]}{B_0} \right\}
\]

\[
-i\frac{\omega}{c} \delta \mathbf{B} \cdot \nabla \left( \frac{\mathbf{b}_0 \cdot \nabla \times \mathbf{B}_0}{B_0} \right)
\]

\[
-i\frac{4\pi}{c} \left[ \nabla \times \mathbf{b}_0 \cdot \nabla \left( \frac{\delta P_{||}}{B_0} \right) + \mathbf{b}_0 \times \nabla B_0 \cdot \nabla \left( \frac{\delta P_{\perp}}{B_0^2} \right) + \frac{\nabla \times \mathbf{b}_0 \cdot \nabla B_0}{B_0^2} \delta P_{\perp} \right]
\]

\[
=0
\]

Electron Models

- For low frequency mode $\omega/k_||<<v_||$, electron response mostly adiabatic
  \[
  \delta f_e = \frac{e \delta \phi}{T_e} f_M (1 + \frac{\omega}{k_||v_|| - \omega})
  \]
- Dynamically evolve non-adiabatic part
  \[
  \delta f^c_e = f_M e^{e\delta \phi/T_e} + \delta g
  \]
- Perturbed potential $\phi = \delta \phi + \Phi(k_||=0)$
  \[
  L \delta g = e^{e\phi/T_e} f_M [\kappa \cdot v_\phi + e \frac{\partial \delta \phi}{\partial t} - e \frac{1}{T_e} (v_\delta \phi + v_d) \cdot \nabla \Phi]
  \]
- Split-weigh scheme [Mamuilskiy & Lee, PoP2000; J. Lewandowski; Y. Chen]

**Fluid-kinetic hybrid model [Lin & Chen, PoP2001]**
- Lowest order: fluid, adiabatic response & non-resonance current
- Higher order: kinetic, resonant contribution
  - Requires $\omega_t>\omega$; neglects $E_\parallel(k_||=0)$ inductive electric field
- Resistive tearing mode demonstrated; collisionless tearing mode pursued
Gyrokinetic Toroidal Code (GTC)


http://phoenix.ps.uci.edu/GTC/
Gyrokinetic Toroidal Code (GTC)

• **GTC physics goal**: first-principles, integrated simulations of microturbulence, energetic particle (EP), magnetohydrodynamic (MHD), neoclassical, radio-frequency (RF) heating/current drive

• Current capability for kinetic MHD simulation: microturbulence + EP + MHD + neoclassical
  ► General geometry & experimental profiles
  ► Kinetic electrons & electromagnetic fluctuations
  ► Gyrokinetic or fully kinetic ions
  ► Equilibrium current
  ► Neoclassical effects

• Next step: collisionless tearing mode

• Ported to GPU (titan) & MIC (tianhe-2)

• Developed by GTC team in US & China in support of ITER

[Graph showing weak scaling of GTC on different architectures]

[Website link: http://phoenix.ps.uci.edu/GTC]
Verification of finite-\(\beta\) Simulation

- Finite-\(\beta\) stabilization of ITG and excitation of KBM

[Holod & Lin, PoP2013]
Linear Simulations of Alfvén Eigenmodes Validated

- GTC, GYRO, and TAEFL simulations of RSAE frequency up-sweeping, and mode structures in good agreement with DIII-D experiments (shot # 142111)

- Verification of GTC simulation of other modes:
  - BAE [H. Zhang et al, PoP2010]
  - TAE [W. Zhang et al, PoP2012]
  - EPM [C. Zhang et al, PoP2013]
  - KBM [Holod & Lin, PoP2013]

GTC simulation of RSAE to TAE transition with real geometry & kinetic electrons

[Deng et al, NF52, 043006 (2012); PoP2010]

RSAE in DIII-D shot # 142111 at 725ms

[D. Spong et al, PoP2012]
GTC Simulation Finds TAE Radial Localization in DIII-D

- TAE radial structure peaks at and moves with EP density gradient
- EP non-perturbative contribution to MHD mode

Z. X. Wang et al, PRL2013

TAE in DIII-D shot
# 142111 at 525ms
Comparison of TAE Mode Structures between Simulation & Experiment

- EP non-perturbative contribution breaks radial symmetry

TAE in DIII-D shot # 142111 at 525ms
GTC Nonlinear Simulations of BAE Find Fast Chirping

- Fast, repetitive, mostly downward chirping, sub-millisecond period
- 90° phase shift between intensity oscillation and frequency chirping
- Simulation features observed in recent NSTX TAE, ASDEX BAE
- Chirping induced by nonlinear dynamics of phase space coherent structures


[M. Podesta et al, NF2011; NF2012]
Backup Slides
# Relevant Time Scales in ETG Turbulence

<table>
<thead>
<tr>
<th>Time Scale</th>
<th>Expression</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\tau_{wp} = \frac{4 \chi_e}{3 \delta v_r^2}$</td>
<td>Effective wave-particle decorrelation time</td>
<td></td>
</tr>
<tr>
<td>$\tau_\parallel = \frac{1}{\Delta k_\parallel v_e}$</td>
<td>Parallel decorrelation time (Electron streaming across wave fields; independence of amplitude)</td>
<td></td>
</tr>
<tr>
<td>$\tau_\perp = \frac{3}{4 s^2 \theta^2 \bar{k}_\theta^2 \chi_e}$</td>
<td>Radial turbulence scattering time (Diffusion across radial width of $m$-harmonics)</td>
<td></td>
</tr>
<tr>
<td>$\tau_{rb} = \frac{4 L_r^2}{3 \chi_e}$</td>
<td>Resonant broadening time (Diffusion across radial streamer length)</td>
<td></td>
</tr>
<tr>
<td>$\tau_{eddy} = \frac{L_r}{\delta v_r}$</td>
<td>Eddy turnover time (Streamer rotation)</td>
<td></td>
</tr>
<tr>
<td>$\tau_{auto}$</td>
<td>Auto-correlation time (eddy life time)</td>
<td></td>
</tr>
<tr>
<td>$\tau_s = \left[ \frac{L_r}{L_\theta} \frac{\partial}{\partial r} \left( \frac{qV_{EXB}}{r} \right) \right]^{-1}$</td>
<td>Zonal flow shearing time</td>
<td></td>
</tr>
<tr>
<td>$1/\gamma$</td>
<td>Linear growth time</td>
<td></td>
</tr>
</tbody>
</table>
Microturbulence: Nonlinear Verification

- GTC results of device size scaling of ITG transport in 2002 verified after 8 years by global particle code ORB5 and continuum code GENE
  - [Lin, Hahm, Ethier, Tang, PRL2002]: GTC device size scaling: Bohm to gyroBohm transition
  - 2004: GYRO agrees with GTC qualitatively: Bohm to gyroBohm transition
  - [Lin & Hahm, PoP2004]: quantitative difference due to s-\(\alpha\) model in continuum codes
  - 2010: [McMillian, PRL2010] ORB5 & GENE agree with GTC qualitatively & quantitatively: Bohm to gyroBohm transition; quantitative difference due to s-\(\alpha\) model in continuum codes


Toward First-Principles Simulation of NTM

- GTC simulation of kink mode in DIII-D finds instability driven by both equilibrium current and pressure gradient.
- GTC simulation finds depression of bootstrap current and suppression of ITG due to flattening of pressure profile in the island center.

GTC simulation of kink in DIII-D #150363 [J. McClanehan, 2013]
GTC simulation of island effects on ITG [P. Jiang, 2013]
GTC simulation of island effects on bootstrap current [G. Dong, 2013]
GTC Simulation of RF Waves in Tokamak

- GTC recovers IBW dispersion relation
- GTC simulation of LHW propagation in cylindrical geometry agrees well with theoretical results
- Poloidal asymmetry in LHW propagation in tokamak

GTC simulation of IBW
[A. Kuley et al, 2013, PoP2013]