Flux-driven turbulence simulation of L-H transition with BOUT++ code

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• Motivation for L-H transition simulation by BOUT++

• L-H transition model by flux-driven RMHD model

• Preliminary results by 2-field L-H transition model

• Summary and Future Works
L-H transition simulation activity in JAEA [M. Yagi CPP2012]

- L-H transition by integrated transport simulation framework
  - TOPICS (1.5D core transport)
  - SONIC (2D SOL transport)
  - CDBM turbulent model
- Spontaneous turbulence quench and pedestal formation
- CDBM turbulent model has free parameters controlling
  - L-H transition power threshold
  - HH-factor

For more self-consistent simulation of L-H transition, CDBM turbulent transport model should be improved
CDBM turbulent transport model is based on
- Current Diffusive Ballooning Mode (CDBM) turbulence
- Turbulence suppression by mean flow shear

CDBM turbulent transport model can be improved by introducing missing physics
- Turbulence suppression by zonal flow shear
- SOL physics (geometry, impurities, neutrals)

BOUT++ framework can handle
- CDBM turbulence (resistive ballooning mode + hyper-resistivity)
- Mean flow shear
- Zonal flow shear
- Complex geometry including open filed (SND,DND,...)

Our goal is to reproduce L/H transition with CDBM turbulence by BOUT++ to get physical insights for improving CDBM turbulence model
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L-H transition simulation by flux-driven 2-field RMHD model


\[ \partial_t \nabla_\perp^2 \phi + \{ \phi, \nabla_\perp^2 \phi \} = -\nabla_\parallel^2 \phi - Gp + \partial_x F_{neo} + \nu_\perp \nabla_\perp^4 \phi \]
\[ \partial_t p + \{ \phi, p \} = \delta_c G \phi + \chi_\parallel \nabla_\parallel^2 p + \chi_\perp \nabla_\perp^2 p + S \]
\[ F_{neo} = -\mu_{neo}(\bar{p})[\partial_x \bar{\phi} - K_{neo}(\bar{p})\partial_x \bar{p}], \]

- **RBM turbulence** described by 2-field RMHD model
- **Strongly sheared** $E_r$ driven by radial force balance with poloidal flow introduced as **poloidal damping** in vorticity equation
- **Dynamic pressure profile** determined by balance among turbulence transport, heat source and sink

If inflow heat flux $Q_0$ exceeds a threshold, strongly sheared $E_r$ is generated and then turbulence transport level is suppressed in the edge region.

=> **Pedestal structure is formed in accordance with a new dynamic heat balance**
Poloidal damping term is derived from parallel viscous force

- Heuristic closure of parallel viscous force [T.A. Gianakon PoP2002]
  \[ \nabla \cdot \vec{\Pi}_{||i} = m_{i} n_{i} \frac{\langle B^2 \rangle}{B_\theta^2} \left( \mu_{i1} \vec{V}_{i\theta} + \mu_{i2} \frac{2q_{i\theta}}{5p_{i}} \right) e_\theta, \quad e_\theta = \sqrt{g} \nabla \zeta \times \nabla \psi, \quad \mathbf{B} = \nabla \zeta \times \nabla \psi + B_\zeta \nabla \zeta \]

- Mean poloidal rotation by radial force balance \( n_i = \text{const.}, \vec{V}_{i\perp} \gg \vec{V}_{i||} \)
  \[ \nabla \vec{P}_i \cdot \nabla \psi = e_i n_i (\vec{E} + \vec{V}_i \times \mathbf{B}) \cdot \nabla \psi \rightarrow \vec{V}_{i\theta} = \frac{\sqrt{g} |\nabla \psi|^2}{B_\zeta} \left[ \frac{d}{d\psi} \left( \frac{\vec{P}_i}{e_i n_i} \right) + \vec{V}_i \zeta \right] \]

- Mean poloidal heat flow by parallel neoclassical force balance
  \[ \frac{2q_{i\theta}}{5p_{i}} = -\frac{\mu_{i1}}{\mu_{i2}} \mathbf{V}_{i\theta}^{\text{nc}}, \quad \mathbf{V}_{i\theta}^{\text{nc}} = \frac{\sqrt{g} |\nabla \psi|^2}{B_\zeta} k_i \frac{d}{d\psi} \left( \frac{\vec{P}_i}{e_i n_i} \right), \quad k_i = -\frac{\mu_{i2}}{\mu_{i1}} \]

Parallel viscous force results in friction force expressing radial force balance with neoclassical poloidal flow \( \langle B^2 \rangle \sim B_t^2 \sim B_0^2 \gg B_p^2 \)

\[ \nabla \cdot \vec{\Pi}_{||i} = m_{i} n_{i} \frac{\langle B^2 \rangle}{B_\theta^2} \mu_{i1} (\vec{V}_{i\theta} - \vec{V}_{i\theta}^{\text{nc}}) e_\theta \simeq m_{i} n_{i} \mu_{i1} \frac{B_0^2}{B_p^2} \left[ \frac{\mathbf{B}_0 \times \nabla \vec{\phi}}{B_0^2} + (1 - k_i) \frac{\mathbf{B}_0 \times \nabla \vec{P}_i}{e_i n_i B_0^2} \right] \]
Physical background of poloidal damping term (2/2)

- Poloidal damping term in torus geometry

\[ b_0 \cdot \nabla \times \left( \nabla \cdot \Pi_{\parallel} \right) = m_i n_i \mu_{nc} \left[ \nabla_{\perp}^2 \frac{\phi}{B_0} - k_{nc} \nabla_{\perp}^2 \frac{P_1}{\epsilon_i n_i B_0} - \nabla_{\perp} k_{nc} \cdot \nabla_{\perp} \frac{P_1}{\epsilon_i n_i B_0} \right] \]

\[ + m_i n_i \nabla_{\perp} \mu_{nc} \cdot \left[ \nabla_{\perp} \frac{\phi}{B_0} - k_{nc} \nabla_{\perp} \frac{P_1}{\epsilon_i n_i B_0} \right], \quad \mu_{nc} = \mu_{i1} \frac{B_0^2}{B_p^2}, \quad k_{nc} = k_i - 1 \]

- First term in RHS is consistent with $\partial_x F_{\text{neo}}$ in slab limit
- This expression is consistent with G.Y. Park’s damping term

- Interpolation formulae for neoclassical flow coefficients
  - Neoclassical parallel viscosity [Callen IAEA 1986]

\[ \mu_{i1} = \frac{0.66 \nu_{*i}^{1/2}}{(1 + 1.03 \nu_{*i}^{1/2} + 0.31 \nu_{*i})(1 + 0.66 \nu_{*i}^{3/2})} \]

- Neoclassical friction coefficient [Hinton & Hazeltine RMP1976]

\[ k_i = \frac{1}{1 + \nu_{*i}^2 \epsilon^3} \left( \frac{1.17 - 0.35 \nu_{*i}^{1/2}}{1 + 0.7 \nu_{*i}^{1/2}} - 2.1 \nu_{*i}^2 \epsilon^3 \right) \]

They are functions of collisionality and vary largely across collisionality regimes

Strongly sheared $E_r$ is generated in the edge region
We have been developing a BOUT++ physics module for L-H transition with CDBM turbulence from elm-pb module.

\[
\frac{\partial U}{\partial t} = - [\phi, U] - \frac{1}{2} \left( - [\phi, U] + [F, \nabla_\perp^2 \phi] + \nabla_\perp^2 [\phi, F] \right)
- [\mu_{nc} (\nabla_\perp^2 \phi - \delta_i k_{nc} \nabla_\perp^2 \vec{p} - \delta_i \nabla_\perp k_{nc} \cdot \nabla_\perp \vec{p}) + \nabla_\perp \mu_{nc} \cdot (\nabla_\perp \phi - \delta_i k_{nc} \nabla_\perp \vec{p})]
- B_0^2 \nabla \parallel J + b_0 \times \kappa \cdot \nabla_\perp p + \mu_\perp \nabla_\perp^2 U + \mu_\parallel \nabla_\parallel^2 U
\]

\[
\frac{\partial Y}{\partial t} = - [\phi, Y] - \frac{1}{B_0} \partial_\parallel (B_0 \phi) + \delta_i \nabla \parallel p + \eta J - \lambda \nabla_\perp^2 J
\]

\[
\frac{\partial p}{\partial t} = - [\phi, p] - \beta_* \left[ 2b_0 \times \kappa \cdot \nabla_\perp \phi + 2\delta_i B_0^2 \nabla \parallel J \right] + \chi_\perp \nabla_\perp^2 p + \chi_\parallel \nabla_\parallel^2 p + S_p - \vec{p} L_p
\]

\[
U = \nabla_\perp^2 F, \quad F = \phi + \delta_i p, \quad J = \nabla_\perp^2 \psi, \quad Y = \psi - d_e^2 J
\]

\[
\beta_* = \frac{\gamma p}{1 + \gamma p/2B_0^2}, \quad d_a = \frac{c}{R_0} \sqrt{\frac{\varepsilon_0 m_a}{n_a e_a^2}}, \quad \delta_a = \frac{d_a}{4B_0}, \quad [f, g] = b_0 \cdot \nabla_\perp f \times \nabla_\perp g,
\]

\[
\partial_\parallel = b_0 \cdot \nabla, \quad \nabla_\parallel = b_0 \cdot \nabla - [\psi,], \quad \nabla_\perp = \nabla - \nabla_\parallel, \quad \nabla_\perp^2 = \nabla^2 - \nabla_\parallel^2
\]
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Simulation setup

- **ES-RBM model w/o 2-fluid effects**

\[
\begin{align*}
\frac{\partial U}{\partial t} &= - [\phi, U] - B_0^2 \nabla_\parallel J + b_0 \times \nabla \perp p + \mu_\perp \nabla_\perp^2 U \\
&\quad - \mu_{nc} [\nabla_\perp^2 \phi - \delta_1 k_{i1}^{nc} \nabla_\perp^2 \bar{p} - \delta_1 \nabla \perp k_{i1}^{nc} \cdot \nabla \perp \bar{p}] \\
\frac{\partial p}{\partial t} &= - [\phi, p] + \chi_\perp \nabla_\perp^2 p + \chi_\parallel \nabla_\parallel^2 p + \alpha_S(t) S - \bar{p} L \\
U &= \nabla_\perp^2 \phi, \quad J = \frac{1}{\eta B_0} \partial_\parallel (B_0 \phi)
\end{align*}
\]

\[\eta = 1.0 \times 10^{-5}, \quad \mu_\perp = \chi_\perp = 1.0 \times 10^{-6}\]

\[\chi_\parallel = 1.0 \times 10^{-1}, \quad n_0 = 1.0 \times 10^{19}[m^{-3}]\]

- **Circular equilibrium w/o shift**

\[q(\rho) = 1.0 + 2.0(\rho/0.95)^3, \quad q = 3 \text{ at } \rho = 0.95\]

\[a_0 = 0.5[m], \quad R_0 = 2.0[m], \quad B = 2.0[T]\]

Source amplitude is set to lower level until turbulence is driven (t<7000t_A).
Source amplitude is ramped up to double (7000t_A < t < 9000t_A) and is kept constant at higher level (9000t_A < t < 15000t_A).
During source intensity ramp-up (7000t_A~9000t_A), turbulence grows up at q=3 and has oscillating structure (9000t_A~11000t_A)

Turbulence is suppressed to lower level and its radial structure is also localized in narrow domain when strongly sheared E_r exits in the edge region (11000t_A~15000t_A) (next slide)
• After source intensity ramp-up, radial electric field at the edge region oscillates with similar amplitudes and then changes drastically
• Radial electric field is strongly sheared at the edge region when turbulence is suppressed
• Pressure gradient front expands to the outer region when turbulence is suppressed to lower level and shrinks to inner region when turbulence is excited to higher level.
• $n < 10$ modes are dominant modes in our simulation
• If our module solves ES-RBM turbulence correctly, dominant modes should be $n = 10\sim20$
• Inflow energy to (0,0)-mode may leak to low-$n$ modes through unphysical channels (numerical nonlinearity in 3rd WENO?)
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Summary

• Some key features related to L-H transition by flux-driven RMHD model are observed
  1. steeply sheared $E_r$ at the edge by poloidal damping effect
  2. turbulence suppression at the edge by mean flow shear
  3. energy deposition at the edge by dynamic energy balance

Future work

• Identification of root causes of energy leak problem
• parameter scan for L-H transition
Back Slides
Sensitivity of neoclassical coefficients to ion temperature

# input parameters
R0 = 2.0 # major radius [m]
a0 = 0.5 # minor radius [m]
n0 = 1.e19 # number density [m^-3]
B0 = 2.0 # magnetic intensity [T]
Ai = 2.0 # ion mass in mp unit
lnA = 20.0 # Coulomb logarithm
Zi = 1.0 # Z effective
q = 3.0 # safty factor at rho = 1
Spatiotemporal structure of mean ion temperature
Unphysical mode transition in linear multi-n RBM simulation: setup (1/2)

• Linear 3-field RBM

\[ \frac{\partial U_1}{\partial t} = -B_0^2 \nabla || J_1 + b_0 \times \kappa \cdot \nabla P_1 \]

\[ \frac{\partial P_1}{\partial t} = -b_0 \cdot \nabla \phi_1 \times \nabla P_{eq} \]

\[ \frac{\partial \psi_1}{\partial t} = -\frac{1}{B_0} \nabla || (B_0 \phi_1) + \eta J_1 \]

\[ U_1 = \nabla \perp \phi_1, \quad J_1 = \nabla \perp \psi_1, \quad \nabla || = b_0 \cdot \nabla \]

• Computational domain

\[ x = \psi - \psi_0, \quad y = \theta, \quad z = \zeta - \int_{\theta_0}^{\theta} \nu(\psi, \theta) d\theta \]

\[ 0.5 \leq \rho(x) \leq 1.0, \quad 0 \leq y \leq 2\pi, \quad 0 \leq z \leq 2\pi/5 \]

\[ N_x \times N_y \times N_z = 516 \times 128 \times 65 \]

• Boundary conditions and toroidal filtering settings

• bdry_all = dirichlet on \( U_1, P_1, \phi_1, J_1 \)

• bdry_xin and _xout = zerolaplace on \( \psi_1 \)

• Inversion flags on \( \phi_1 \) are dirichlet on both x boundaries

• low_pass_z = 8 (n=0,5,...,40)

Simulation results in this slides were calculated by rmhd-lh.cxx not by elm-pb.cxx

Two BOUT.inp employing same parameters except parameters of differencing scheme are used in this slide (next slide)
Unphysical mode transition in linear multi-n RBM simulation: setup (2/2)

- w/ Upwind-case (default)
  - advection terms declared by \texttt{b0xGrad_dot_Grad} are calculated by 3rd order WENO
  - advection terms declared by \texttt{bracket} are also calculated by 3rd order WENO

```plaintext
# derivative methods
[ddx]
first = C4 # order of first x derivatives (options are 2 or 4)
second = C4 # order of second x derivatives (2 or 4)
upwind = W3 # order of upwinding method (1, 4, 0 = TVD (DO NOT USE), 3 = WENO)

[ddy]
first = C4
second = C4
upwind = W3

[ddz]
first = C4 # Z derivatives can be done using FFT
second = C4
upwind = W3
```

- w/o Upwind-case
  - advection terms declared by \texttt{b0xGrad_dot_Grad} are calculated by 4th order central differencing
  - advection terms declared by \texttt{bracket} are calculated by ARAKAWA scheme

```plaintext
# derivative methods
[ddx]
first = C4 # order of first x derivatives (options are 2 or 4)
second = C4 # order of second x derivatives (2 or 4)
upwind = C4 # order of upwinding method (1, 4, 0 = TVD (DO NOT USE), 3 = WENO)

[ddy]
first = C4
second = C4
upwind = C4

[ddz]
first = C4 # Z derivatives can be done using FFT
second = C4
upwind = C4

# BRACKET_METHOD flags:
# 0:BRACKET_STD; derivative methods will be determined
# by the choices C or W in this input file
# 1:BRACKET_SIMPLE; 2:BRACKET_ARAKAWA; 3:BRACKET_CTCU.

bm_exb_flag = 0
bm_mag_flag = 0
```
Growth rates of $\phi$ in w/ and w/o upwind-case

- Unphysical mode jumps occur in w/ Upwind-case but those don’t occur in w/o Upwind-case
- Unphysical mode jump may be driven by nonlinear up-winding scheme (System is linearized theoretically but has nonlinearity numerically)
- In w/o Upwind-case numerical oscillation of $n=0$ mode grows but it may be stabilized by dissipation term
Temporal evolution of $\phi$ in w/ and w/o upwind-case

- w/ Upwind-case (default)
- w/o Upwind-case
Poloidal slice of $\phi$ before and after mode transition in w/ and w/o upwind-case

- w/ Upwind n=10 before transition (t=300t$_A$)

- w/o Upwind n=10 before transition (t=300t$_A$)

- w/ Upwind n=10 after transition (t=600t$_A$)

- w/o Upwind n=10 after transition (t=600t$_A$)
Implementation of 3\textsuperscript{rd} WENO in BOUT++

BOUT/src/sys/derivs.cxx

// 3rd-order WENO scheme
BoutReal VDDX_WENO3(stencil &v, stencil &f) {
    BoutReal deriv, w, r;
    if(v.c > 0.0) {
        // Left-biased stencil
        r = (WENO_SMALL + SQ(f.c - 2.0*f.m + f.mm)) / (WENO_SMALL + SQ(f.p - 2.0*f.c + f.m));
        w = 1.0 / (1.0 + 2.0*r*x);
        deriv = 0.5*(f.p - f.m) - 0.5*w*(-f.mm + 3.0*f.m - 3.0*f.c + f.p);
    } else {
        // Right-biased
        r = (WENO_SMALL + SQ(f.pp - 2.0*f.p + f.c)) / (WENO_SMALL + SQ(f.p - 2.0*f.c + f.m));
        w = 1.0 / (1.0 + 2.0*r*x);
        deriv = 0.5*(f.p - f.m) - 0.5*w*(-f.m + 3.0*f.c - 3.0*f.p + f.pp);
    }
    return v.c*deriv;
}

// 3rd-order CWENO. Uses the upwinding code and split flux
BoutReal DDX_CWENO3(stencil &f) {
    BoutReal a, ma = fabs(f.c);
    // Split flux
    a = fabs(f.m); if(a > ma) ma = a;
    a = fabs(f.p); if(a > ma) ma = a;
    a = fabs(f.mm); if(a > ma) ma = a;
    a = fabs(f.pp); if(a > ma) ma = a;
    stencil sp, vp, sm, vm;
    vp.c = 0.5; vm.c = -0.5;
    sp = f + ma;
    sm = ma - f;
    return VDDX_WENO3(vp, sp) + VDDX_WENO3(vm, sm);