BOUT Simulations of Drift Resistive Ballooning L-mode Turbulence in the Edge of the DIII-D Tokamak

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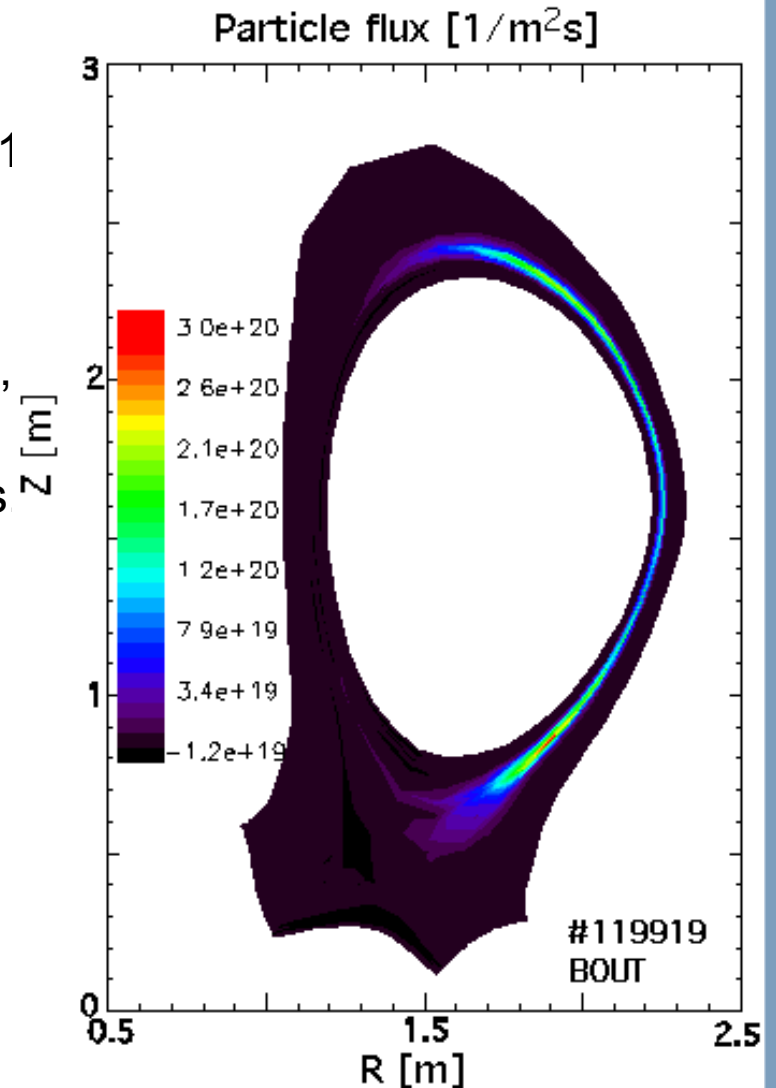
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BOUT Simulations of Drift Resistive Ballooning L-mode Turbulence in the Edge of the DIII-D Tokamak - Outline

1. Introduction -- Overview, definitions of suite of BOUT simulations and equations used

2. BOUT simulations of shot #119919 (with T_e & with/without T_i fluctuations)

3. Plunging probe data for shot #119919 and comparison to BOUT simulation

4. BES data for shot #119919, GKV software synthetic diagnostic adjustment for spatial resolution of BES, and comparison to BOUT simulation

5. Inclusion of model $E_{radial}(r)$ fitted to probe and CER data in BOUT simulations of #119919: sheared E_{radial}xB is stabilizing

6. Summary of comparisons between BOUT and experimental data from probe and BES for shot #119919

7. Simulations of colder, lower density shot #119934, with and without E_{radial}

8. Conclusions

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BOUT Simulations of Resistive Drift Ballooning Turbulence in Edge Region for DIII-D Well Characterized L-Mode Shots

- Simulations of electromagnetic resistive drift ballooning in DIII-D L-mode shots #119919, 119921, 119930, and 119934, with full geometry and magnetic shear, crossing the separatrix

- Nonlinear BOUT equations for ion density, vorticity, electron and ion velocities, electron and ion temperatures, Ohm's law, and Maxwell's equations

- **Simulation results for various physics models and validation against probe and BES data**

- **BOUT obtains steady-state turbulence with fluctuation amplitudes and transport that compare reasonably to DIII-D probe and BES data. Sheared rotation due to $E_{radial}(r)$ can be stabilizing.**

Particle flux [1/m²s]

3 0e+20
2 6e+20
2.1e+20
1.7e+20
1 2e+20
7 9e+19
3.4e+19
-1.2e+19

#119919
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Example of Drift Resistive Ballooning Equations in BOUT06

- Consider the following simplified Braginskii + reduced Maxwell eqns with drift ordering in the BOUT06 framework (**Case #4**):

$$\frac{d\tilde{N}_i}{dt} + \nabla N_i \tilde{V}_\parallel = \left(\frac{2c}{eB}\right) b_0 \times \kappa \bullet (\nabla \tilde{P}_e - N_i e \nabla \varphi) + \nabla_\parallel (\tilde{j}_\parallel / e)$$

$$\frac{d\varpi}{dt} = 2\omega_{ci} b_0 \times \kappa \bullet \nabla \tilde{P} \quad + \quad N_{i0} Z_i e \frac{4\pi V_A^2}{c^2} \nabla_\parallel \tilde{j}_\parallel$$

$$\frac{d\tilde{V}_{\parallel e}}{\partial t} = -\frac{e}{m_e} E_\parallel - \frac{1}{N_{i0} m_e} (T_{e0} \nabla_\parallel \tilde{N}_i) + 0.51 \nu_{ei} \tilde{j}_\parallel$$

$$\frac{d\tilde{V}_{\parallel i}}{dt} = -\frac{1}{N_{i0} M_i} \nabla_\parallel \tilde{P},$$

$$\frac{d\tilde{T}_e}{dt} = \frac{2}{3N_{i0}} \nabla \bullet \left(\kappa_\parallel^e \nabla_\parallel \tilde{T}_e\right), \quad \kappa_\parallel^e = 3.2 \frac{N_{i0} T_{e0} \tau_{e0}}{m_e}$$

$$\mathbf{E} = -\frac{1}{c}\frac{\partial}{\partial t} A_\parallel - \nabla \varphi, \quad -\nabla_\perp^2 A_\parallel = \frac{4\pi}{c} j_\parallel, \quad \mathbf{B} = \nabla \times A_\parallel + \mathbf{B}_0$$

$$\varpi = \nabla \cdot [e Z_i N_i \nabla \varphi] \approx e Z_i N_{i0} \nabla^2 \varphi \quad \nabla_\parallel = \mathbf{b}_0 \cdot \nabla + \tilde{\mathbf{b}} \cdot \nabla \quad Z_i = 1$$

$$\frac{d}{dt} = \frac{\partial}{\partial t} + (\mathbf{V}_{E0} + \tilde{\mathbf{V}}_E) \bullet \nabla \quad N_i = N_{i0} + \tilde{N}_i, \quad T_s = T_{s0} + \tilde{T}_s, \dots$$

$$\tilde{P} = N_{i0}(\tilde{T}_e + \tilde{T}_i) + \tilde{N}_i(T_{e0} + T_{i0}), \quad T_{i0} = T_{e0}, \quad \tilde{T}_i = 0, \quad V_{\parallel s0} = 0$$

- Electromagnetic with
$$\nabla_\parallel = \mathbf{b}_0 \cdot \nabla + \tilde{\mathbf{b}} \cdot \nabla \text{ in } -\nabla_\parallel \phi \text{ and } \nabla_\parallel \tilde{j}_\parallel$$

- Actual DIII-D geometry

- Radial bdry conditions: Von Neumann on fluid fluctuations, Dirichlet on $A_\parallel$ & ϕ
  Fluctuations decay to 0 at outer bdry & not necessarily at inner bdry

- DIII-D - like fixed background profiles for shots #119919 and 119934

- **Case #4** includes δT_e (δT_i=0) and parallel heat conduction

- **Case #5** includes all of the above and evolution equation for δT_i

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BOUT Simulation of Resistive Drift Ballooning Turbulence for DIII-D L-mode Shots - Outline

- Electromagnetic simulations of resistive ballooning turbulence in single-null DIII-D geometry (Braginskii equations + drift ordering):

  Case #1: Simplest model for resistive drift ballooning ($\delta T_{e,i}=0$)

  Case #2: Adds δT_e

  Case #3: Adds δT_e and electron parallel thermal conduction

  → **Case #4:** Adds δT_e, electron parallel thermal conduction, and
  $$\nabla_{\parallel} = \mathbf{b}_0 \cdot \nabla + \tilde{\mathbf{b}} \cdot \nabla \text{ in } -\nabla_{\parallel}\phi \text{ and } \nabla_{\parallel} j_{\parallel}$$

  → **Case #5:** Adds δT_e and δT_i, etc.

  → **Case #6:** Adds δT_e and δT_i, etc., and imposed E_r

- Comparison to probe and BES data for DIII-D shots #119919,30,34. These shots are well-characterized L-mode shots exhibiting steady-state turbulence.

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History of rms fluctuation amplitudes in midplane at separatrix with electron parallel thermal conduction and magnetic flutter, showing saturated turbulence in BOUT for shot #119919

BOUT Cases 4 and 5
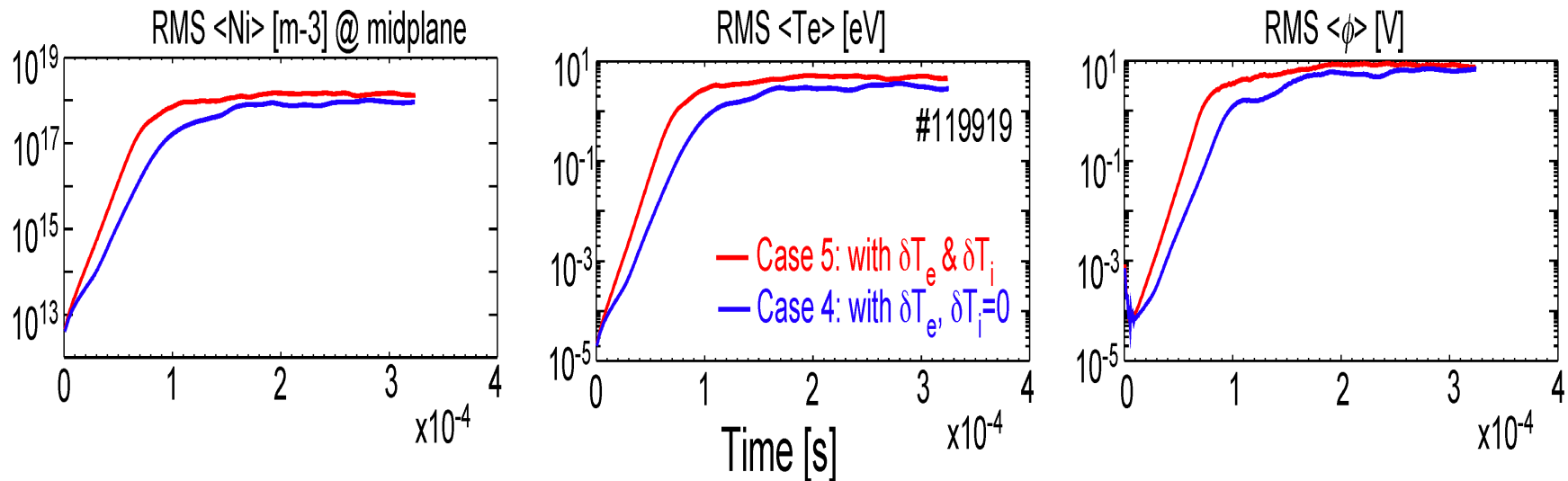
- With δT_e, electron parallel thermal conduction, convective nonlinearities,
  $\nabla_\parallel = \mathbf{b}_0 \cdot \nabla + \tilde{\mathbf{b}} \cdot \nabla$ in $-\nabla_\parallel \phi$ and $\nabla_\parallel j_\parallel$ (and δT_i in Case 5)

- Temperature and density fluctuations saturate given fixed background profiles

- Including δT_i increases fluctuation amplitudes modestly

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Time-averaged ion density fluctuations in the midplane saturate at ~10-30% and peak near R_{sep}
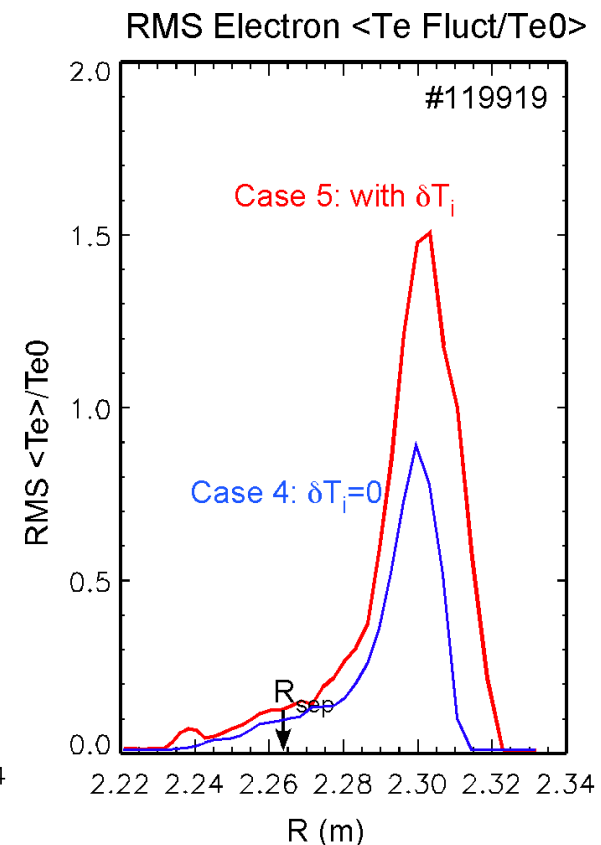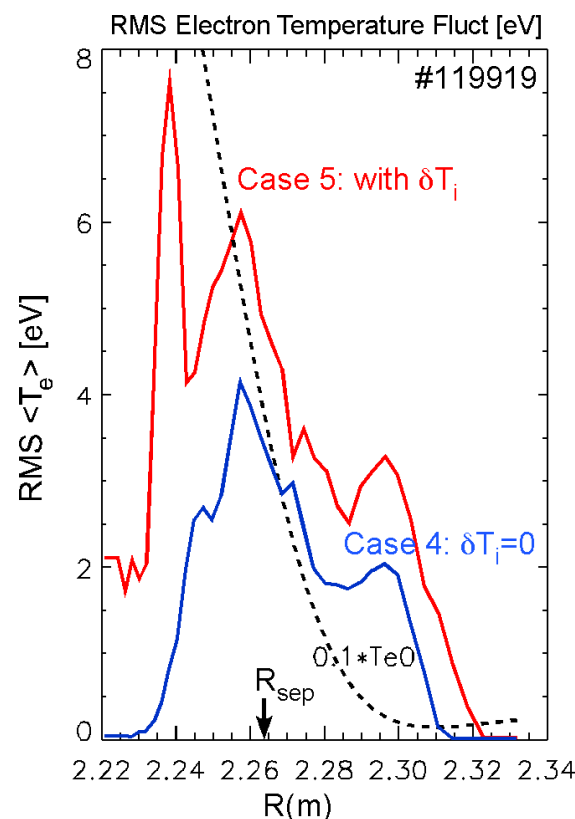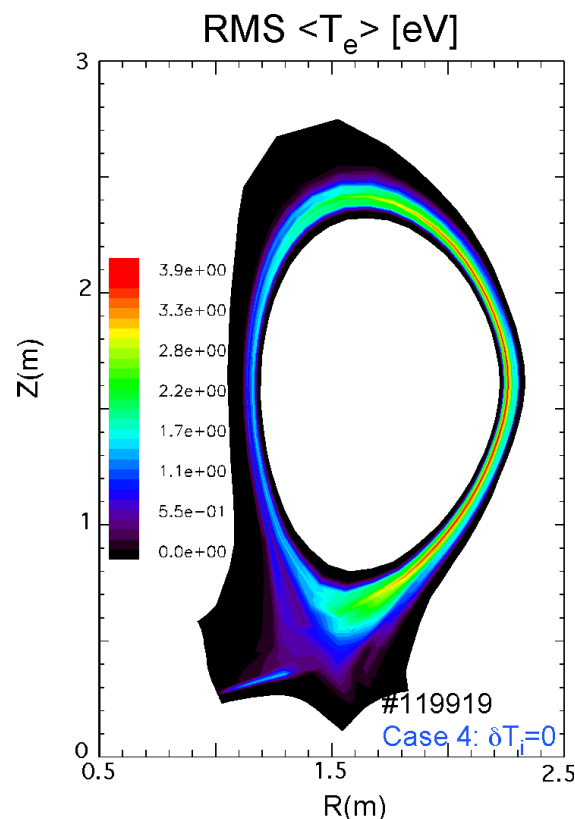
BOUT Cases 4 and 5
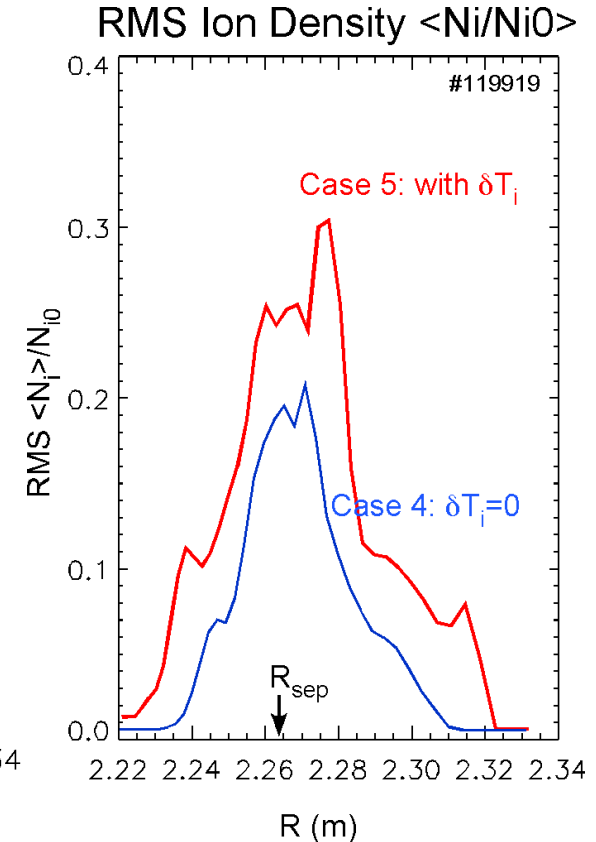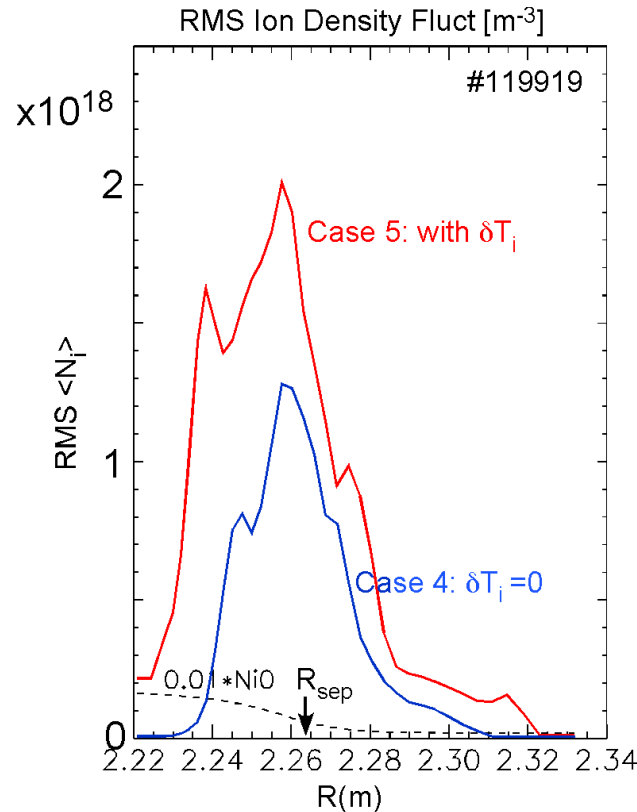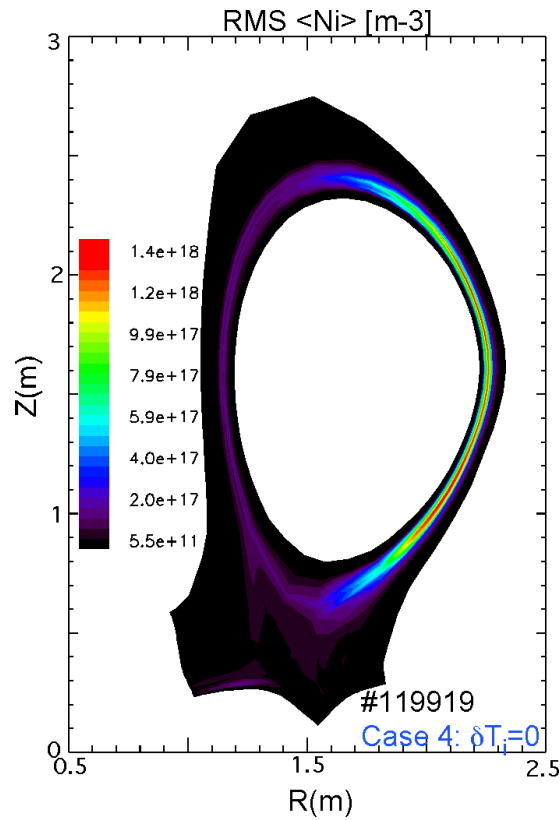
@ midplane

- With T_e fluctuations, electron parallel thermal conduction, and $\nabla_{\parallel} = \mathbf{b}_0 \cdot \nabla + \tilde{\mathbf{b}} \cdot \nabla$
- Including T_i fluctuations leads to ~50% higher fluctuation amplitudes
- There is a poloidal asymmetry wrt midplane in the fluctuations due X-pt & shear

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Time-averaged T_e fluctuations in the midplane peak near the R_{sep} and saturate at ±50% relative amplitude at $R > R_{sep}$
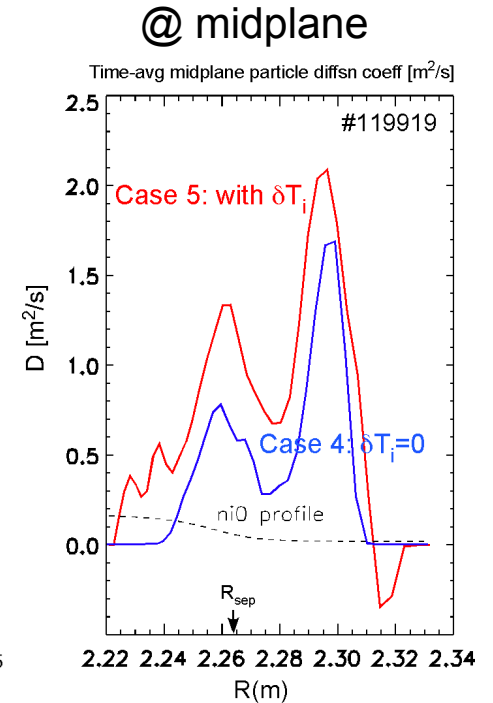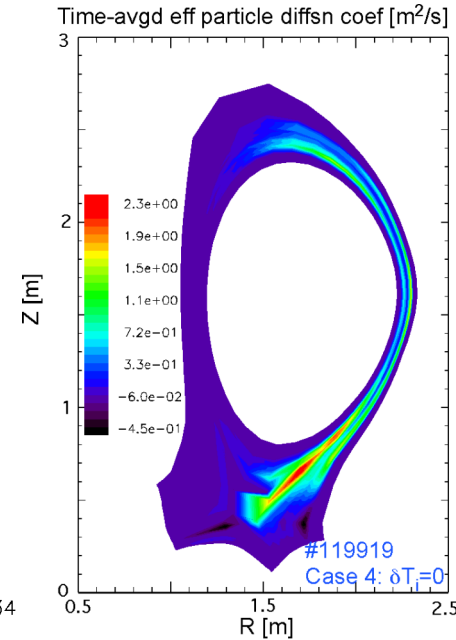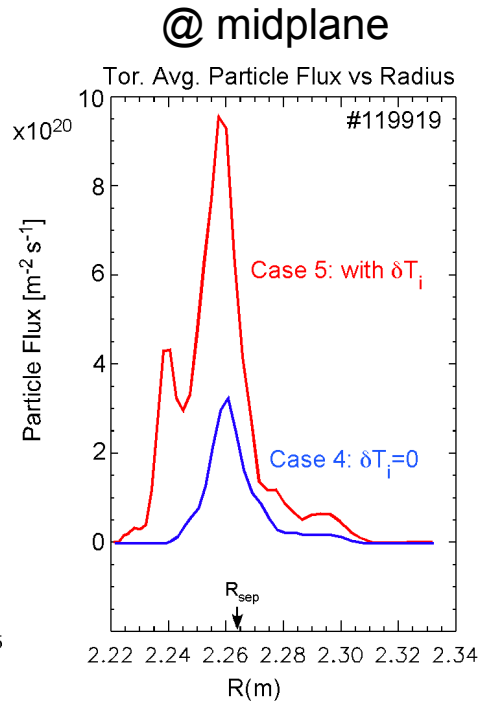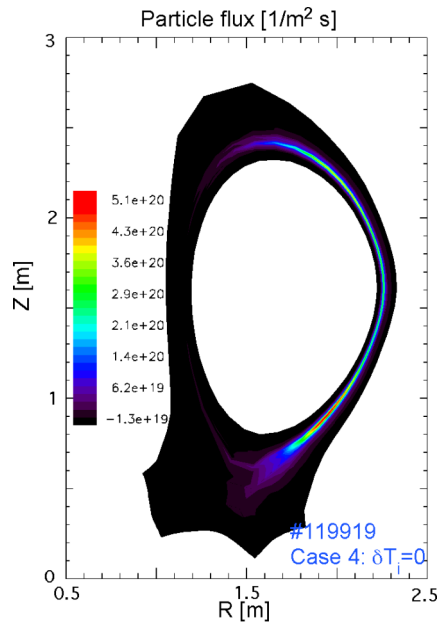
BOUT Cases 4 and 5

@ midplane

- With T_e fluctuations, electron parallel thermal conduction, nonlinear convection, and $\nabla_{\parallel} = \mathbf{b}_0 \cdot \nabla + \tilde{\mathbf{b}} \cdot \nabla$
- T_e fluctuations are ~10% near R_{sep} and are generally higher with finite δT_{ii}

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Time-averaged ion particle diffusion coefficient in the midplane saturates at ~1.5-2 m²/s

BOUT Cases 4 and 5

@ midplane

@ midplane
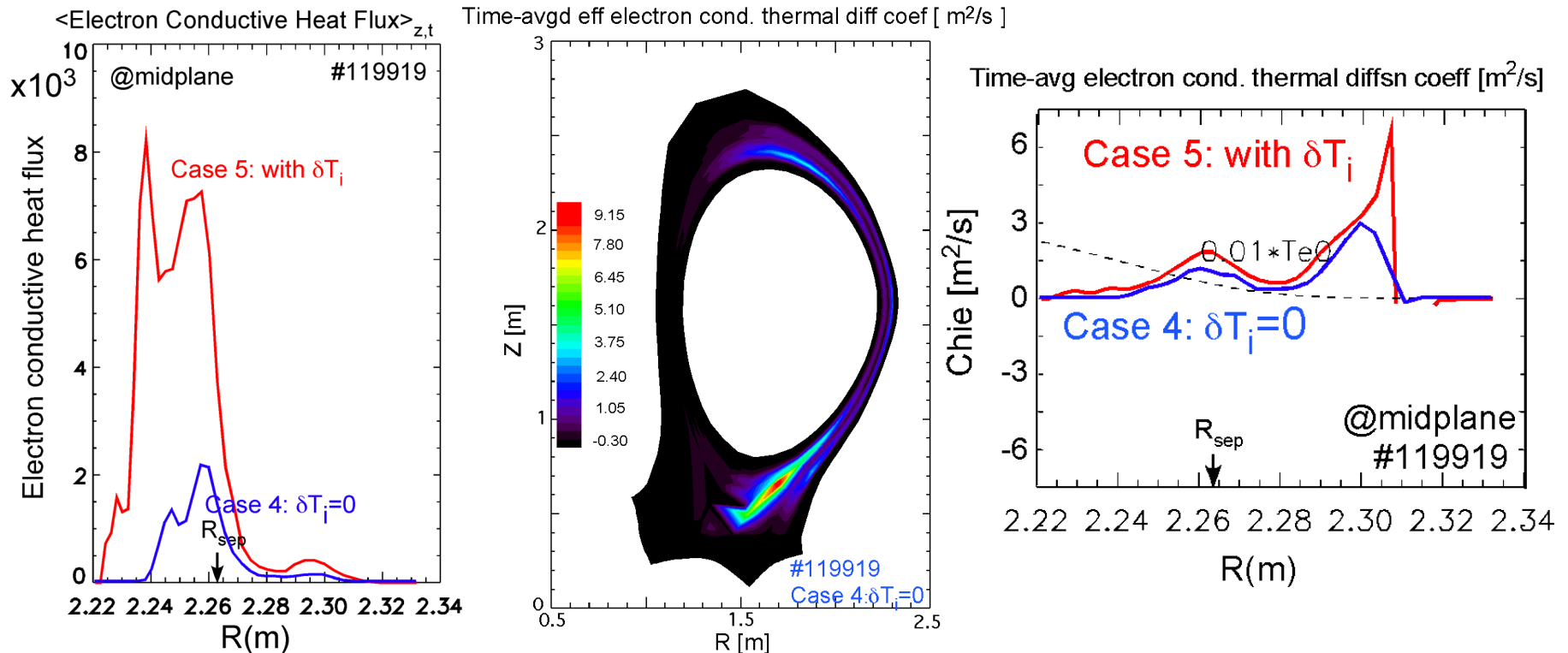

- With T_e fluctuations, electron parallel thermal conduction, nonlinear convection, and $\nabla_{\parallel} = \mathbf{b}_0 \cdot \nabla + \tilde{\mathbf{b}} \cdot \nabla$
- Including T_i fluctuations leads to higher particle fluxes and diffusion coefficient

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Time-averaged electron conductive thermal diffusion coefficient in the midplane saturates at ~2-6 m²/s

BOUT Cases 4 and 5

Note: Here heat flux (conductive) $= \frac{3}{2} N_0 < \delta v_r \delta T_e >_{tor,t}$, and $\chi_e = -\frac{3}{2} N_0 < \delta v_r \delta T_e >_{tor,t} / N_0 \nabla T_{e0}$

- With T_e fluctuations, electron parallel thermal conduction, nonlinear convection, and $\nabla_{\parallel} = \mathbf{b}_0 \cdot \nabla + \tilde{\mathbf{b}} \cdot \nabla$
- Including T_i fluctuations leads to higher heat fluxes and diffusion coefficient
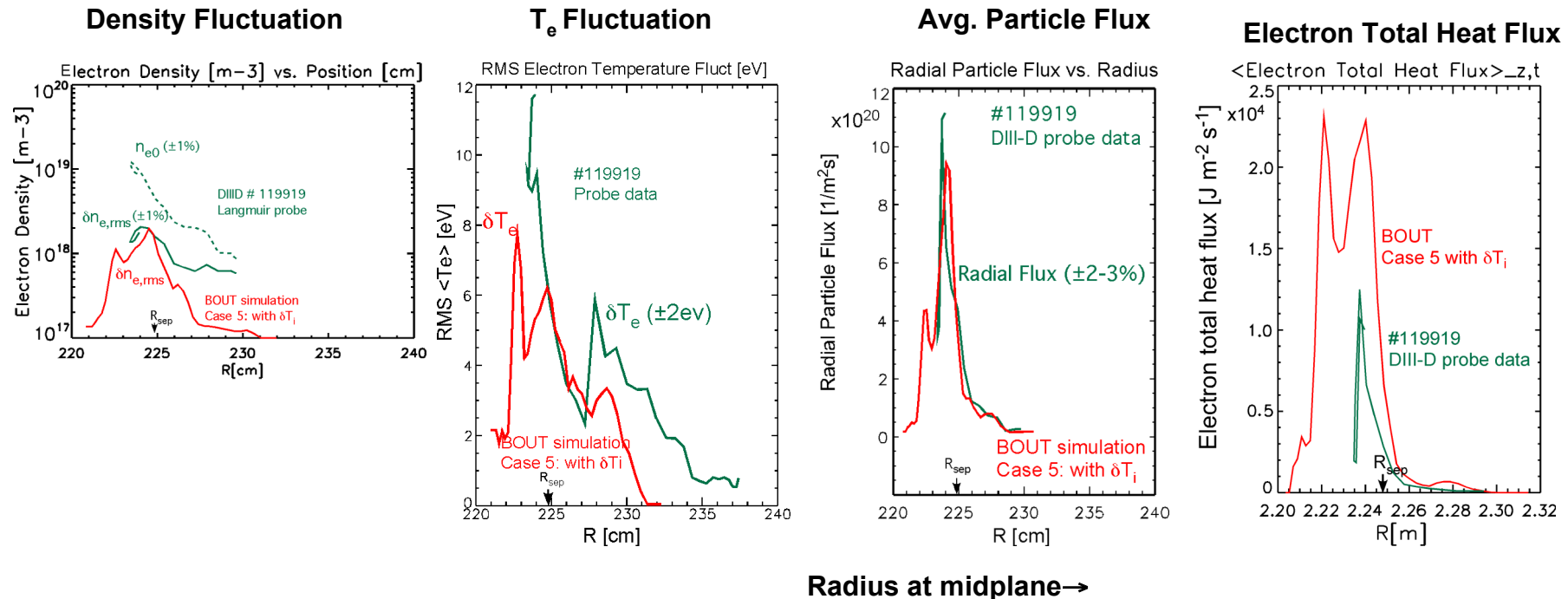
Electron density and radial particle flux vs. radius near midplane -- relative density fluctuations exceed ~20% and temperature fluctuations exceed ~50%
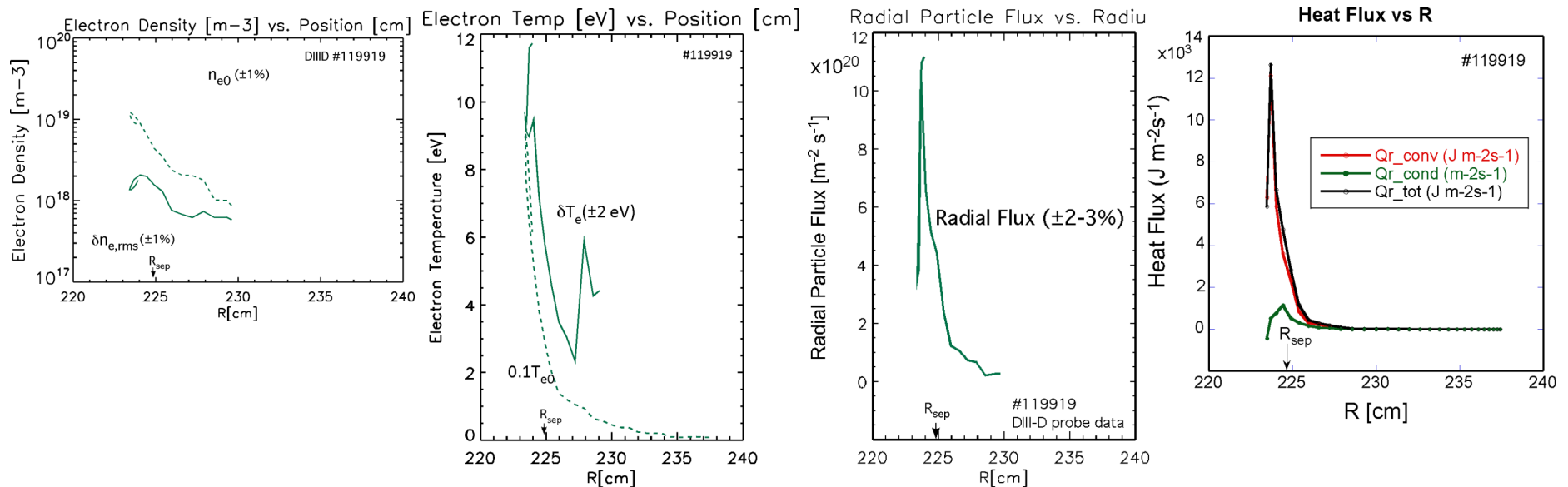
- Probe signals decrease below noise levels for R > 229 cm, and stop for R < 223 cm
- Typical experimental rms δn_e and δT_e fluctuations at the separatrix exceed ~20% & ~50%
- δn_e, δT_e and the probe fluxes in the midplane peak near the separatrix as in BOUT results
- Here the convective heat flux is much larger than the conductive heat flux

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There is reasonable agreement between BOUT simulation and Langmuir probe data for DIII-D #119919 with respect to peak fluctuation amplitudes, particle and heat fluxes, and spatial localization

- BOUT with T_e & T_i fluct'ns, electron parallel thermal conduction, convective nlrity, $\nabla_\parallel = \mathbf{b}_0 \cdot \nabla + \tilde{\mathbf{b}} \cdot \nabla$

Density Fluctuation **T_e Fluctuation** **Avg. Particle Flux** **Electron Total Heat Flux**
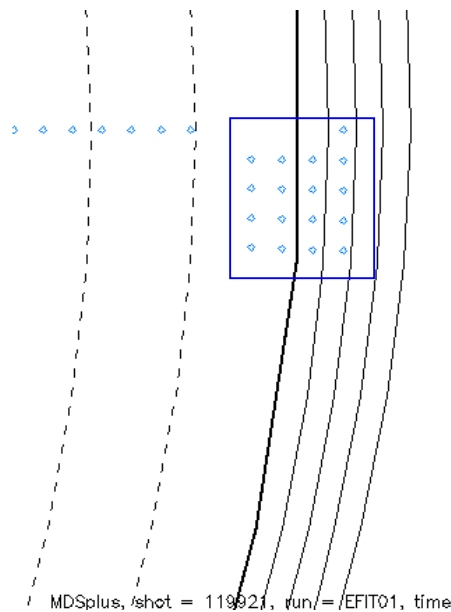

Radius at midplane→

- BOUT simulation results and Langmuir probe data generally agree well on peaking near separatrix and relative spatial localization, and within factors of two on peak amplitudes for density and T_e fluctuations, and fluxes for 2.23m ≤ R ≤ 2.29m.

- Agreement on total heat flux is better than on conductive heat flux

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ñ/n Amplitude Profile
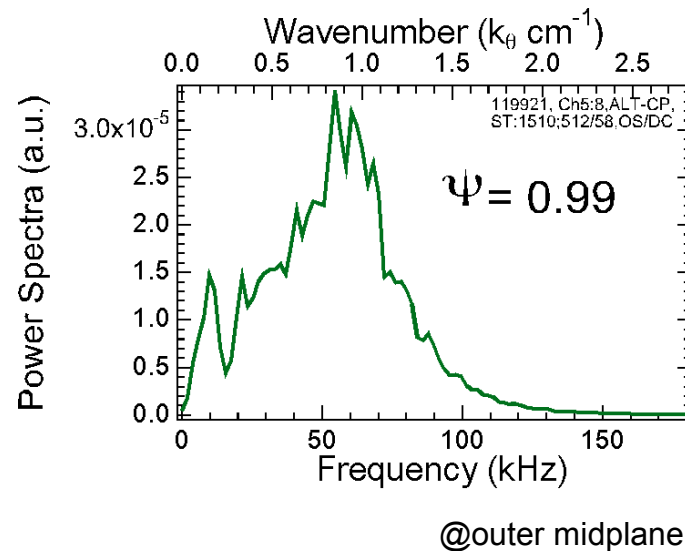
- Short beam-blips injected to obtain BES data during L-mode plasma conditions

BES 4x4 Grid

Density Fluctuation Spectrum

$\Psi = 0.99$

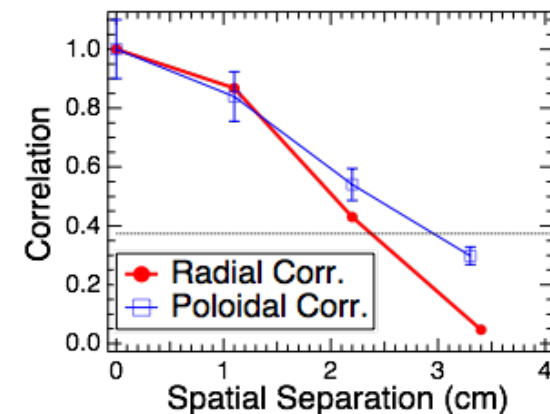@outer midplane

MDSplus, shot = 119921, run = /EFIT01, time

(Preliminary Analysis)

Spatial Correlation

Radial Corr.
Poloidal Corr.

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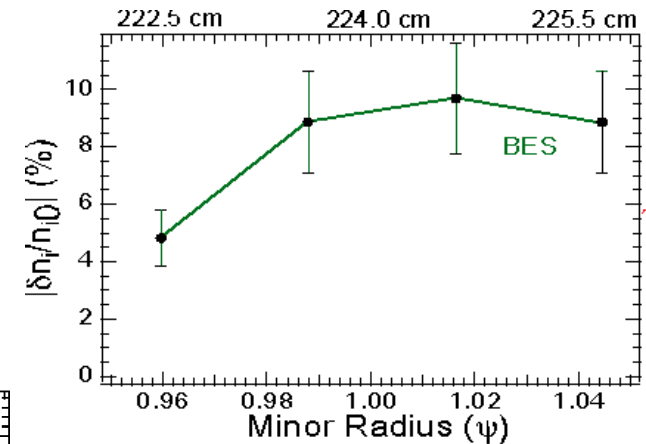
# Synthetic Simulation Diagnostics Using GKV Suite to Match BES Data

- GKV is a suite of IDL routines built by Bill Nevins to analyze data from simulations or experiments. GKV computes power spectra, correlation functions, etc., makes plots.

  refs: W.M. Nevins, "GKV User's Manual", UCRL-TR-206016 (Aug. 2004); W.M. Nevins *et al*., Phys. Plasmas **13**, 122306 (2006).

- We construct synthetic diagnostics using the GKV suite of IDL routines to compare to BES data.

- Spatial filtering (1D or 2D) required in simulation diagnostics to model the Δx=1 cm limit on spatial resolution in the BES grid in R and Z. Filtering is applied to both the radial and binormal coordinates thru the convolution

$$f_{smooth}(x) = \int dx' w(x - x') f(x'), \text{ where } w(x) = \begin{cases} \frac{1}{2\Delta x}\left[1 + \cos\left(\frac{\pi x}{\Delta x}\right)\right] & \left|\frac{x}{\Delta x}\right| < 1 \\ 0 & \left|\frac{x}{\Delta x}\right| > 1 \end{cases}$$

- Correlation functions are defined by normalized integrals:

$$C(f; x, t) = \frac{\iint dx' \, dt' \, f(x', t') f(x' - x, t' - t)}{\iint dx' \, dt' \, f(x', t') f(x', t')}$$

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Reasonable Agreement between BOUT Simulation and BES Data for DIII-D #119921 with respect to Peak Fluctuation Amplitude, Localization, Spatial Correlation Width, and Spectral Width

ñ/n Amplitude Profile

Spatial Correlation

Density Fluctuation Spectrum

@outer midplane
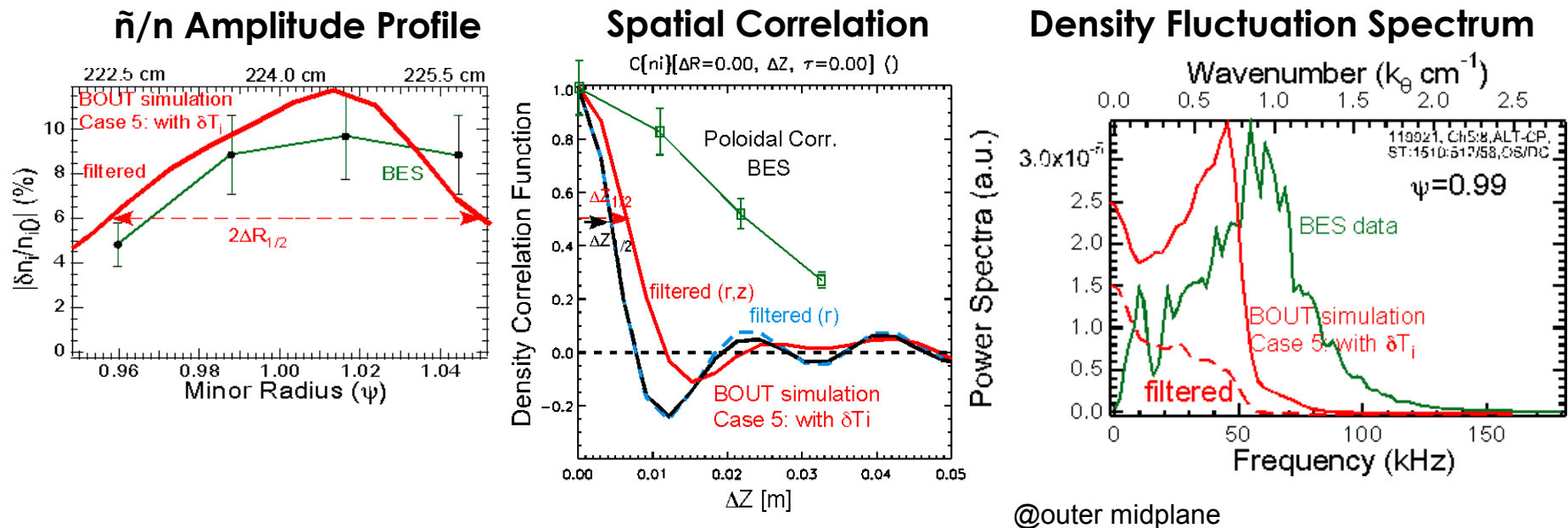
- Spatial filtering of the BOUT diagnostics reduces and spatially spreads peaks

- There is reasonable agreement between BOUT and BES to within factors of two or three, or less

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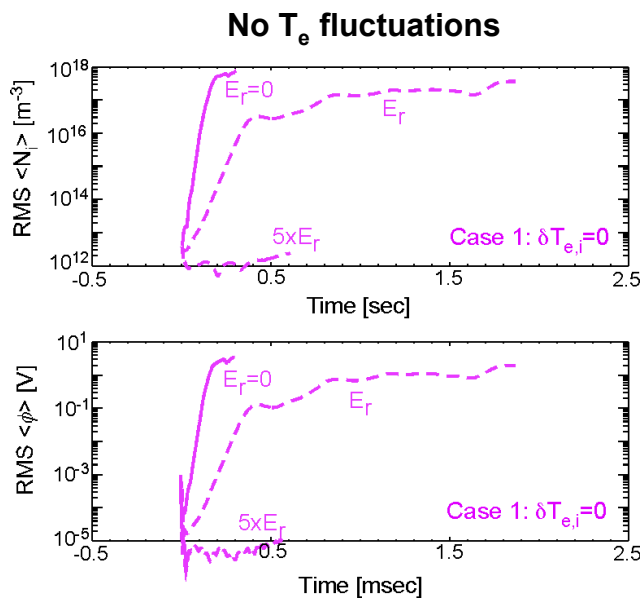# Model the Experimental Radial Electric Field to Study the Effects of Imposed $E_0 \times B$ Shearing on Simulated Turbulence

- An equilibrium radial electric field E_0 is included in BOUT simulations for Cases 1, 4, & 5, using fits to the experimental probe and CER data near the midplane

- L-mode plasmas typically have weakly sheared ExB flows. In our fit to probe and CER data the ExB shearing rate is $< 2.4 \times 10^4$ (1/s) $<$ BOUT growth rates$\sim O(1) \times 10^5$ (1/s)

- We expect that with imposed sheared ExB flow there will be weaker linear instability and some reduction of the saturated turbulence
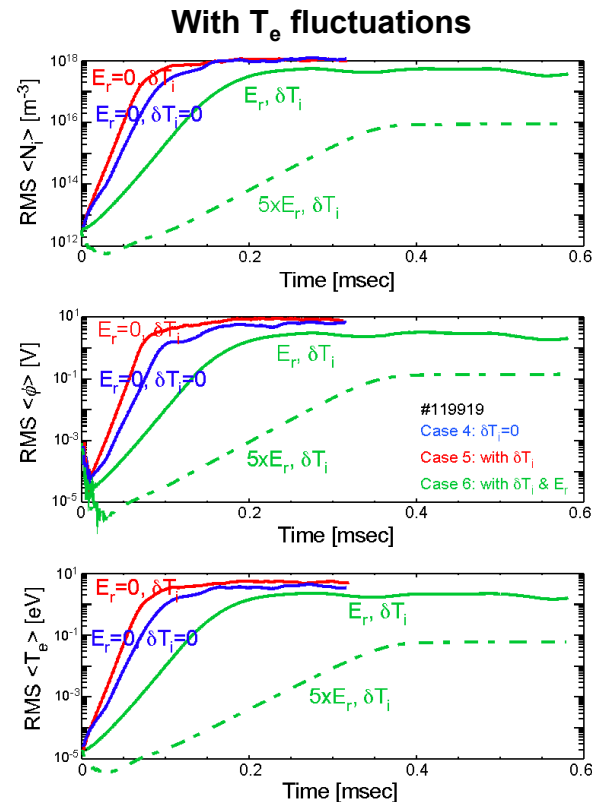

E_r Profiles for DIIID #119919 from 2 - 4 s (CER Data)

- An equilibrium radial electric field E_0 is included in BOUT simulations for #119919 **Cases 1, 4, 5, & 6**, using fits to the experimental probe and CER data near midplane

- In our fit to probe and CER data the ExB shearing rate is $< 2.4 \times 10^4$ (1/s) $<$ BOUT growth rates $\sim O(1) \times 10^5$(1/s)
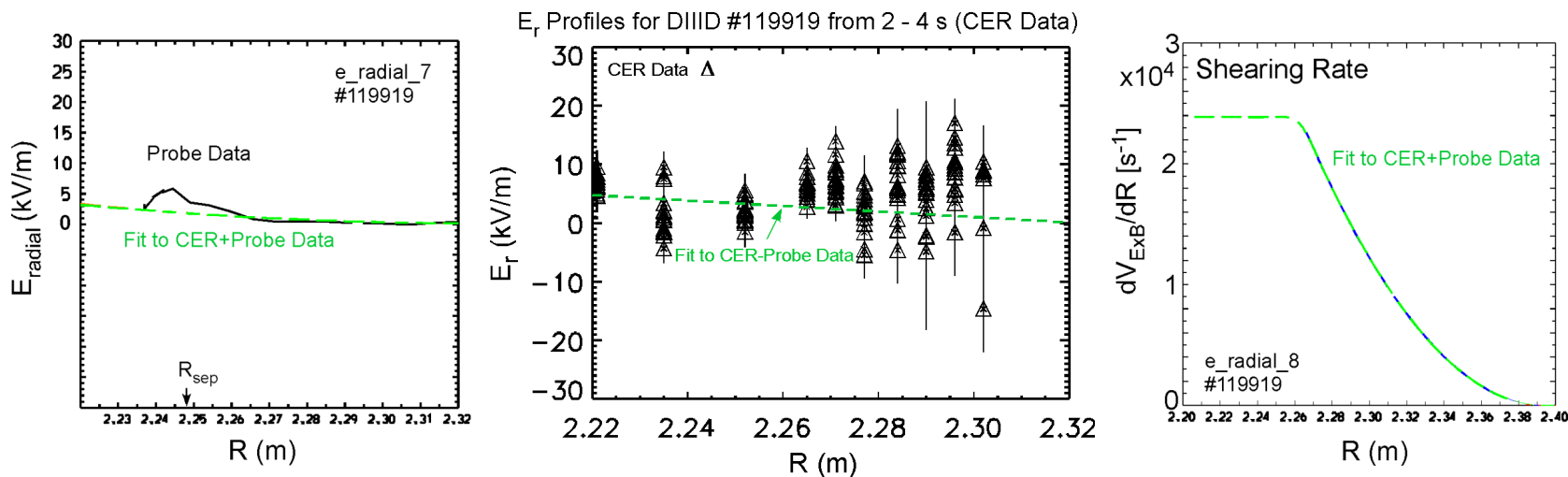
- Imposed sheared ExB flow weakens linear growth rates, and saturation is much delayed and perhaps at lower amplitudes, while $5 \times E_r$ is much more stabilized. Have the simulations with E_r run long enough?

No T_e fluctuations

With T_e fluctuations

Case 1: $\delta T_{e,i}=0$

$E_r=0$

E_r

$5 \times E_r$

$E_r=0, \delta T_i$

$E_r=0, \delta T_i=0$

$E_r, \delta T_i$

$5 \times E_r, \delta T_i$

#119919
Case 4: $\delta T_i=0$
Case 5: with δT_i
Case 6: with δT_i & E_r

@midplane

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As the Physics Model Becomes More Complete, the Agreement of BOUT Results with DIII-D Probe Data Improves - Summary

- Comparison of suite of BOUT simulations to shot #119919: peak values in midplane at saturation near R_{sep} ($E_{rad}=0$, $E_{rad}\neq0$, $E_{rad}=5E_{rad}$ with δT_e convective nonlinearity)

Bout simulation	$\langle\delta N_i\rangle_{rms}$ (10^{18} m^{-3})	$\langle\delta T_e\rangle_{rms}$ (eV)	Radial Particle Flux (10^{20} /m^2 s)	D_r (m^2/s) local	Conductive Radial Heat Flux $=\frac{3}{2}N_0\langle\delta\tilde{v}_r\delta\tilde{T}_e\rangle$ (10^3 J/m^2 s)	χ_e(m^2/s), local conductive
#1: $\delta T_e=0$	0.95	N/A	1.8	0.4	N/A	N/A
w/E$_r$**	0.37	N/A	0.07	0.02	N/A	N/A
#4: $\delta T_e\neq0$ $\kappa_{\|e}\neq0$ & $\tilde{\mathbf{b}}\cdot\nabla$	1.3	4.0	3.3	1.7	3.3	2.7
#5 & w/δT_i	2.0	7.5	9.5	2	10 (total=22)	2.2 (~4)
#6 & w/E$_r$	0.7	5.5	0.8	0.27	2.5 (4.6)	0.32 (0.39)
& w/5E$_r$	0.3	3.5	0.18	0.035	0.75	0.036
DIII-D #119919 probe data	2.0	10	11.0	~0.2-1 ‡	1.2 (total=12)	~1-2 ‡

**Cases #1 w/E$_r$ has not saturated at end of simulation (1.8 ms)   *total heat flux = cond. +conv. fluxes
‡Typical, flux-surface-averaged values for shot #119919 inferred from UEDGE reconstruction

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- Comparison of suite of BOUT simulations to shot #119921 BES data: fluctuation frequency spectra, peak density amplitude radial half-width, correlation lengths
- Factor of 2 or better agreement seen between simulation synthetic diagnostics with filtering and the DIII-D #119921 BES data (with or without sheared $E_0 \times B$ velocity included in BOUT)

Bout simulation	$\langle \delta N_i/N_i \rangle_{rms}$ peak vs. R raw/filtered	$\Delta R_{half-max}$ of $\langle \delta N_i/N_i \rangle_{rms}$ (cm) raw/filtered	$\Delta Z_{corr,half-max}$ of density (cm) raw/filtered	Peak freq in density fluct'n spect, raw/filtered (10^5 rad/s)	Freq half-max in density fluct'n spect, raw/filtered (10^5 rad/s)
#1: $\delta T_e = 0$ w/E_r**	0.13 / 0.07 0.065/0.045	1.2 / 1.5 0.7/0.8	0.6 / 0.9 2.0 / 2.3	3 / 0.5 0 / 0	4 / 2 1/ 0.7
#4: $\delta T_e \neq 0$ $\kappa_{\|e} \neq 0$ & $\tilde{\mathbf{b}} \cdot \nabla$	0.20 / 0.12	1.4 / 1.2	0.4 / 0.7	3.0 / 1.5	2 / 1.2
#5: w/δT_i	0.21 / 0.12	1.7 / 2	0.4 / 0.7	3.0 / 0&1.5	1 / 1.5
#6: & w/E_r & $E_r = 5E_r$	0.07 / 0.05 0.011/0.006	1.5 / 1.7 0.5 / 0.6	0.8/ 0.9 3/3	0.5 / 0.5 3.8 / 3.8	0.5 / 0.5 0.25 / 0.25
DIII-D #119921 BES data	0.09 ±0.2	2 ±0.2	2 ±0.2	3.8	1.3 ±0.2

**Cases #1 w/E_r has not saturated at end of simulation (1.8 ms)

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Comparison of Probe Data from Lower Density and Colder Plasmas in Shots #119930 and 119934 vs. 119919

- Edge plasmas in L-mode shots #119930 and 119934 are colder and have lower densities than in shot #119919

- The growth rate for resistive ballooning is proportional to $\eta^{1/3}\beta^{2/3}/n^{1/3} \propto T^{1/6}n^{1/3}$

- A factor of two lower temperature and density decreases the drive for resistive balloonining by $O(1/\sqrt{2})$ if all else is fixed in shots #119930/119934 vs. #119919


Outer midplane profiles
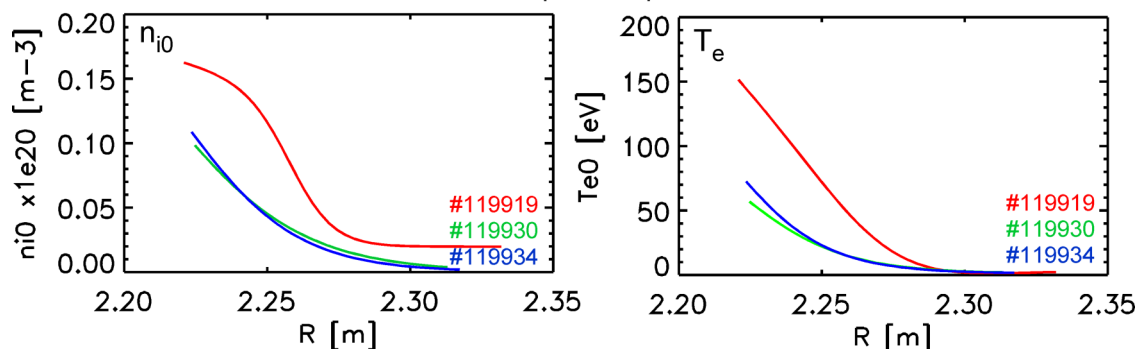
N_{i0} & T_{e0} profiles
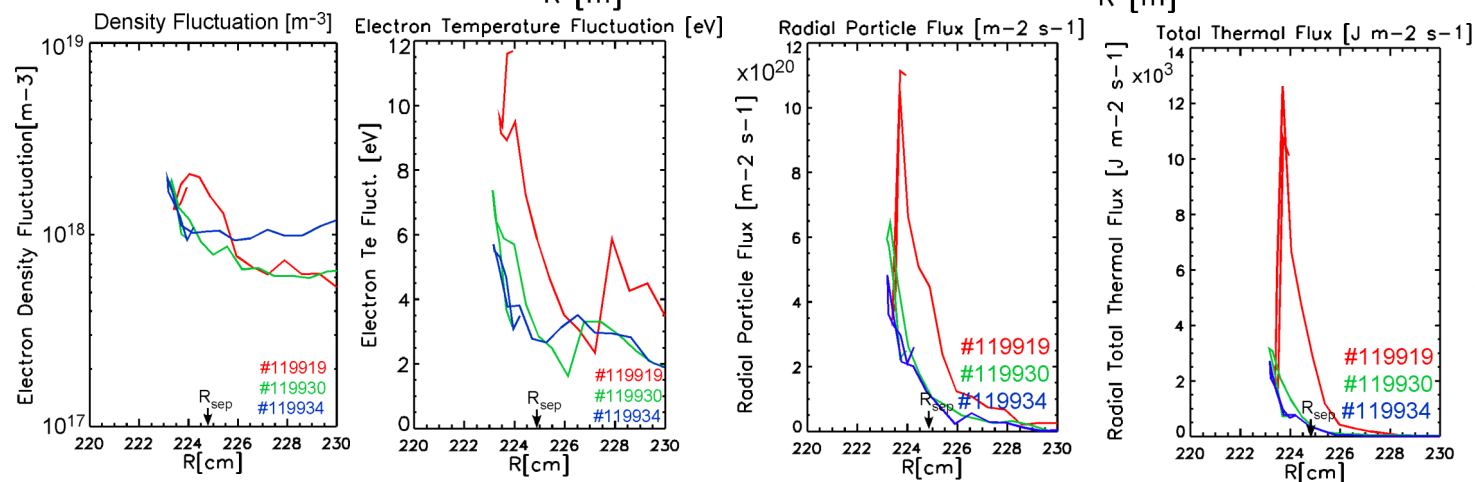
Probe data

Reduced N_{i0} & T_{e0} lead to **reduced fluctuation amplitudes and radial fluxes** in #119930 & 119934

@midplane

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BOUT Simulations of Shots #119919 and 119934 Show Reduced Turbulence with Lower Equilibrium Temperature and Density

- BOUT simulations of **Case 5** with no E_{radial} for shots # 119919 and 119934 show that the linear growth rate for the resistive-drift ballooning instability is reduced by ~20% in 119934, roughly consistent with inference from the linear dispersion relation for resistive ballooning

- The absolute values of the fluctuation amplitudes are reduced in #119934 accompanying the reduction in growth rate of the instability
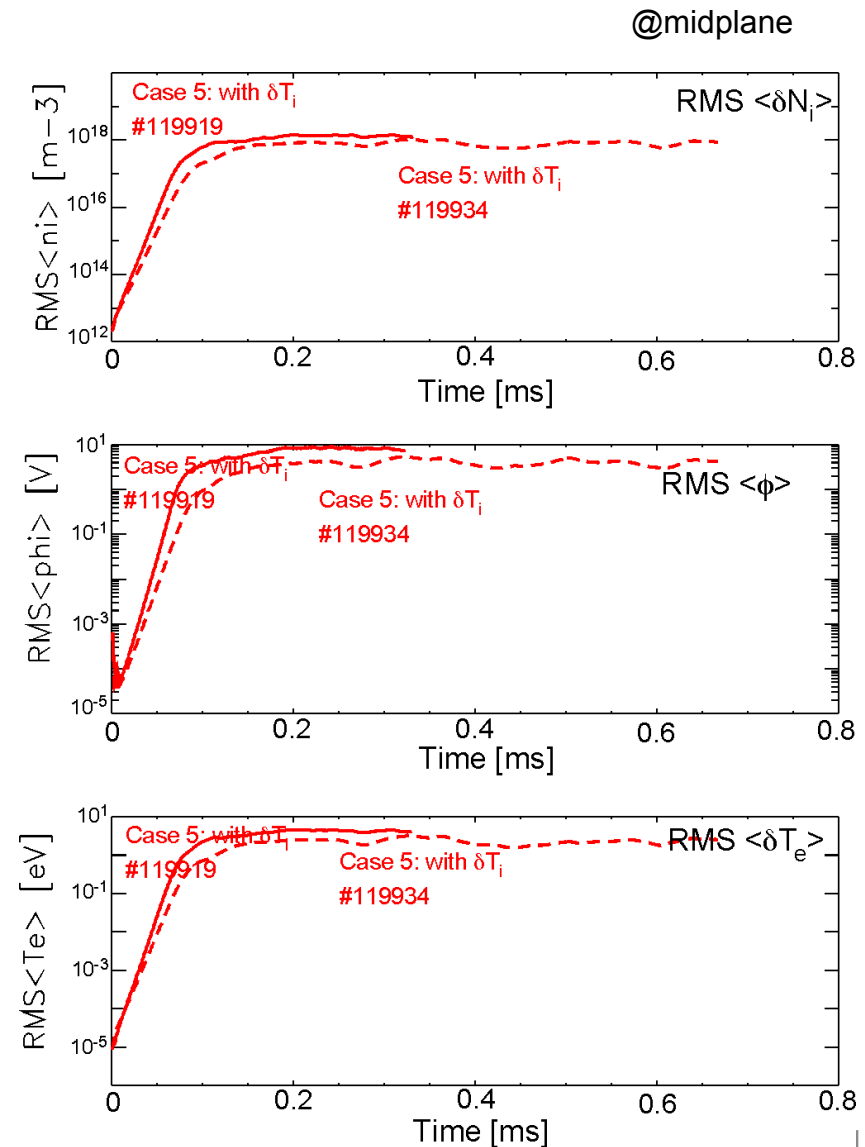
@midplane


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- Probe data from L-mode shot #119934 is compared with BOUT simulation including T_e and T_i fluctuations (**Case 5**). Turbulence in #119934 is reduced from that in #119919

- Density and temperature fluctuations, and radial fluxes tend to peak near the separatrix

- Saturated density and T_e fluctuations, and electron particle and total thermal fluxes agree with probe data within factors of ~2 for 2.23m ≤ R ≤ 2.29m

Case 5 #119934

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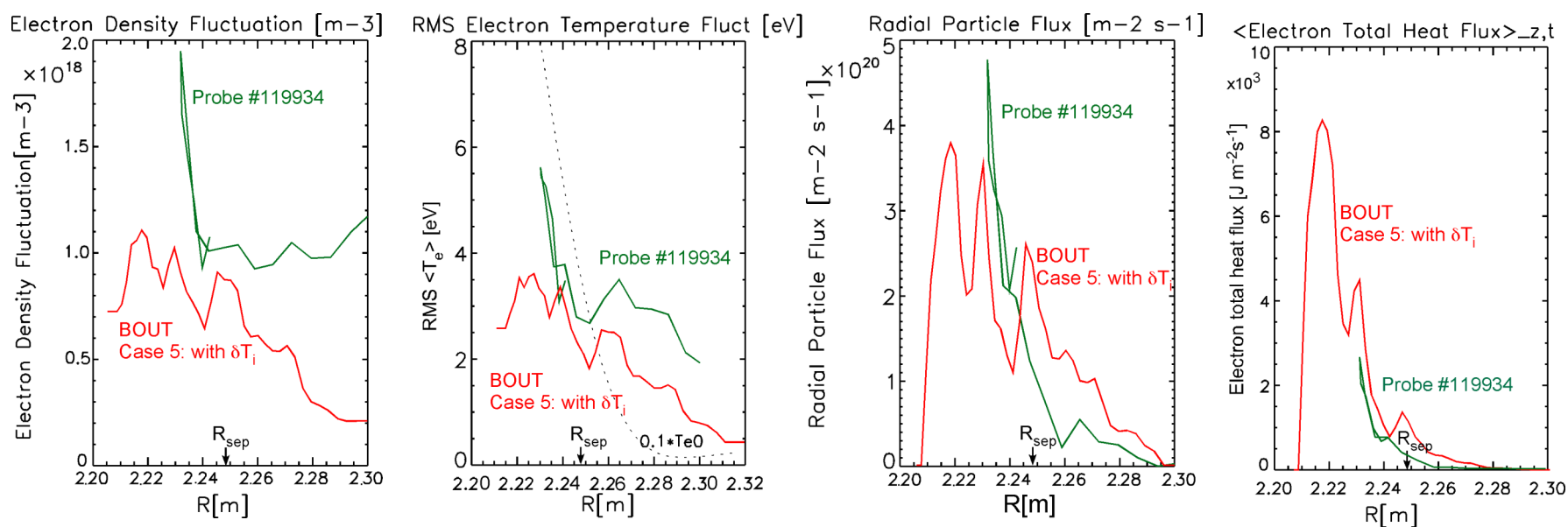
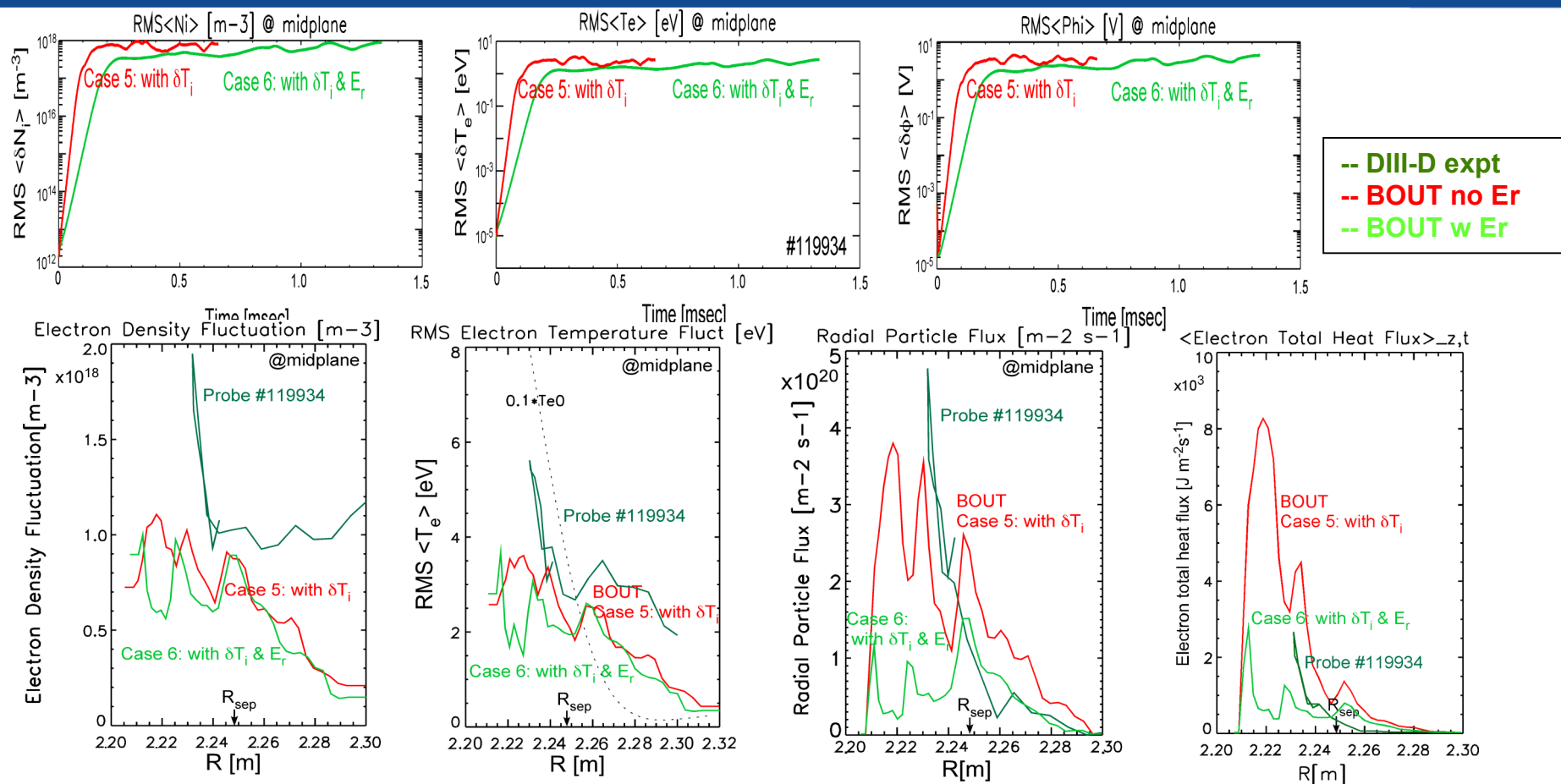# Radial Electric Fields Fitted to Probe and CER Data for Simulations of Shots #119919, 119930 and 119934

- The electric potential and radial electric field are well determined from the probe data in the SOL, but have a lot of structure inside the last close flux surface.

- The radial electric field determined by CER extends to smaller radii and suggests that E_r is relatively flat within significant temporal scatter.

- Our model preserves 1st and 2nd radial derivatives of ϕ across R_{sep}. Alternative models for E_r need to be studied. Self-consistent zonal flows will be included.
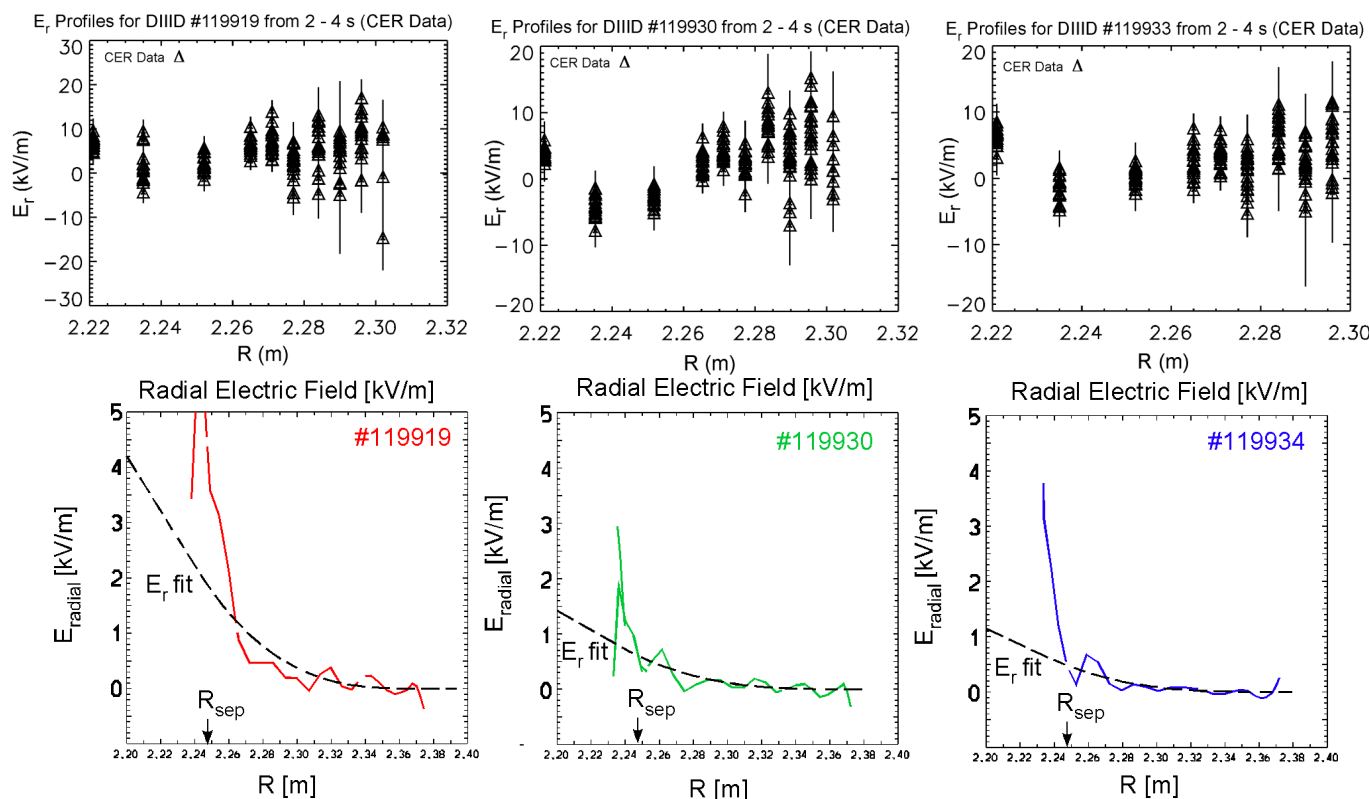
E_{radial} CER data

E_{radial} probe data & fit to probe/CER data

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- Inclusion of E_r reduces linear growth rates, but fluctuation amplitudes slowly rise to near those with $E_r=0$.
- Simulation agreement with probe for 2.23m ≤ R ≤ 2.29m remains reasonably good with/without E_r

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Summary: Relative Agreement in Comparison of BOUT Results with DIII-D Probe and BES Data on Shots #119919/..21/..34...

- Comparison of suite of BOUT simulations to shots #119919/119921 probe and BES data -- fluctuation frequency spectra, peak density amplitude radial half-width, correlation lengths, fluxes and diffusion rates are in reasonable agreement.

- RMS peak density and temperature fluctuation amplitudes measured with Langmuir probe agree (#119919 & 119934) within factors of 2 or less with simulations as the physics model improves. Observed radial particle diffusivities and thermal diffusivities in simulation are consistent with typical L-mode inferred values.

- Spatial filtering of synthetic simulation diagnostics is needed to model the 1 cm spatial resolution of the BES data. The spatial filtering spreads and reduces peaks in the raw data.

- There is factor-of-2 or better agreement seen between simulation synthetic diagnostics and the DIII-D #119921 BES data for the relative ion density and T_e fluctuation amplitudes, particle flux, spatial widths, and spectral frequency widths.

- Inclusion in simulation of radial E_r modeling expt introduces sheared ExB flow that reduces growth rates, and reduces saturation amplitudes in simulation of #19 but *not* in #34. More work is in progress to understand the effects of E_r and self-consistent zonal flows.

- Colder, lower density edge plasmas are less unstable and have smaller fluctuation levels.

- **Resistive drift ballooning modes are a reasonable candidate for L-mode edge plasma turbulence in DIII-D shots.**

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