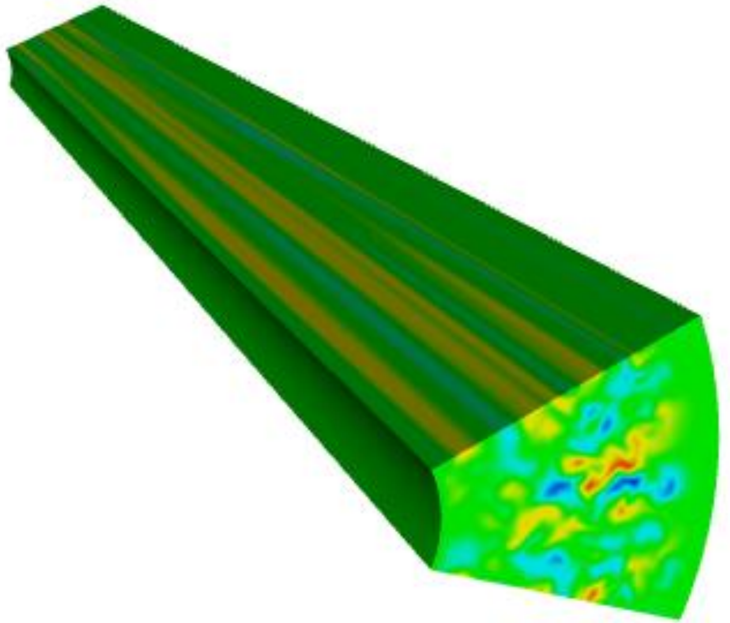
Energy Dynamics in a Simulation of LAPD Turbulence



Brett Friedman[1]

Collaborators: Troy Carter[1], Maxim Umansky[2], David Schaffner[1], Ilon Joseph[2], Ben Dudson[3]

[1]UCLA Department of Physics and Astronomy
[2]LLNL, [3]University of York

Summary

- A two fluid plasma model is used to simulate zero mean flow drift wave turbulence in the Large Plasma Device (LAPD). The code used is BOUT++. The simulated turbulence is qualitatively and quantitatively similar to the experimental turbulence.

- We use spectral energy dynamics to analyze the turbulent simulation. Spectral energy dynamics analysis shows where in wavenumber-space energy is injected, transported, and dissipated.

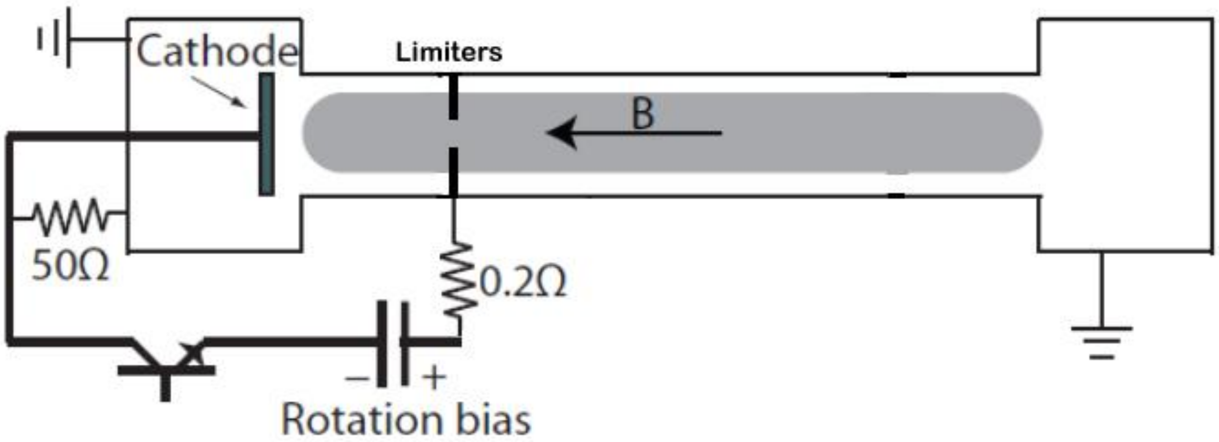
- We find that although a linear drift wave instability exists in the system, a nonlinear instability provides the dominant turbulent drive mechanism. The nonlinear instability relies upon axial wavenumber transfer between finite and infinite wavelength modes.

- The nonlinear instability is robust to axial periodic and conducting plate boundary conditions.

A Plasma Fluid Model is Suitable for Long-Wavelength Turbulence in LAPD

Machine and plasma size:
Plasma column length $\sim 17\ m$
Plasma radius $\sim 30\ cm$

Typical LAPD operational parameters:
$0.4 < B_0 < 2\ kG$
$10^{11} < n_e < 4 \times 10^{12}\ cm^{-3}$
$1 < T_e < 10\ eV$
$0.5 < T_i < 1.5\ eV$
$\omega_{ci} \sim 1\ MHz$
$\nu_{in} \sim 2\ KHz$
$\nu_{ei} \sim 5\ MHz$
$\frac{\omega}{k_\parallel} \leq v_{the}$



$\lambda_{ei}/L_\parallel \sim \omega^*/\nu_e \sim 0.01$ (parallel kinetics)

$k_\perp \rho_i \sim 0.1$
$\nu_i/\omega_{ci} \sim 1$ (perpendicular kinetics)

$\beta \sim 10^{-4}$ (electromagnetic)

Electrostatic fluid justifications

Partially Linearized Model Equations Solved in BOUT++

Continuity equation

$$\frac{\partial N}{\partial t} = -\mathbf{v}_E \cdot \nabla N_0 - N_0 \nabla_\| v_{\|e} + \mu_N \nabla_\perp^2 N + S_N + \{\phi, N\}$$

Parallel electron force balance

$$\frac{\partial v_{\|e}}{\partial t} = -\frac{m_i}{m_e}\frac{T_{e0}}{N_0}\nabla_\| N - 1.71\frac{m_i}{m_e}\nabla_\| T_e + \frac{m_i}{m_e}\nabla_\| \phi - \nu_e v_{\|e} + \{\phi, v_{\|e}\}$$

Energy evolution equation

$$\frac{\partial T_e}{\partial t} = -\mathbf{v}_E \cdot \nabla T_{e0} - 1.71\tfrac{2}{3}T_{e0}\nabla_\| v_{\|e} + \frac{2}{3N_0}\kappa_{\|e}\nabla_\|^2 T_e - \frac{2m_e}{m_i}\nu_e T_e + \mu_T \nabla_\perp^2 T_e + S_T$$
$$+\{\phi, T_e\}$$

Charge conservation / Vorticity equation

$$\frac{\partial \varpi}{\partial t} = -N_0 \nabla_\| v_{\|e} - \nu_{in}\varpi + \mu_\phi \nabla_\perp^2 \varpi + \{\phi, \varpi\}$$

$$\varpi = \nabla_\perp \cdot (N_0 \nabla_\perp \phi)$$

Previous published verification and validation studies:

P. Popovich, M. V. Umansky, T. A. Carter, and B. Friedman, Phys. Plasmas 17, 102107 (2010).

P. Popovich, M. V. Umansky, T. A. Carter, and B. Friedman, Phys. Plasmas 17, 122312 (2010).

M. V. Umansky, P. Popovich, T.A. Carter, et al, Phys. Plasmas 18, 055709 (2011).

B. Friedman, M. V. Umansky, and T. A. Carter, Contrib. Plasma Phys. 52, 412 (2012).

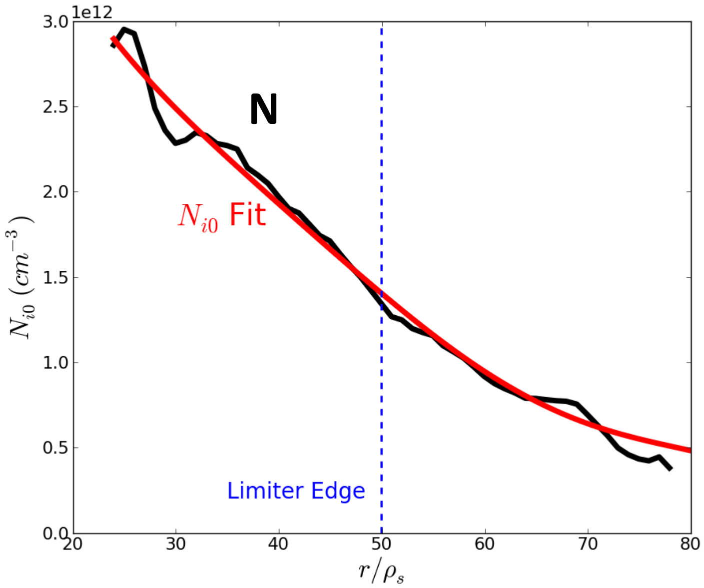
Experimental Profiles Used in Simulation

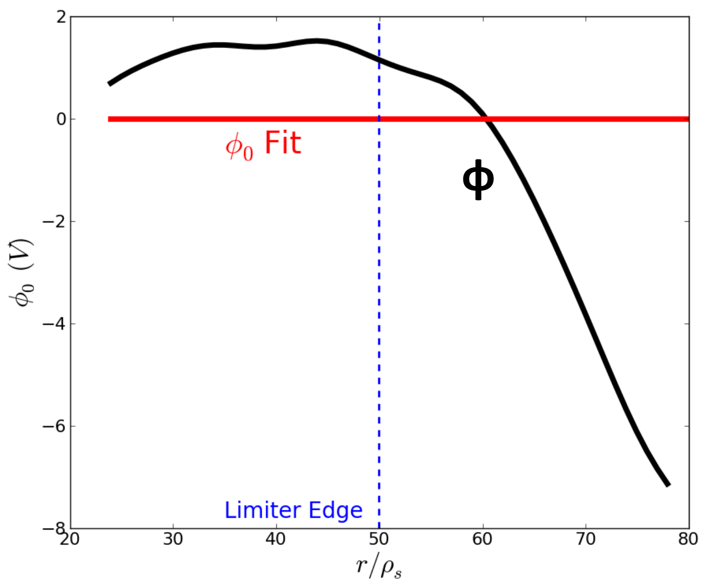
- **Density equilibrium profile fit to experiment**
- **Density source – suppresses m=0 density fluctuation component**

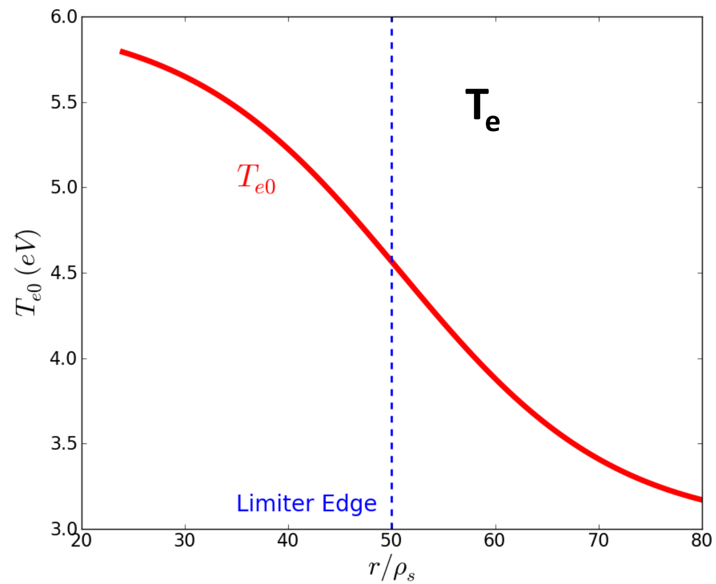
- **T_e is a typical looking tanh fit**
- **T_i << T_e**

- **Zero mean potential profile (min flow biasing)**
  Only linear drift waves supported
- **Zonal flows evolved**

- **Periodic axial BC, Zero-value radial BC**

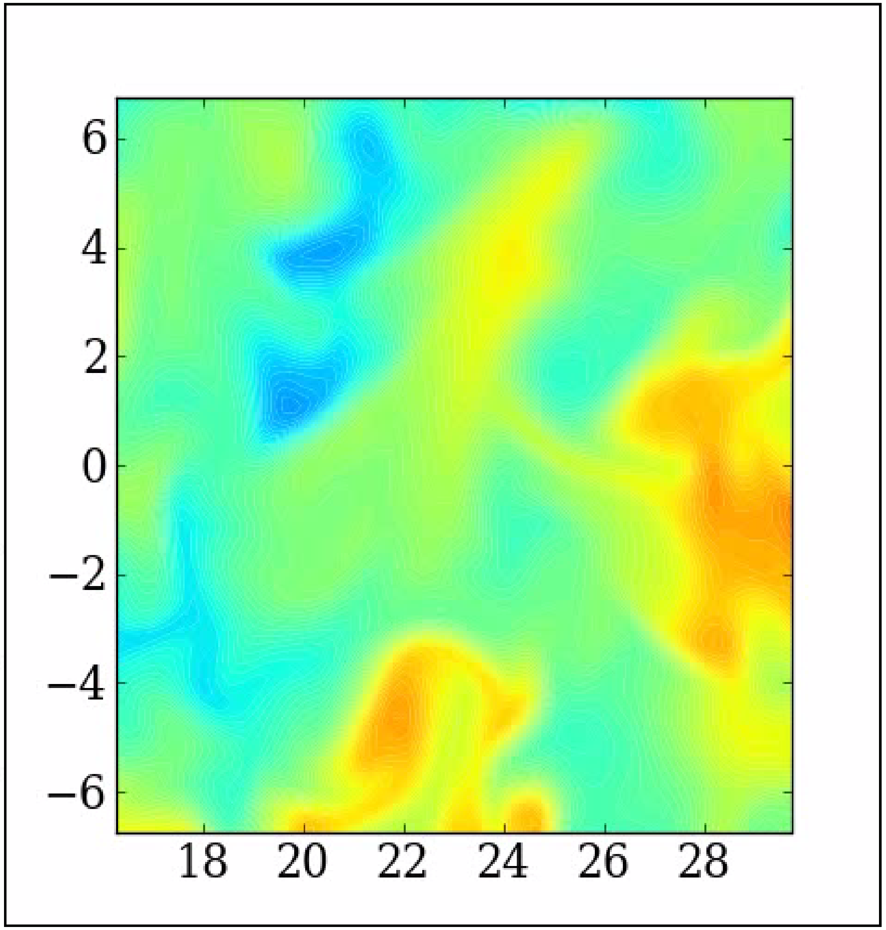




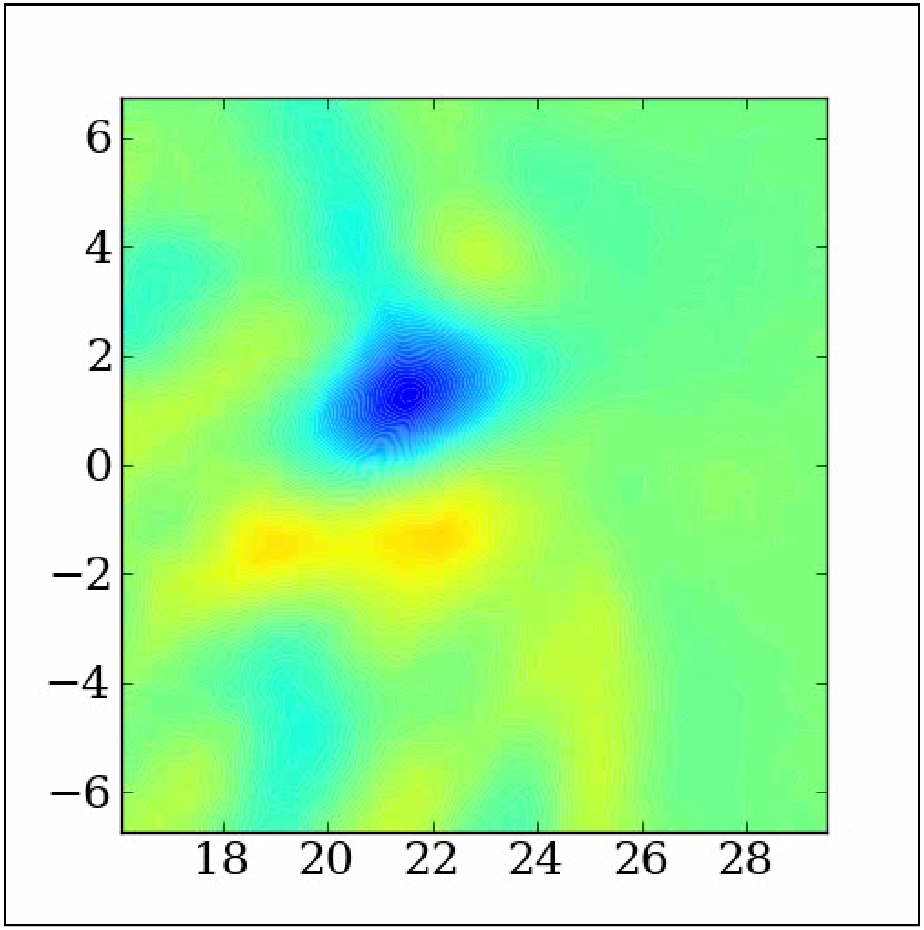


Simulation vs Experimental Camera Turbulence

Simulation (Density Fluctuations)



Experiment (Visible Light Fluctuations)

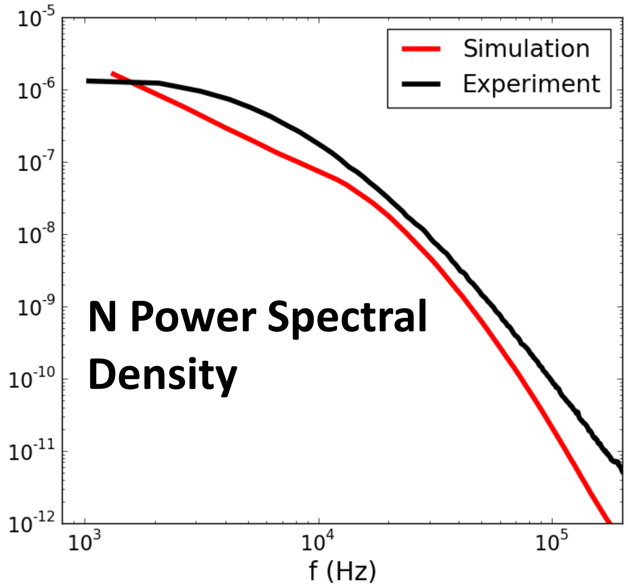


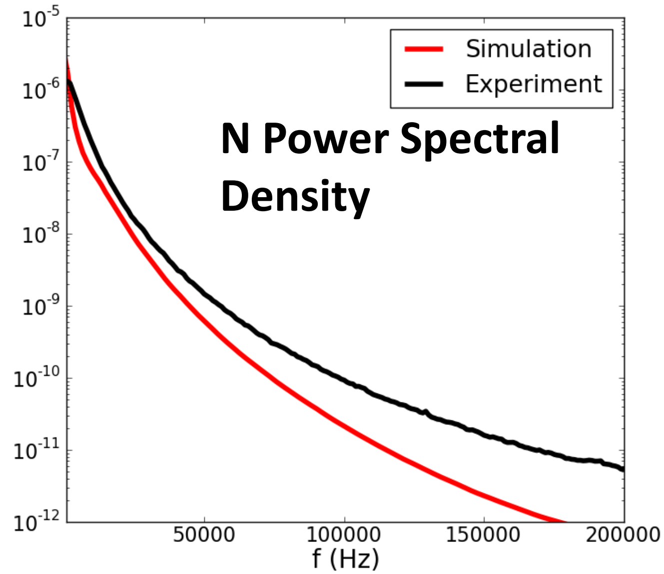
XY plane in cm

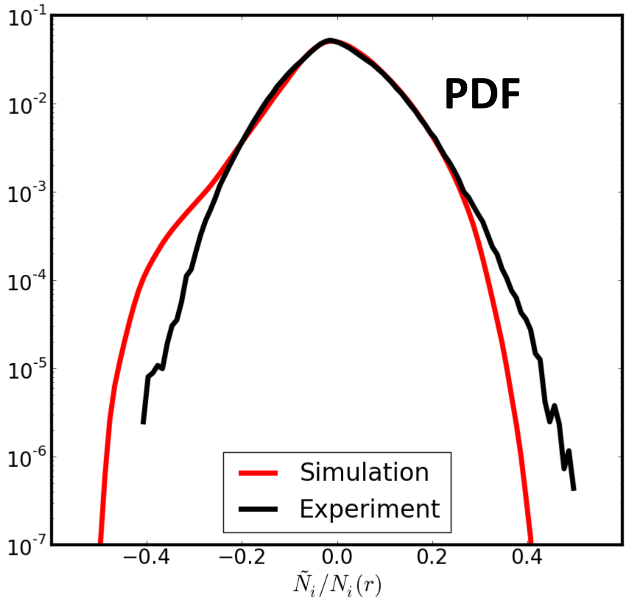
Red = positive fluctuations

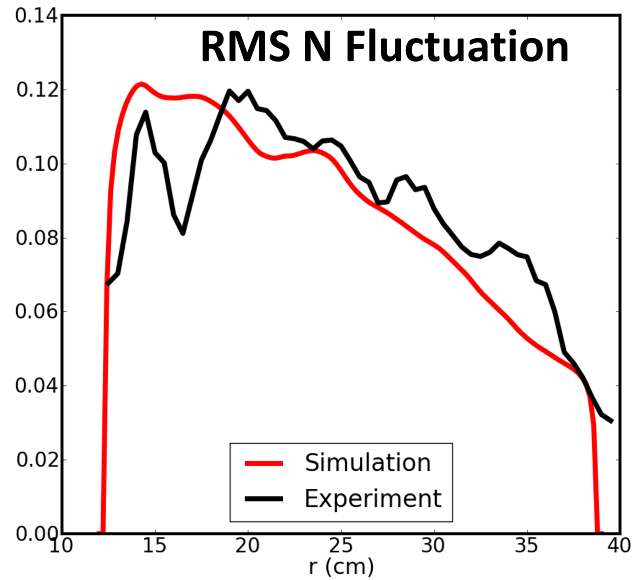
Blue = negative fluctuations

BOUT++ and LAPD Experimental Density Fluctuations Have Remarkably Similar Statistical Properties

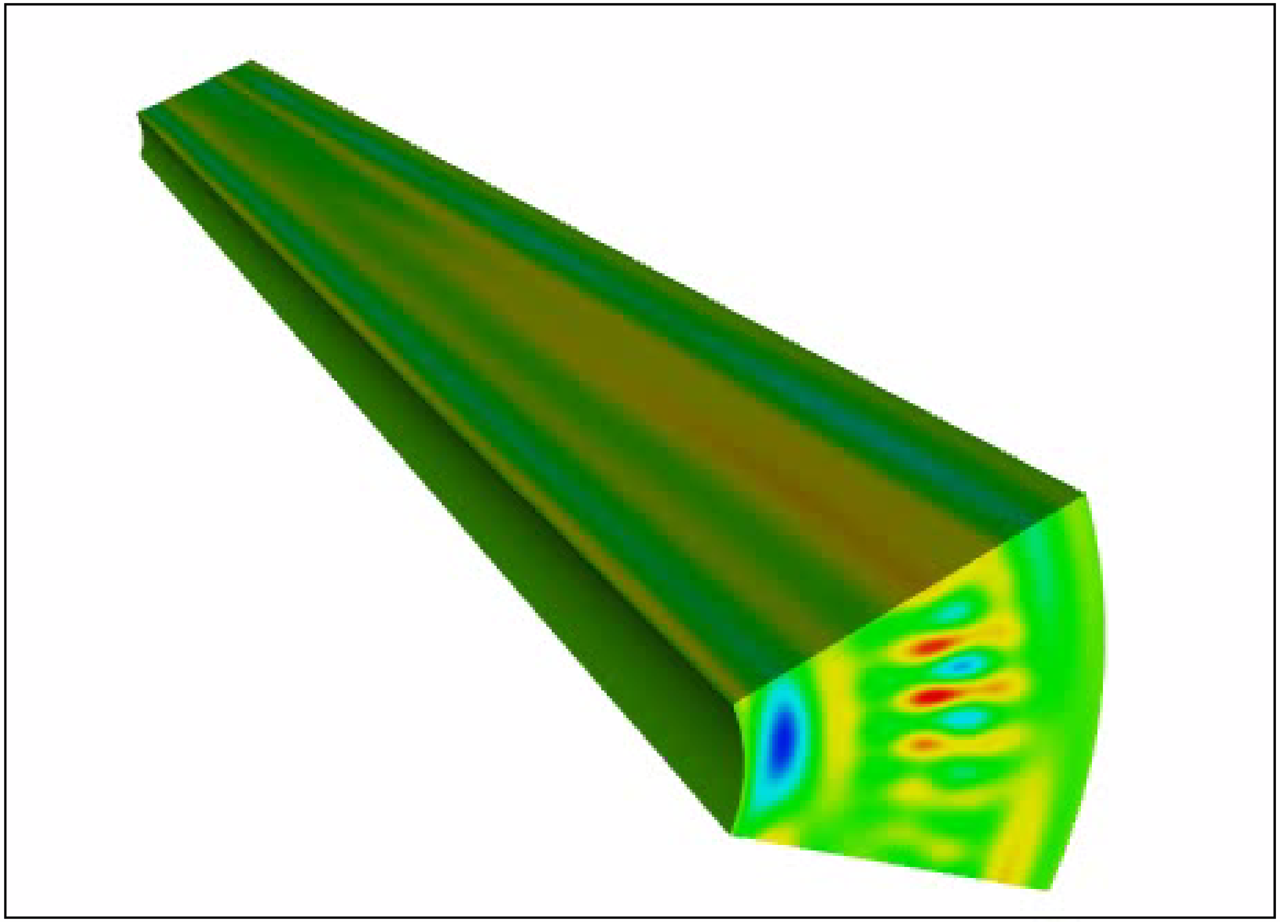








The Simulation Shows an Apparent Dominance of $k_{||} = 0$ Structures in the Saturated State



- **Note transition from linear drift waves with finite $k_{||}$ to $k_{||} = 0$ structures in the nonlinear stage**

Possible causes:

- **Primary Flute Instability**
- **Secondary/Tertiary Instability (Three-wave Transfer)**
- **Nonlinear Instability**

How Does Energy Get Into $k_{||}=0$ Structures? An Energy Dynamics Analysis Reveals the Answer

Total Fluctuation Energy - B. Friedman et al, Phys. Plasmas 19, 102307, 2012

$$E = \sum_k E_{tot}(k) = \frac{1}{2}\left\langle N^2 + \frac{3}{2}T_e^2 + \frac{m_e}{m_i}v_{\parallel e}^2 + N_0(\nabla_\perp \phi)^2 \right\rangle$$

Dynamical Energy Equation Forms

$$\frac{\partial E(k)}{\partial t} = Q(k) + C(k) + D(k) + \sum_{k'} T(k,k')$$

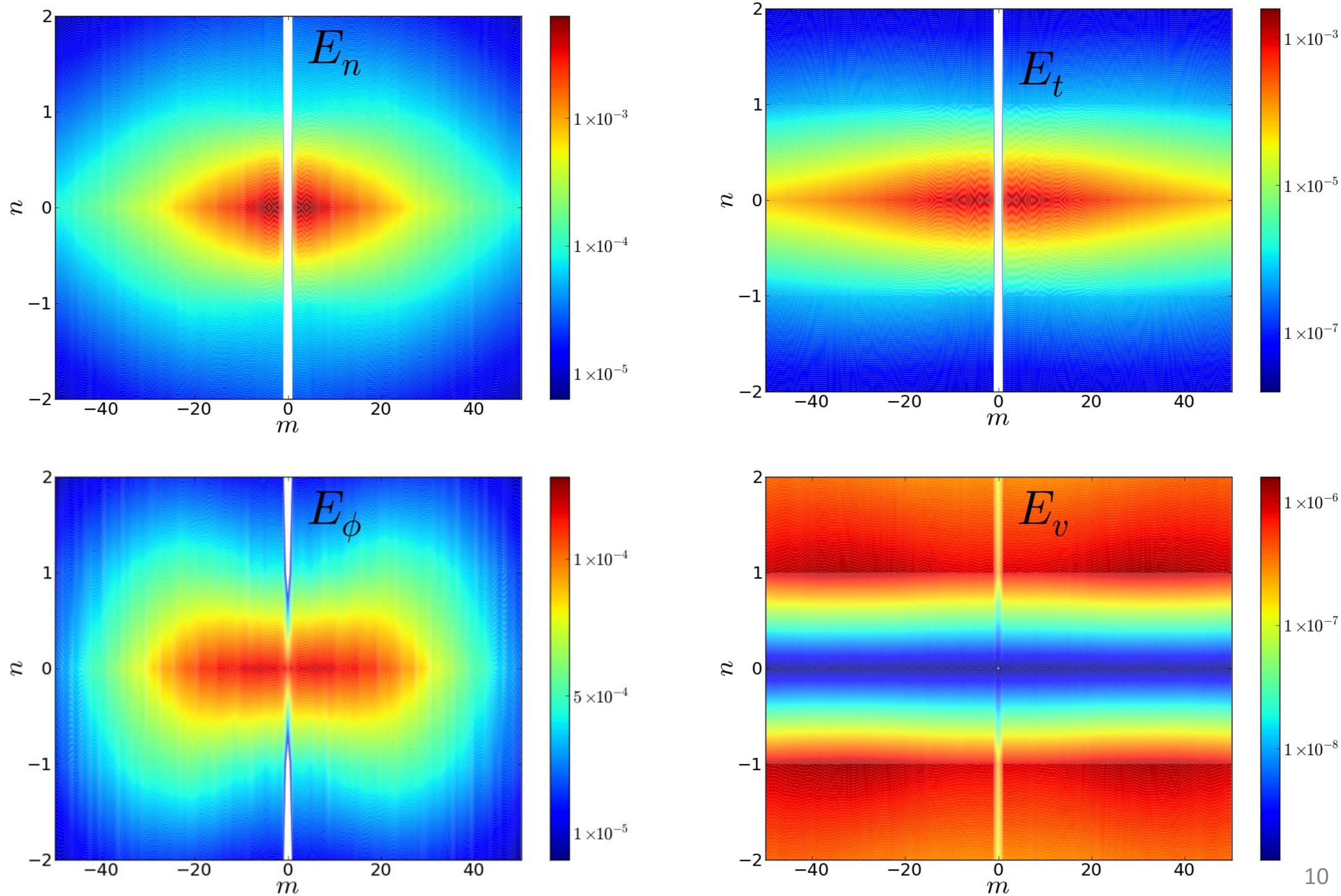
Free Energy Sources	Axial compression transfer channel	Dissipation	Three-wave transfers

Advective Nonlinearities Conserve Total Fluctuation Energy:

$$\int_V f\{f,g\}dV = 0 \quad \rightarrow \quad \sum_{k,k'} T_f(k,k') = 0$$

- **Most finite difference schemes do not conserve this property for finite grid spacing**
  - **use Arakawa advection scheme that conserves energy for any grid spacing**
- **Energy sources and dissipation do not conserve fluctuation energy**
  - **exchange energy with background gradients**

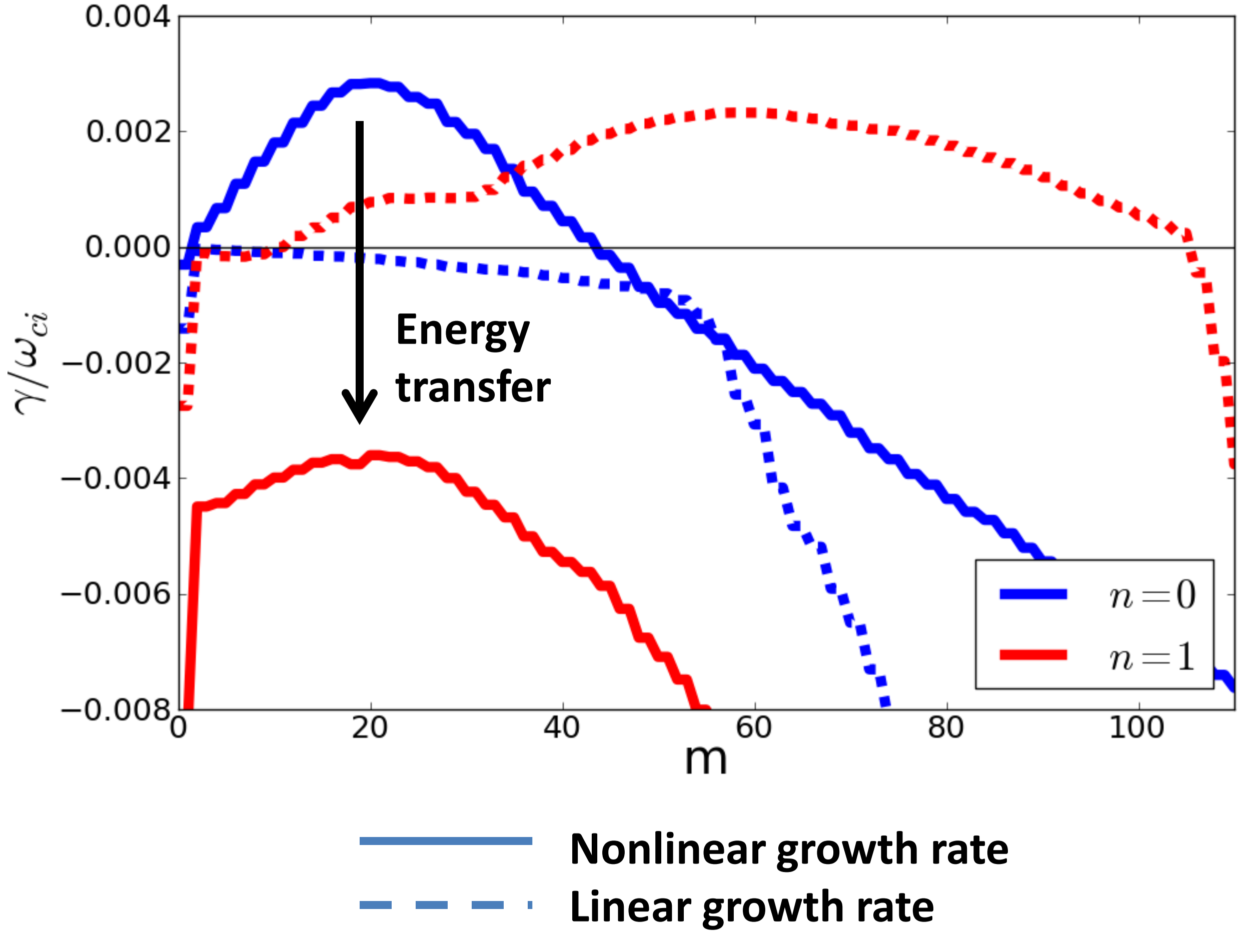
Most of the Energy Clusters in n=0 ($k_{||}$=0) Flute Structures Despite Linear Drift Wave Instability Indications



The "Nonlinear" Growth Rate is Positive for n=0 and Negative for n=1, Opposite of the Linear Growth Rate

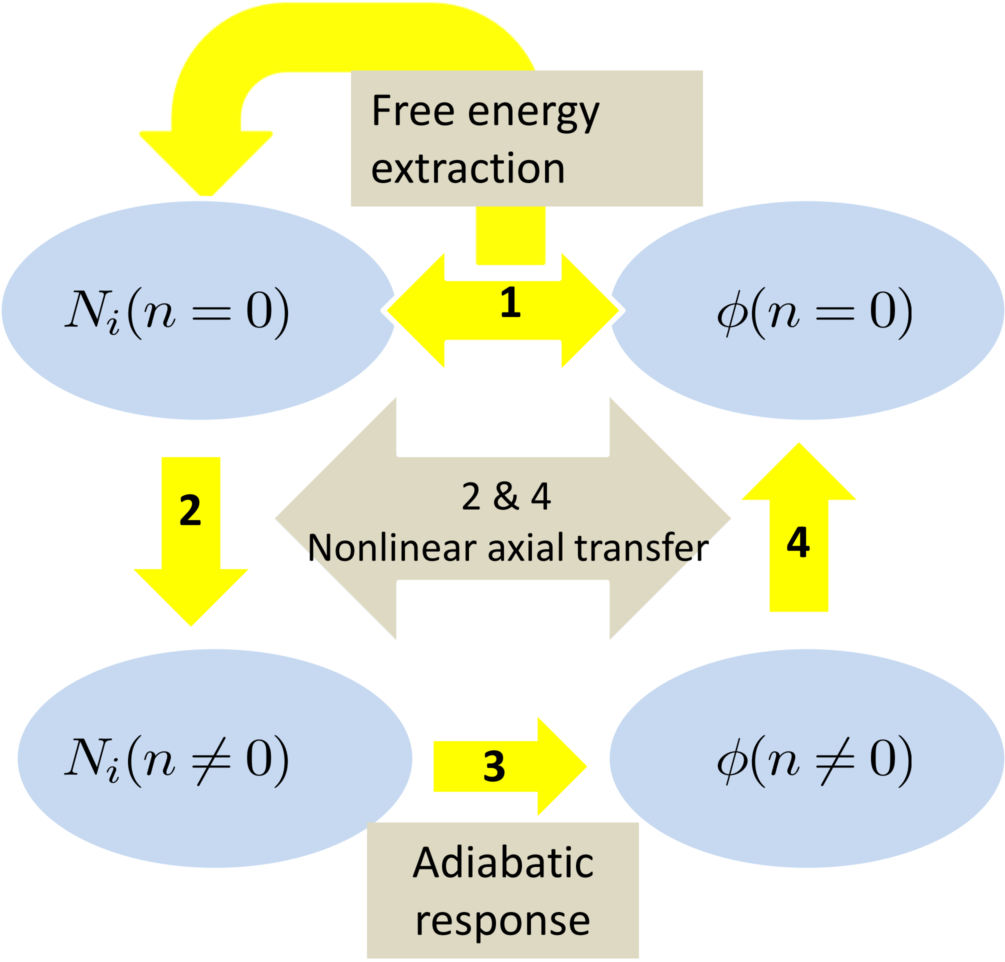
$$\gamma(\mathbf{k}) = \frac{\partial \mathbf{E_{tot}}(\mathbf{k})/\partial \mathbf{t}}{2\mathbf{E_{tot}}(\mathbf{k})}$$

(Neglecting three-wave contributions)



Notice the difference in m-space between the linear and nonlinear growth rates

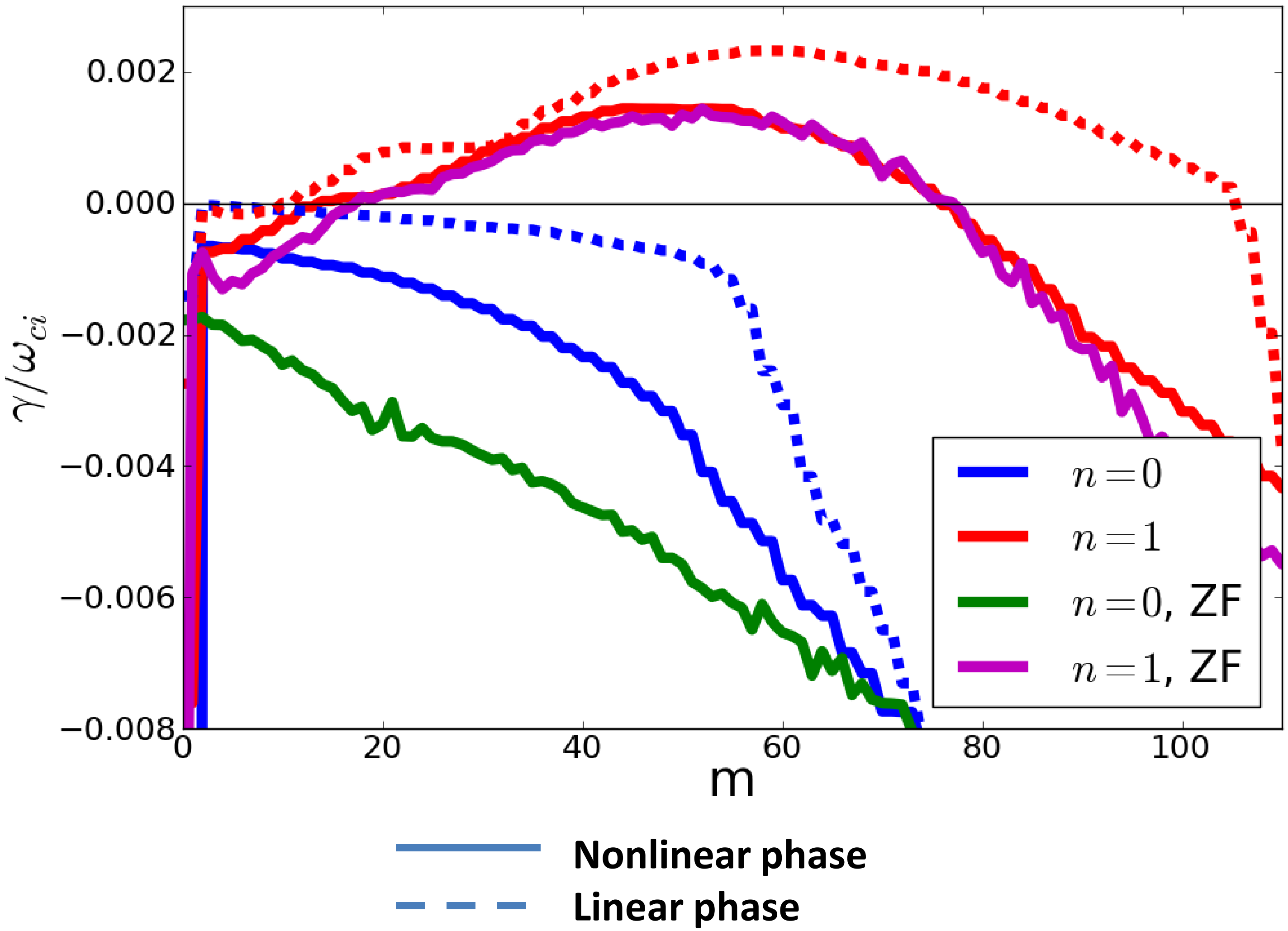
Dominant Mechanism is a Nonlinear Instability Driven by n=0 Density and Potential Fluctuations with Nonlinear Transfer Providing Access to the Adiabatic Response



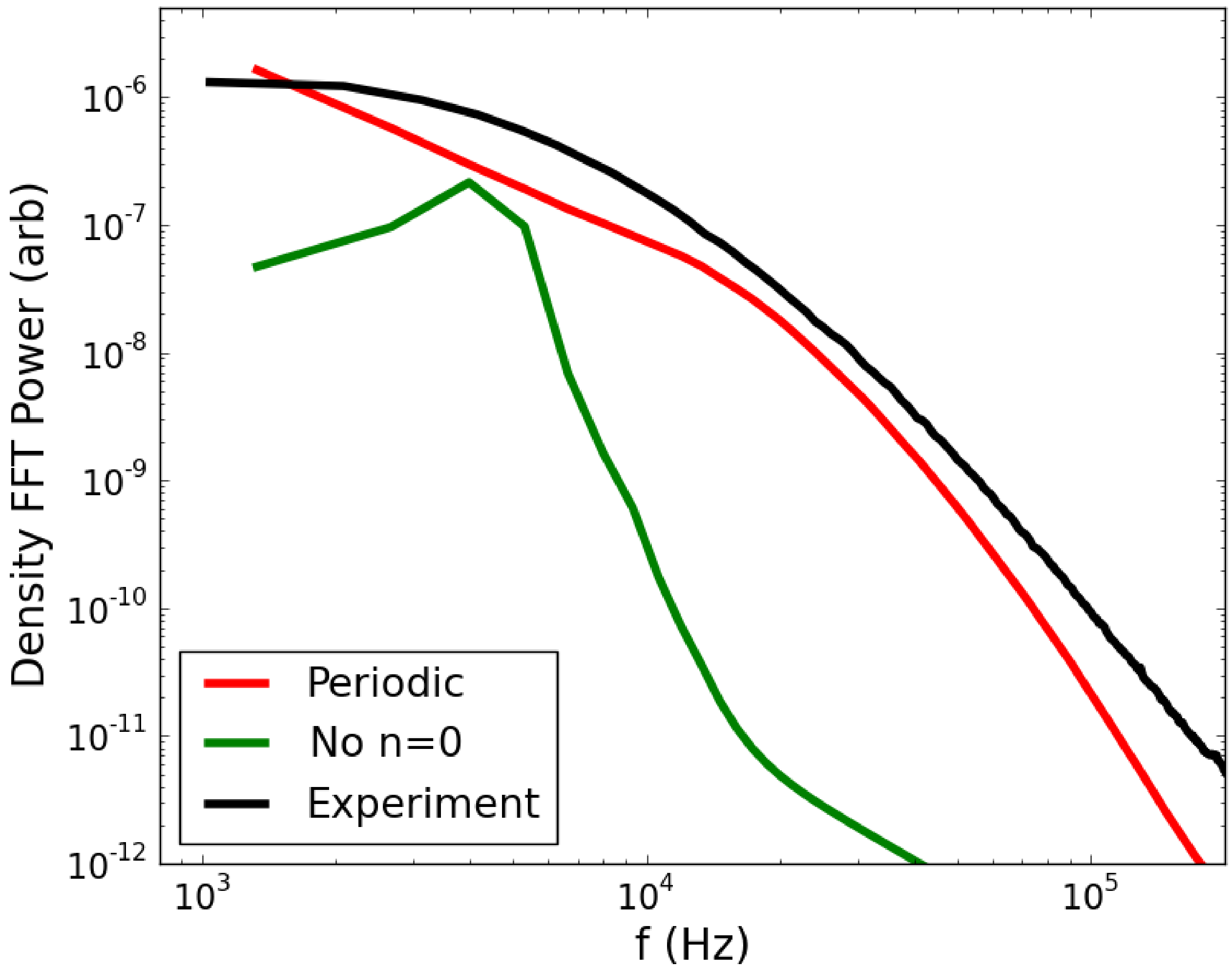
<u>Nonlinear instabilities in Plasma Physics</u>

- **Waltz 1985**
- **Scott 1990, 1991, 1992, 2002, 2003, 2005**
- **Drake 1995**
- **Biskamp 1995**
- **Zeiler 1996, 1997**
- **Korsholm 1999**
- **Dimits 2000**
- **Baver 2002**
- **Ernst 2004**
- **Highcock 2012**

The Removal of n=0 Modes Causes the Linear Instability to Dominate the Dynamics

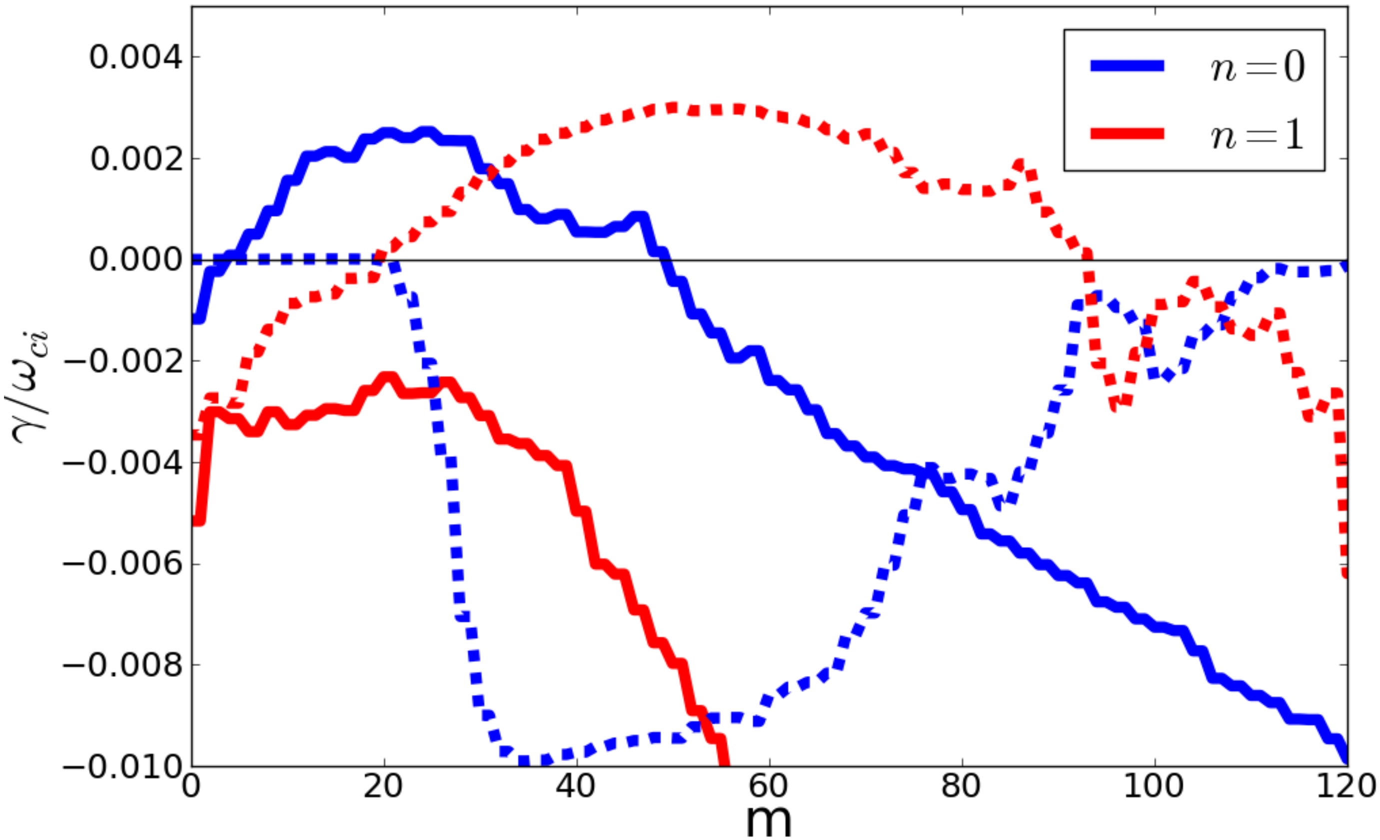


The Removal of n=0 Modes Causes an Experimentally Inconsistent Peak in the Frequency Spectrum



The Nonlinear Instability is Robust to Conducting Wall (Sheath) Boundary Conditions on the Axial Machine Ends

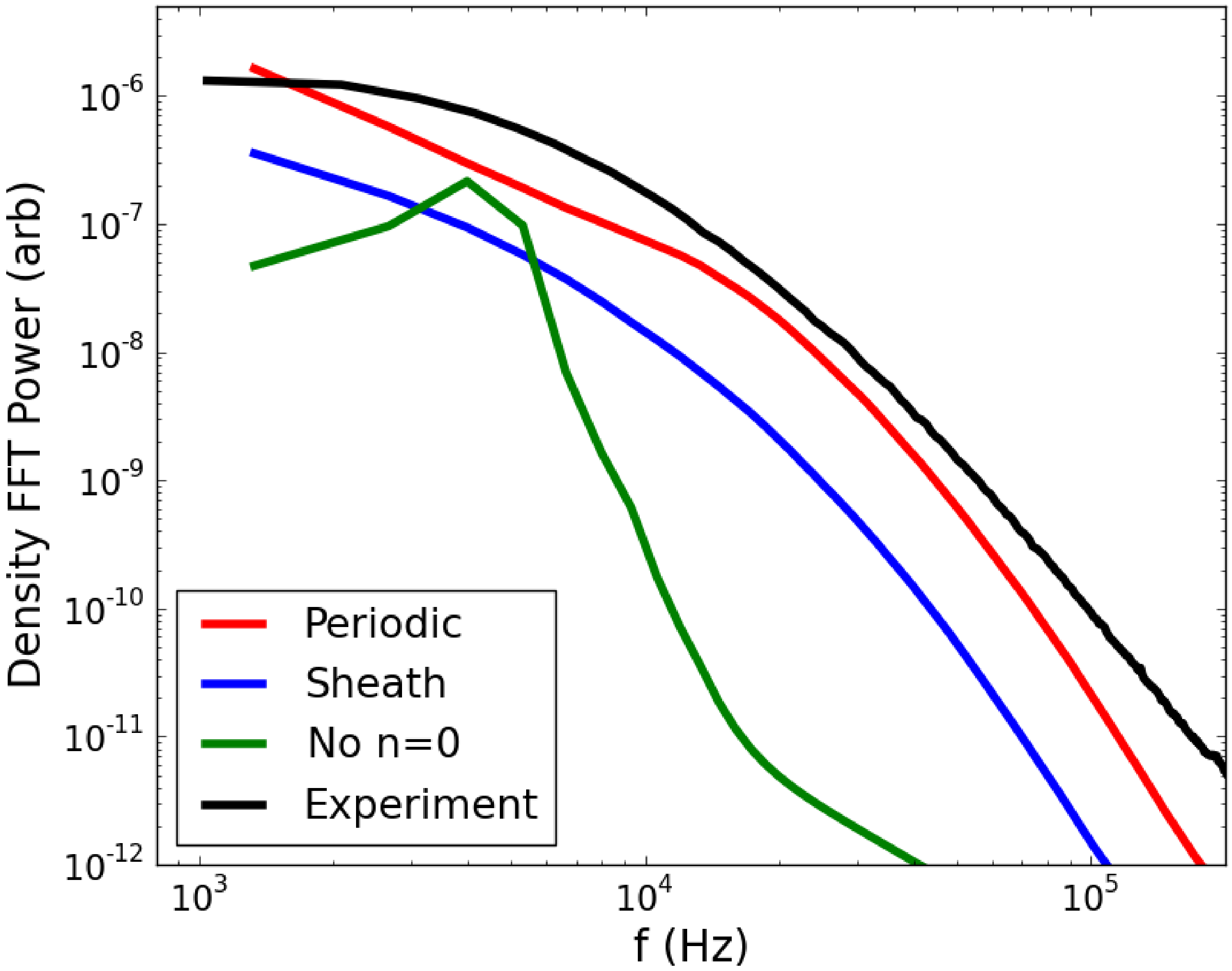
$$j_{\|} = \pm e N_o C_s \left(\phi + \log \sqrt{\frac{4\pi m_e}{m_i}} T_e \right)$$



Nonlinear growth rate

Linear growth rate

Sheath Boundary Conditions Maintain the Shape of the Frequency Spectrum Consistent with Experiment But Suppress the Total Energy Somewhat



Conclusion

- **The drift wave turbulence produced in a simulation of a low-flow LAPD scenario has statistical properties that are qualitatively and quantitatively similar to those of the experimental turbulence.**

- **An energy dynamics analysis reveals that a nonlinear instability provides the dominant turbulent drive mechanism. The instability preferentially drives n=0 structures, but relies upon the nonlinearities and n=1 structures to access the adiabatic response.**

- **The nonlinear instability is robust to axial periodic and sheath boundary conditions.**

- **Nonlinear instability can be relevant in other plasmas (like the tokamak edge) and assumptions based on linear instability analysis can be misleading when nonlinear instabilities are present.**

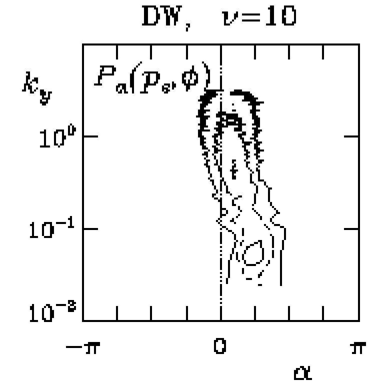
Backup Slides

Nonlinear Instabilities Can Overwhelm Linear Instabilities, Affecting Turbulent Characteristics

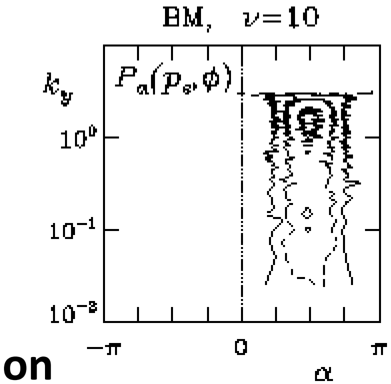
$$\alpha_{p\phi}(l) = \mathrm{Im}\ \ln(\tilde{p}^*_{e_l}\tilde{\phi}_l)$$

Cross phase - measure of the instability drive and transport.

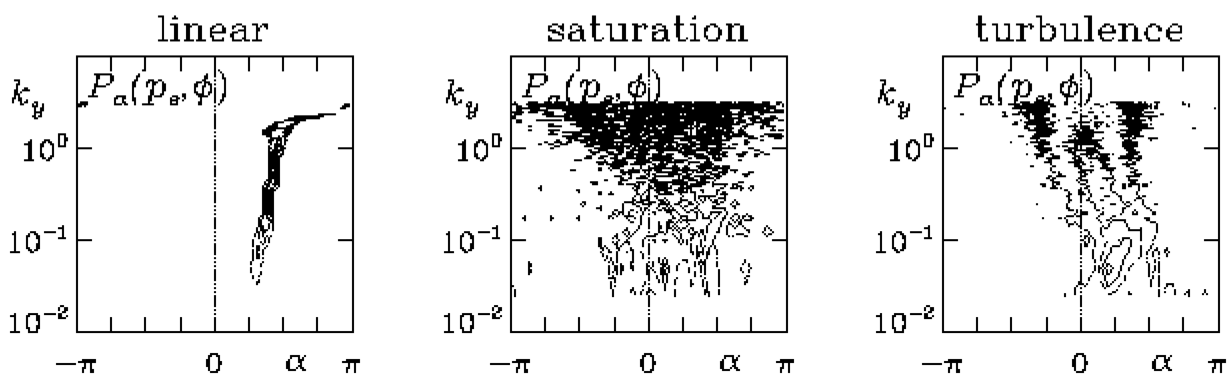
Pure drift wave turbulence



Pure ballooning mode turbulence

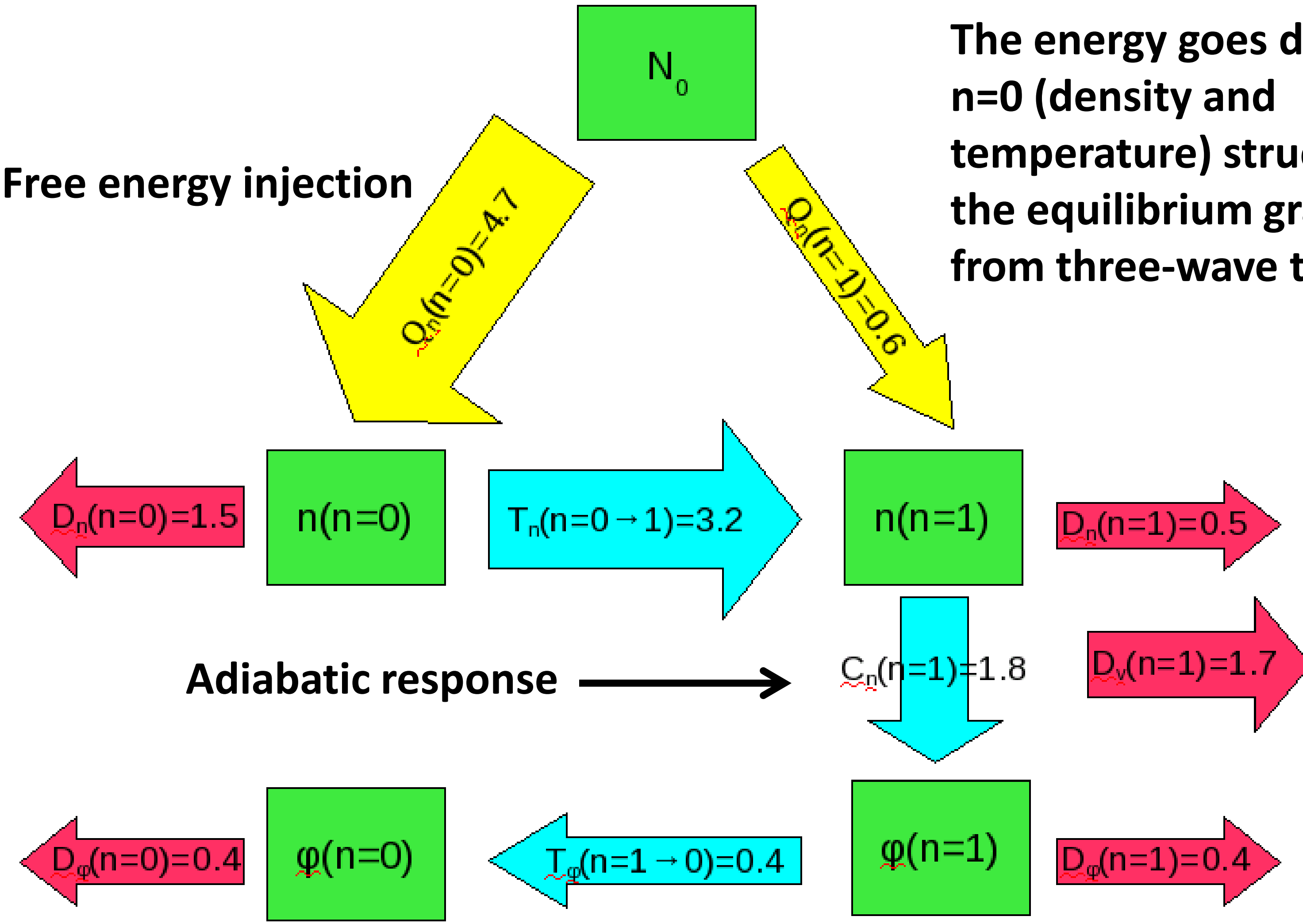


Drift-ballooning physics starting from small perturbation



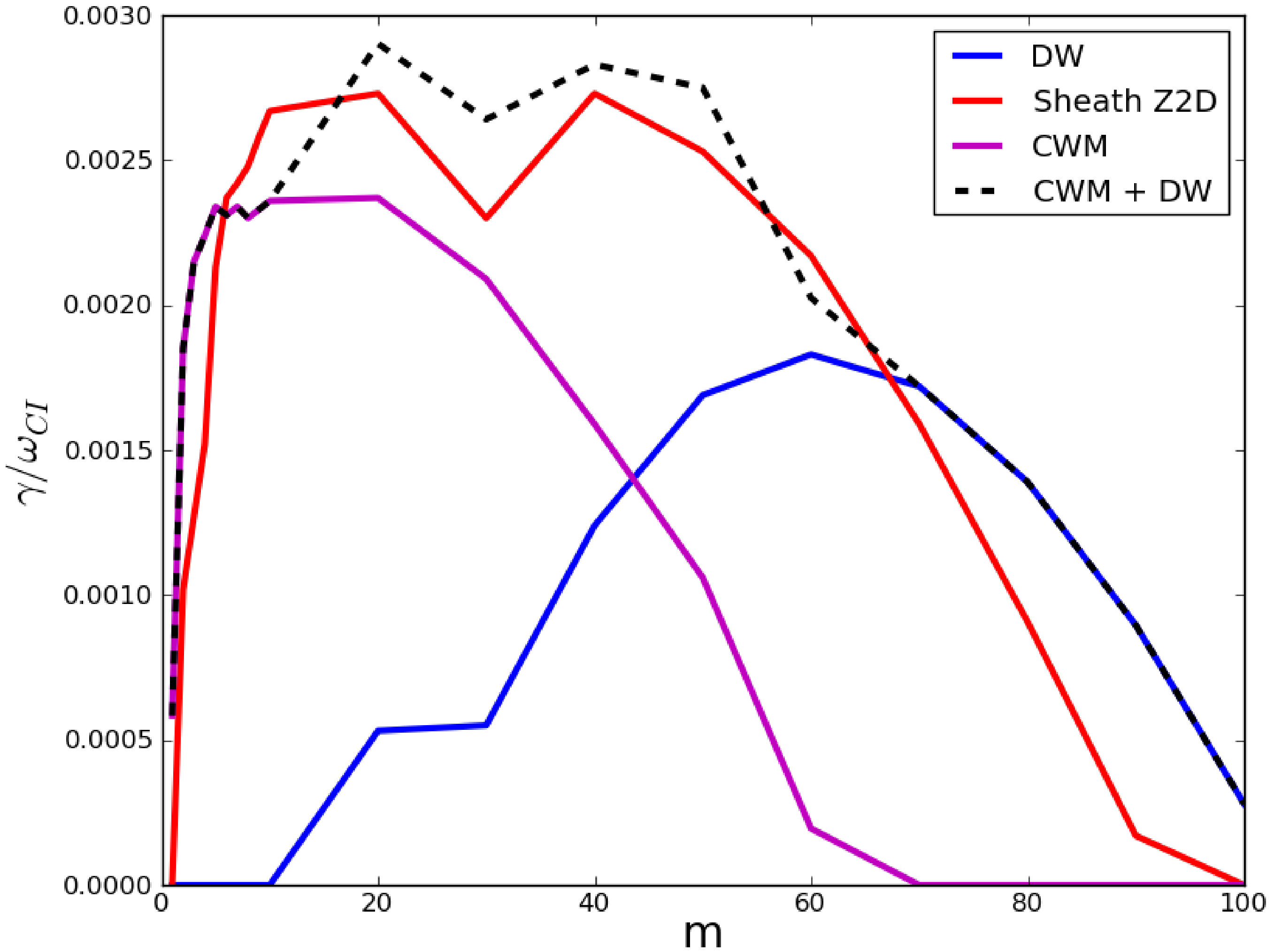
B.D. Scott (2005)

Energy Dynamics Overview Reveals the Direction of Three-Wave Transfer For Density Fluctuations from n=0 to n=1

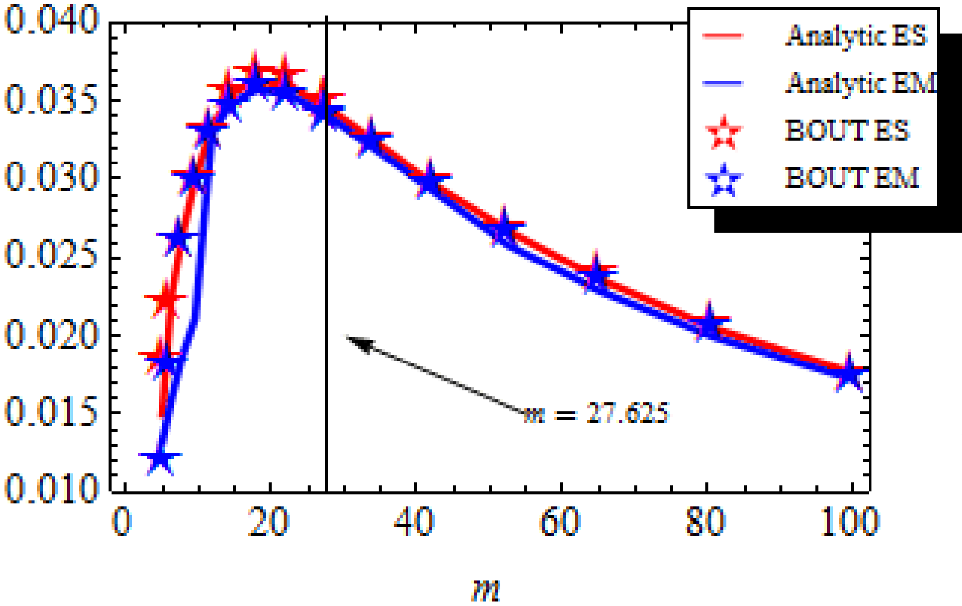


The energy goes directly into n=0 (density and temperature) structures from the equilibrium gradients, not from three-wave transfer!

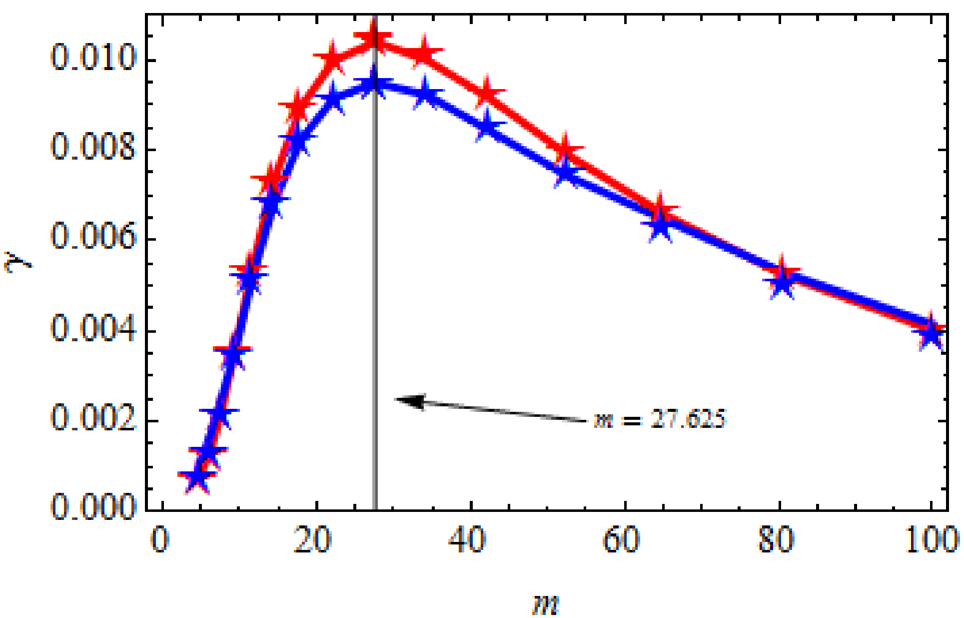
Linear Growth Rates of Drift Waves, the Conducting Wall Mode, and the Full Sheath Model



Electromagnetics Has Minimal Effect on Linear Physics



$$n_0 = 2.5 \times 10^{12} cm^{-3}$$
$$T_e = 5eV$$
$$B_0 = 400G$$
$$\phi_0 = 0V$$



BOUT solutions agree well with analytic solutions

Peak mode number predictions consistent with LAPD experiment