Mapping Alcator C-Mod’s EDA H-Mode Stability Boundaries with BOUT++

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Outline

• Motivation
  – Enhanced $D_\alpha$ (EDA) H-Mode
  – Peeling-Ballooning (PB) modes
• BOUT++ framework and capabilities
• BOUT++ computed linear stability boundaries
  – Ideal
  – Diamagnetic (FLR) effects
  – Resistivity
• Initial nonlinear simulations
• Conclusions and future work
Motivation

• The Enhanced D\(\alpha\) (EDA) H-mode\(^{[1,2]}\) exhibits:
  – Excellent energy confinement
  – Reduced impurity confinement
• EDA pedestal regulated by a quasi-coherent mode (QCM) oscillation \(\sim\)100 kHz
• \(\nu^* > 1\)

Peeling-Ballooning (PB) modes believed to constrain ELMy pedestals

- Peeling and ballooning modes *couple* at intermediate $5 \leq n \leq 25$ to drive ELMs
  - Ideal MHD instability
- Experimentally, ELMs are routinely observed when crossing the PB threshold
- Codes such as ELITE\textsuperscript{[3]} can assess stability to PB modes

ELITE calculations indicate ideal PB modes do **not** constrain EDA pedestals.
Nonideal and nonlinear physics can be investigated with the BOUT++ code\textsuperscript{[4]}

- 3D, nonlinear, initial value \textit{fluid} code in a general geometry
  - Realistic X-point geometry
- Nonideal affects (resistivity, FLR, etc) can easily be implemented
- MPI parallelization allows ideal strong scaling to \~10,000 cores

The nonideal reduced MHD equations\cite{Hazeltine2003} solved in BOUT++

\[
\frac{\partial \mathbf{\omega}}{\partial t} + \mathbf{v}_E \cdot \nabla \mathbf{\omega} = B_0^2 \nabla \times \left( \frac{j_\parallel}{B_0} \right) + 2b_0 \times \kappa \cdot \nabla p,
\]

\[
\frac{\partial P}{\partial t} + \mathbf{v}_E \cdot \nabla P = 0,
\]

\[
\frac{\partial A_\parallel}{\partial t} = -\nabla \times (\varphi + \Phi_0) + \frac{\eta}{\mu_0} \nabla^2 A_\parallel,
\]

\[
\mathbf{E}_r = \frac{1}{N_i Z_i e} \nabla \perp P_{i0}
\]

\[
\mathbf{E} = \nabla \varphi + \frac{1}{n_0 Z_i e} \nabla^2 P_i,
\]

\[
\mathbf{P} = P_0 + p
\]

\[
\mathbf{j}_\parallel = J_{\parallel 0} - \frac{1}{\mu_0} \nabla_\perp A_\parallel,
\]

\[\mathbf{v}_E = \frac{1}{B_0} b_0 \times \nabla (\varphi + \Phi_0)\]

Non-ideal physics:
- Include resistivity
- After gyroviscous cancellation, the diamagnetic drift modifies the vorticity
- Using force balance and assuming no net rotation,

\[E_{r0} = \frac{1}{N_i Z_i e} \nabla_\perp P_{i0}\]

\cite{Hazeltine2003} B.D. Hazeltine and J.D. Meiss, *Plasma Confinement* (Dover, Mineola, NY, 2003)
Profiles with self-consistently varied pressure gradient used as BOUT++ input

Model VARYPED Pressure Profiles

- 55%
- 75
- 100
- 115
- 135
- 145
- Exp

P [kPa]

\( \psi \)
Profiles with self-consistently varied pressure gradient used as BOUT++ input
BOUT++ indicates a “growing” mode at experimental values of $\nabla P$. . .
BOUT++ marginal stability thresholds and growth rate trends agree with ELITE.
However, FLR effects can stabilize these ideal MHD modes\cite{6}

• Kinetic theory gives

\[ \gamma_{MHD}^2 = -\omega \left( \omega - \omega_{*\text{eff}} \right), \quad \omega_{*\text{eff}} = \frac{\omega_{*i,\text{max}}}{2} \]

• The ideal MHD mode will be *stabilized* when

\[ \gamma_{MHD} < \frac{\omega_{*\text{eff}}}{2} \]

• The diamagnetic frequency can be roughly calculated as

\[ \omega_{*i} = \vec{k} \cdot \vec{V}_{di} = \vec{k} \cdot \left( \frac{\hat{b} \times \vec{\nabla} p_i}{en_i B} \right) \approx \frac{nq|\vec{\nabla} p|}{2\sqrt{\kappa an_i eB_\phi}} \]

\cite{6} P.B. Snyder et al., *Nucl. Fusion* **51**, 103016 (2011)
The diamagnetic shearing stabilizes the modes at experimental $\nabla P$.
Resistivity can drive Instability

$n=15$ Growth Rate vs. Pedestal Resistivity

Normalized linear growth rate, $\gamma/\omega$ vs. Normalized Resistivity, i.e. $S^{-1}$

$\gamma \propto \eta^{1/3}$

Typ. EDA H-mode

~ ideal MHD

$\sim 10^{-9}$  $10^{-8}$  $10^{-7}$  $10^{-6}$  $10^{-5}$
Resistivity substantially enhances growth rates above the FLR stability criterion.

![Graph showing FLR Stability Threshold for Ideal MHD modes](image)
Stability picture if PB modes constrained the EDA Pedestal

EDA Stability Map

Pedestal Current

Pedestal Pressure Gradient

P-B UNSTABLE
P-B STABLE

EDA operational point
Stability picture if resistive-ballooning modes constrain the EDA pedestal
Nonlinear simulations have reached turbulent steady-state.
The mode spans the separatrix and exists in the high gradient region at the outboard midplane.
Nonlinear simulations show a quasi-coherent oscillation *qualitatively* similar to EDA’s QCM.
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The nonlinear quasi-coherent oscillation is peaked about $n \sim 16$, similar to EDA’s QCM.
The pressure and potential fluctuations are out of phase, in contrast to recent QCM measurements.
Conclusions

- EDA H-mode is a steady-state high performance operational regime
- PB theory fails to constrain the EDA pedestal
- BOUT++ is an extensible nonlinear fluid code suitable for investigating the EDA H-mode
- BOUT++ ideal linear growth rates are in good agreement with those from ELITE
- Inclusion of realistic pedestal resistivity drives RBMs that may ultimately constrain EDA
- Nonlinear simulations are ongoing and share some qualitative features with EDA’s QCM