Fluid models of free energy cascade dynamics

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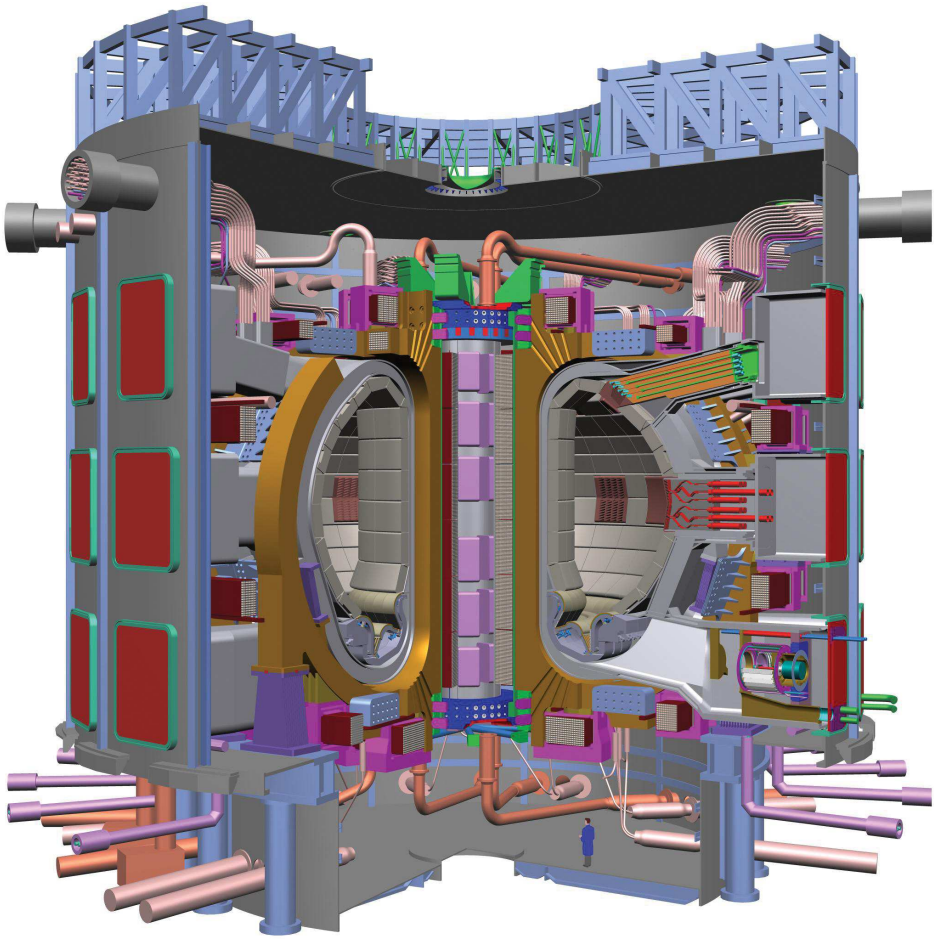
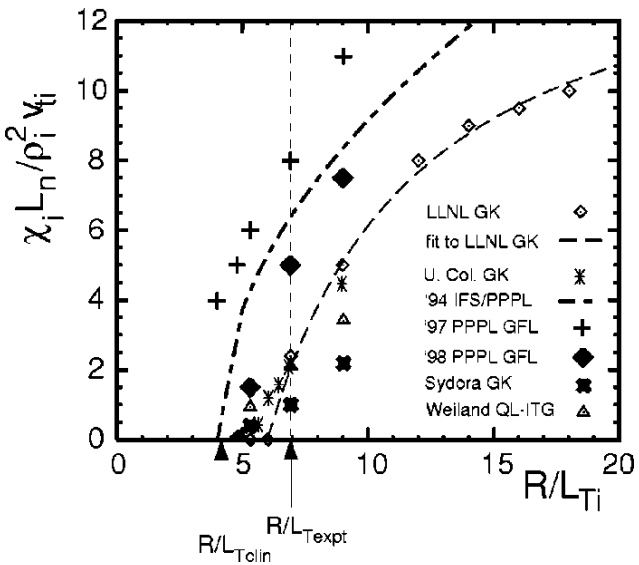
Tokamak Turbulence



Gyrofluid Models for Tokamak Turbulence

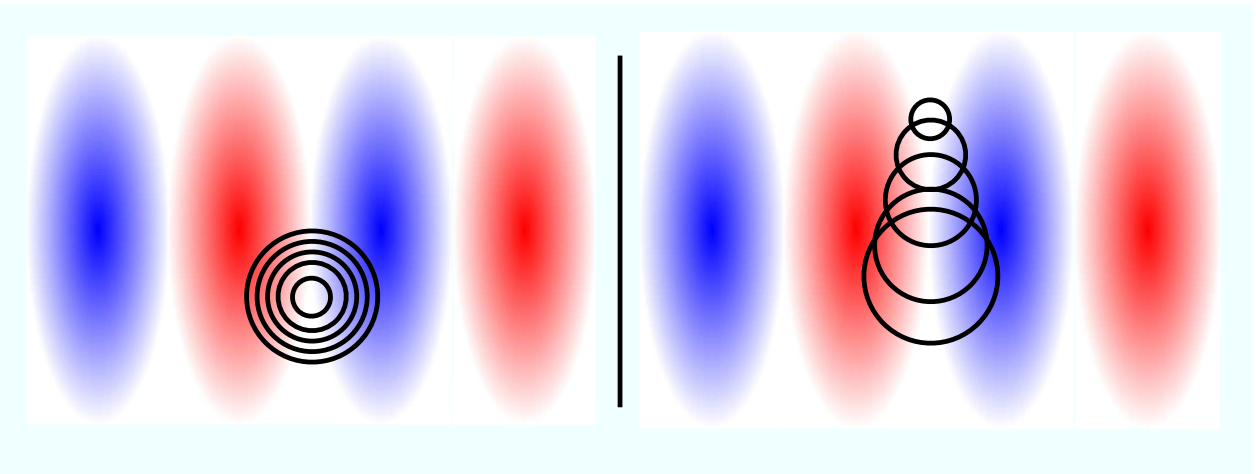
- Gyrofluids are computationally efficient
- Problem with models in the past



"Comparisons and physics basis of tokamak transport models and turbulence simulations", Dimits, et al. Physics of Plasmas, 2000.

Nonlinear Phase Mixing - Physical Space

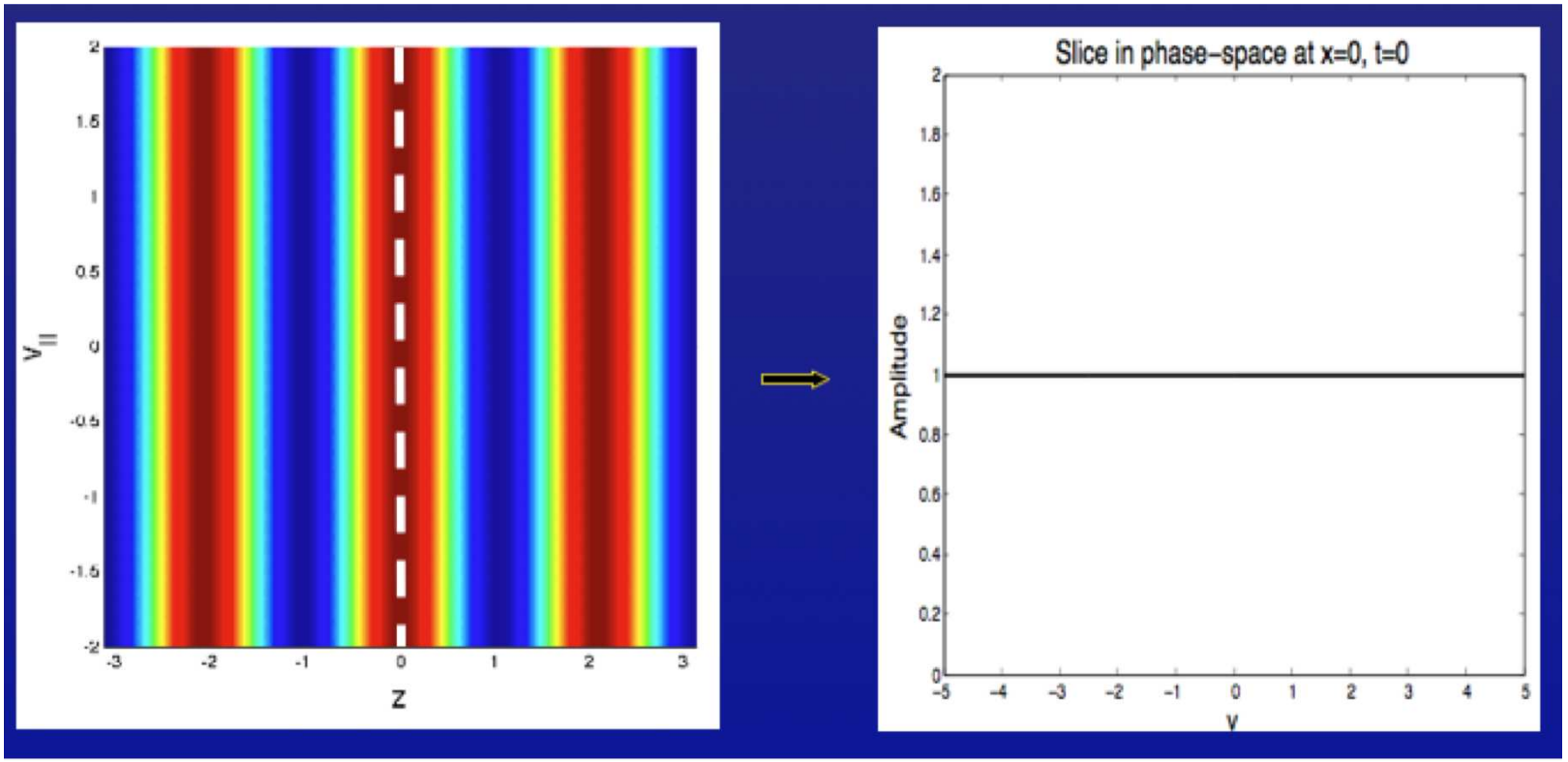
- Contours of a sinusoidal electrostatic potential
- Density perturbation with Maxwellian velocity distribution



"Nonlinear Phase Mixing and Phase-Space Cascade of Entropy in Gyrokinetic Plasma Turbulence", Tatsuno, T., et al., PRL 2009.

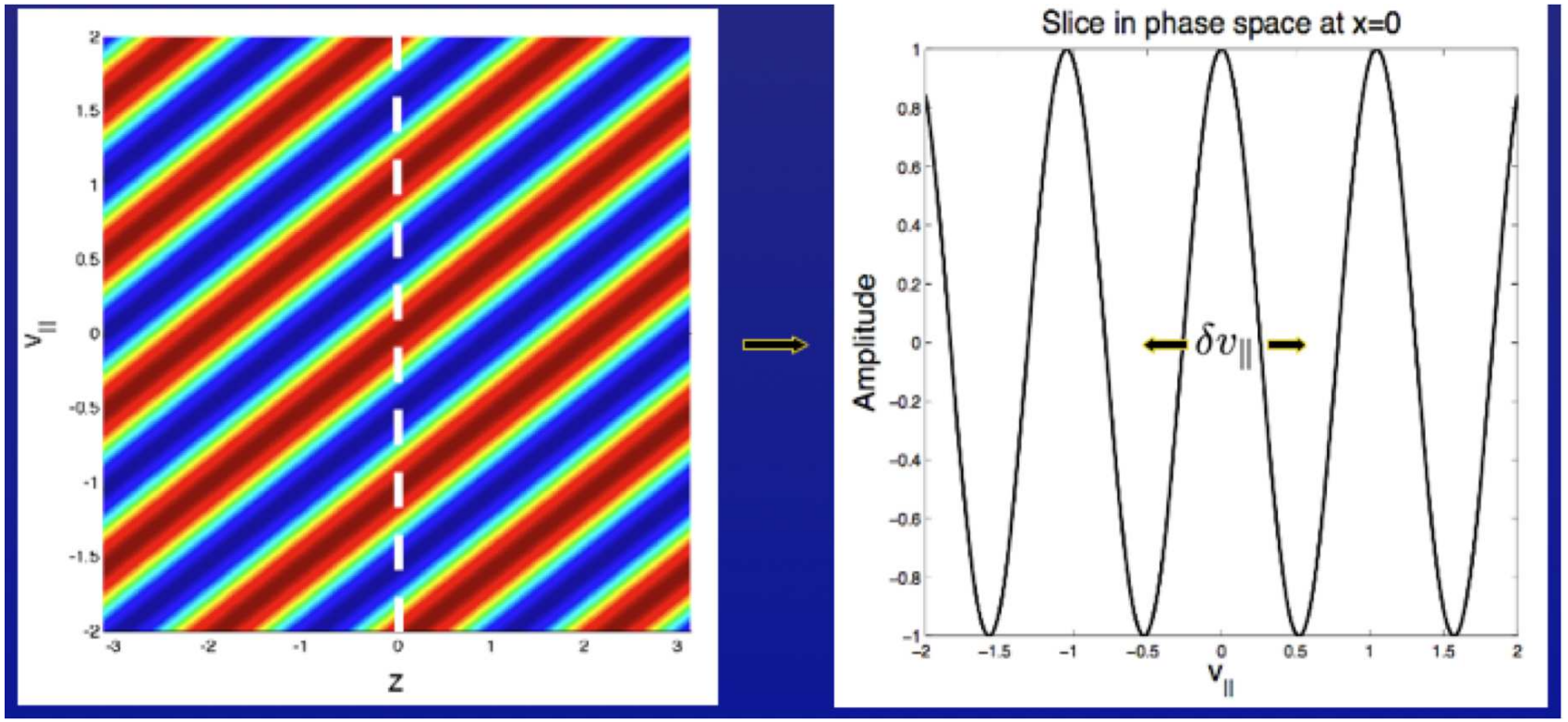
Linear (Parallel) Phase Mixing - Velocity Space

- Contours of $\delta f / F_0 \propto \cos\left(k_z\left(z - v_z t\right)\right)$



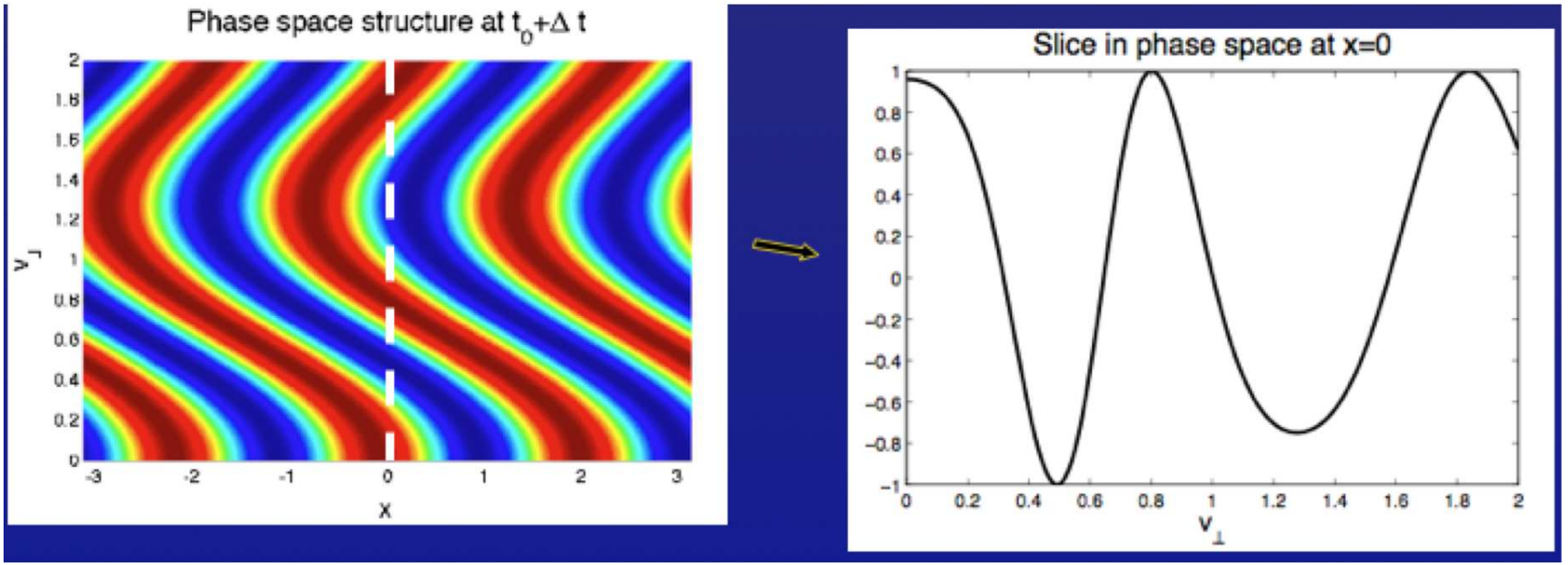
Linear (Parallel) Phase Mixing - Velocity Space

- Structure in z becomes structure in v_z



Nonlinear Phase Mixing - Velocity Space

$$\frac{\partial \delta f}{\partial t} + \langle \mathbf{v}_E \rangle \cdot \nabla \delta f = 0 \; ; \; \delta f \propto \cos\left(k_y \left(y - \langle v_E \rangle t\right)\right)$$



Closure Terms

$$\frac{\partial q_\parallel}{\partial t} + \mathbf{v}_\Psi \cdot \nabla q_\parallel + \left(3 + \beta_\parallel\right) \nabla_\parallel T_\parallel + \sqrt{2} D_\parallel \left|k_\parallel\right| q_\parallel + i\omega_d \left(-3q_\parallel - 3q_\perp + 6u_\parallel\right)$$
$$+ \left|\omega_d\right| \left(\nu_5 u_\parallel + \nu_6 q_\parallel + \nu_7 q_\perp\right) = -\nu_{ii} q_\parallel$$

$$\frac{\partial q_\perp}{\partial t} + \mathbf{v}_\Psi \cdot \nabla q_\perp + \left[\frac{1}{2}\hat{\nabla}_\perp^2 \mathbf{v}_\Psi\right] \cdot \nabla u_\parallel + \left[\hat{\hat{\nabla}}_\perp{}^2 \mathbf{v}_\Psi\right] \cdot \nabla q_\perp + \nabla_\parallel \left(T_\perp + \frac{1}{2}\hat{\nabla}_\perp^2 \Psi\right)$$
$$+ \sqrt{2} D_\perp \left|k_\parallel\right| q_\perp + \left(T_\perp - T_\parallel + \hat{\hat{\nabla}}_\perp{}^2 \Psi - \frac{1}{2}\hat{\nabla}_\perp^2 \Psi\right) \nabla_\parallel \ln B$$
$$+ i\omega_d \left(-q_\parallel - q_\perp + u_\parallel\right) + \left|\omega_d\right| \left(\nu_8 u_\parallel + \nu_9 q_\parallel + \nu_{10} q_\perp\right) = -\nu_{ii} q_\perp$$

Closure terms to model parallel phase mixing

Closure Terms

$$\frac{\partial q_\parallel}{\partial t} + \mathbf{v}_\Psi \cdot \nabla q_\parallel + \left(3+\beta_\parallel\right)\nabla_\parallel T_\parallel + \sqrt{2}D_\parallel \left|k_\parallel\right| q_\parallel + i\omega_d\left(-3q_\parallel - 3q_\perp + 6u_\parallel\right)$$
$$+ \left|\omega_d\right|\left(\nu_5 u_\parallel + \nu_6 q_\parallel + \nu_7 q_\perp\right) = -\nu_{ii} q_\parallel$$

$$\frac{\partial q_\perp}{\partial t} + \mathbf{v}_\Psi \cdot \nabla q_\perp + \left[\frac{1}{2}\hat{\nabla}_\perp^2 \mathbf{v}_\Psi\right]\cdot \nabla u_\parallel + \left[\hat{\hat{\nabla}}_\perp{}^2 \mathbf{v}_\Psi\right]\cdot \nabla q_\perp + \nabla_\parallel\left(T_\perp + \frac{1}{2}\hat{\nabla}_\perp^2 \Psi\right)$$
$$+ \sqrt{2}D_\perp \left|k_\parallel\right| q_\perp + \left(T_\perp - T_\parallel + \hat{\hat{\nabla}}_\perp{}^2 \Psi - \frac{1}{2}\hat{\nabla}_\perp^2 \Psi\right)\nabla_\parallel \ln B$$
$$+ i\omega_d\left(-q_\parallel - q_\perp + u_\parallel\right) + \left|\omega_d\right|\left(\nu_8 u_\parallel + \nu_9 q_\parallel + \nu_{10} q_\perp\right) = -\nu_{ii} q_\perp$$

Closure terms to model toroidal phase mixing by curvature and ∇B drifts

Closure Terms

$$\frac{\partial q_\parallel}{\partial t} + \mathbf{v}_\Psi \cdot \nabla q_\parallel + \left(3+\beta_\parallel\right)\nabla_\parallel T_\parallel + \sqrt{2}D_\parallel \left|k_\parallel\right| q_\parallel + i\omega_d\left(-3q_\parallel - 3q_\perp + 6u_\parallel\right)$$
$$+ \left|\omega_d\right|\left(\nu_5 u_\parallel + \nu_6 q_\parallel + \nu_7 q_\perp\right) = -\nu_{ii} q_\parallel$$

$$\frac{\partial q_\perp}{\partial t} + \mathbf{v}_\Psi \cdot \nabla q_\perp + \left[\frac{1}{2}\hat{\nabla}_\perp^2 \mathbf{v}_\Psi\right]\cdot \nabla u_\parallel + \left[\hat{\hat{\nabla}}_\perp{}^2 \mathbf{v}_\Psi\right]\cdot \nabla q_\perp + \nabla_\parallel\left(T_\perp + \frac{1}{2}\hat{\nabla}_\perp^2 \Psi\right)$$
$$+ \sqrt{2}D_\perp \left|k_\parallel\right| q_\perp + \left(T_\perp - T_\parallel + \hat{\hat{\nabla}}_\perp{}^2 \Psi - \frac{1}{2}\hat{\nabla}_\perp^2 \Psi\right)\nabla_\parallel \ln B$$
$$+ i\omega_d\left(-q_\parallel - q_\perp + u_\parallel\right) + \left|\omega_d\right|\left(\nu_8 u_\parallel + \nu_9 q_\parallel + \nu_{10} q_\perp\right) = -\nu_{ii} q_\perp$$

Missing: closure terms to model nonlinear phase mixing by $\mathbf{E}\times\mathbf{B}$ drifts

Nonlinear Phase Mixing Closure

$$q_{\parallel} \propto \int_{-\infty}^{\infty} v_{\parallel}^{3} \left[\nabla \cdot \left(F \frac{c}{B} \hat{\mathbf{b}} \times \nabla \Phi \right) - \nabla \cdot \left(F \frac{c}{B} \hat{\mathbf{b}} \times \nabla \frac{k_{\perp}^{2} v_{\perp}^{2}}{4\Omega_{i}^{2}} \Phi \right) + \nabla \cdot \left(F \frac{c}{B} \hat{\mathbf{b}} \times \nabla \frac{k_{\perp}^{4} v_{\perp}^{4}}{64\Omega_{i}^{4}} \Phi \right) + \cdots \right] d^{3}v$$

Consider the effect of the first two terms

$$\frac{\partial f}{\partial t} + \hat{\mathbf{b}} \times \left(1 - \frac{k_{\perp}^{2} v_{\perp}^{2}}{4\Omega^{2}} \right) \nabla \Phi_{0} e^{ik_{x}x} \cdot \nabla f = 0$$

$$f = F_{M} e^{ik_{y}y} e^{\left(\frac{k_{\perp}^{2} v_{\perp}^{2}}{4\Omega^{2}} \right) ik_{x}\Phi k_{y} t}$$

$$q_{\parallel}(t) \propto \frac{e^{-ik_{y}(k_{x}\Phi)t}}{1 - k_{y} k_{x}^{2} \rho_{i}^{2} (k_{x}\Phi) t}$$

Model with a damping term: $\nu_{pm} \left| \frac{1}{2} \hat{\nabla}_{\perp}^{2} \mathbf{v}_{\Psi} \cdot \nabla \right| q_{\parallel}$

NLPM Closure

$$\frac{\partial}{\partial t} \rightarrow \frac{\partial}{\partial t} + \nu_{pm} \left| \frac{1}{2} \hat{\nabla}_{\perp}^{2} \mathbf{v}_{\Psi} \cdot \nabla \right|$$

Simplify further to

$$\nu'_{pm} = \nu_{pm} \sum_{k_x} \frac{1}{2} \left| \hat{\nabla}_{\perp}^{2} v_y \right|$$

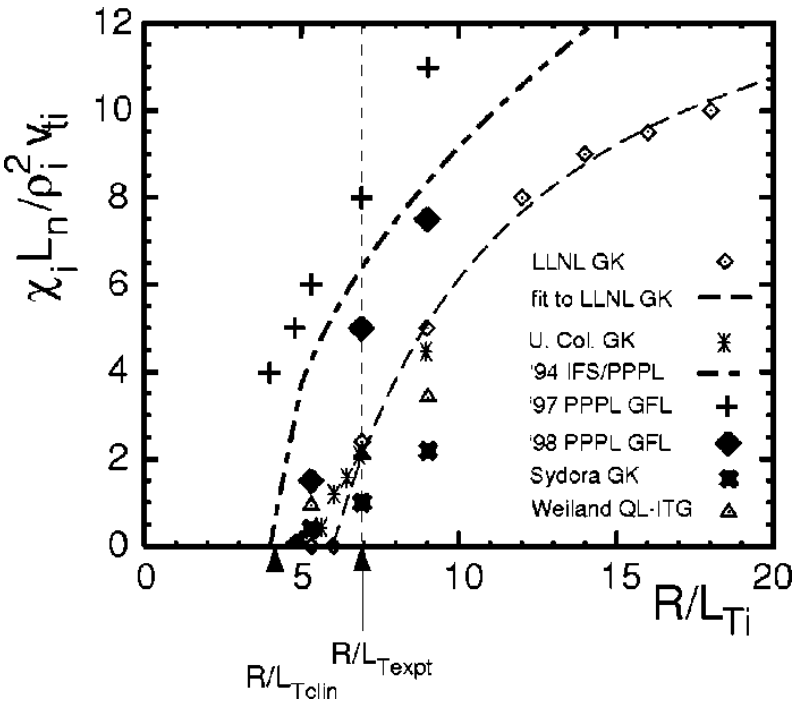
$$\frac{\partial}{\partial t} \rightarrow \frac{\partial}{\partial t} + \nu'_{pm} \left| k_y \right|$$

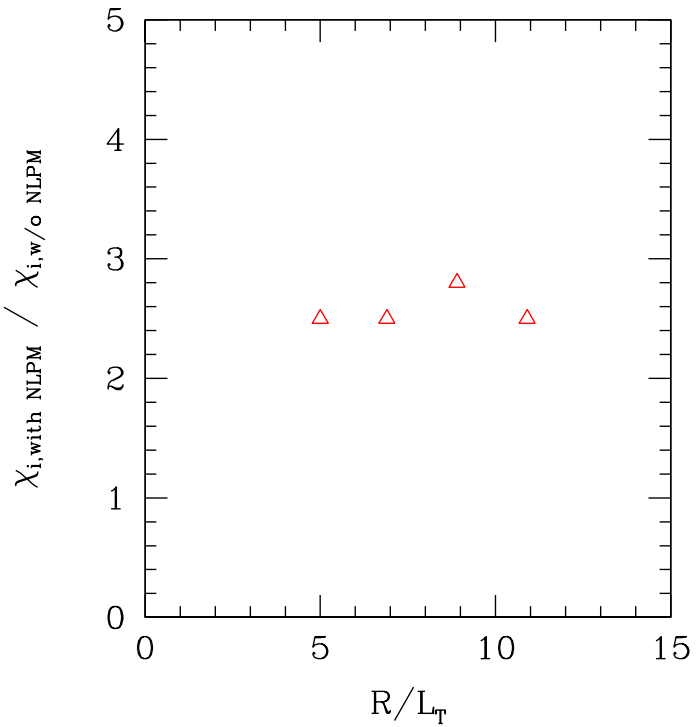
- Proportional to $k_{\perp}^{4}$
- Proportional to magnitude of v_y
- Direction of the flow is irrelevant

NLPM Results

- Cyclone Base Case - ITG Turbulence
- Trapped Particle Scan - ε (r/R) scan
- ETG
- General Geometry Case
- Local vs. Global

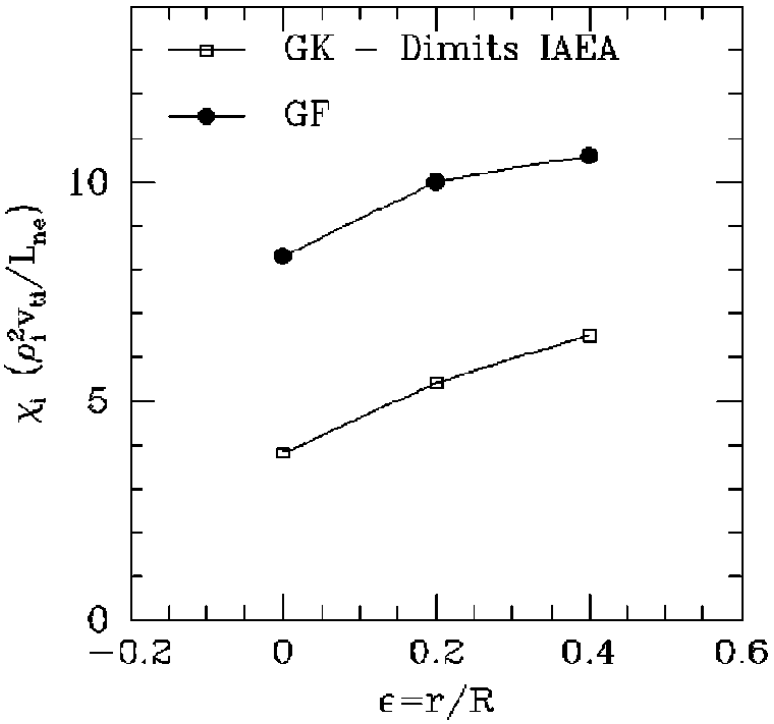
Cyclone Base Case (ITG)

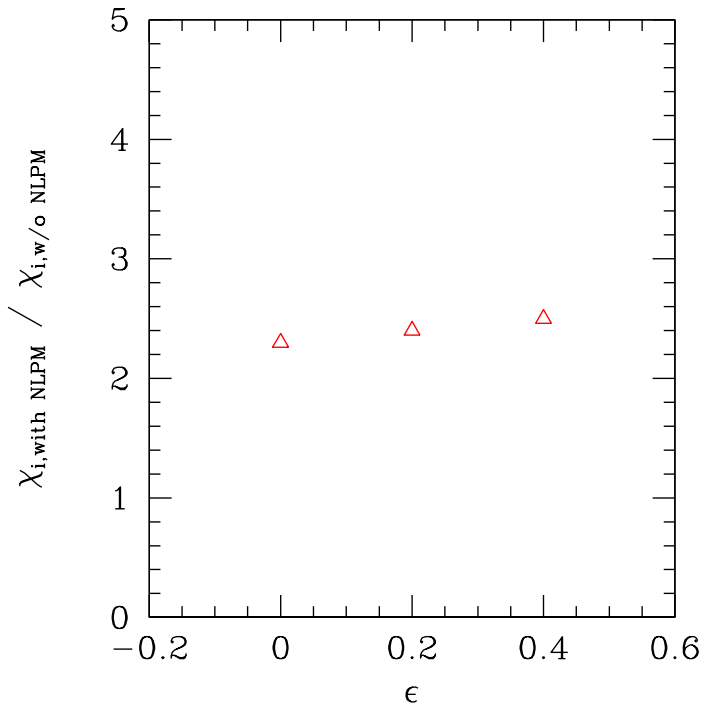




"Comparisons and physics basis of tokamak transport models and turbulence simulations", Dimits, et al. Physics of Plasmas, 2000.

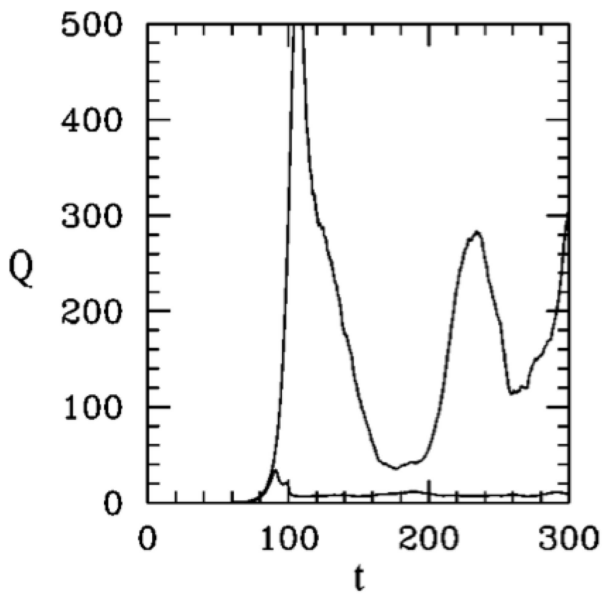
Trapped Particle Scan

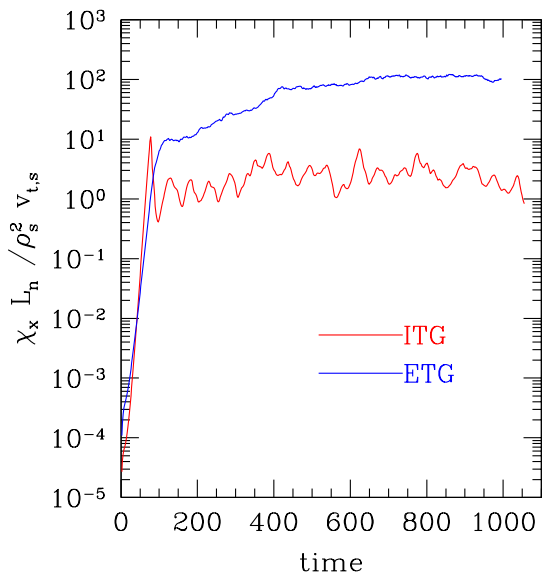




"Comparisons and physics basis of tokamak transport models and turbulence simulations", Dimits, et al. Physics of Plasmas, 2000.

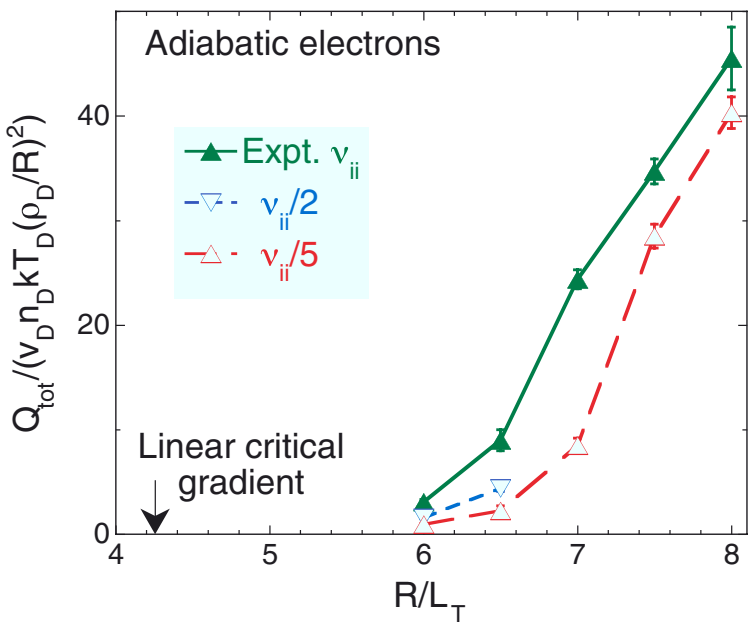
ETG

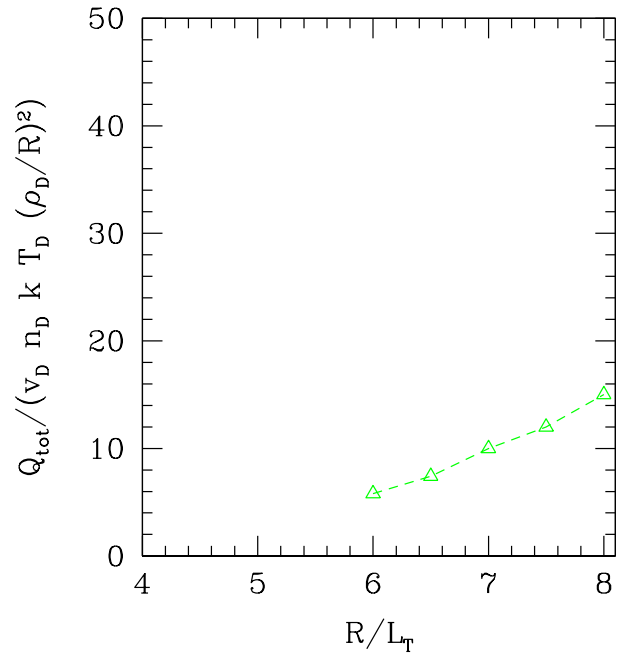




"Electron temperature gradient driven turbulence", Jenko, F., et al. Physics of Plasmas, 2000.

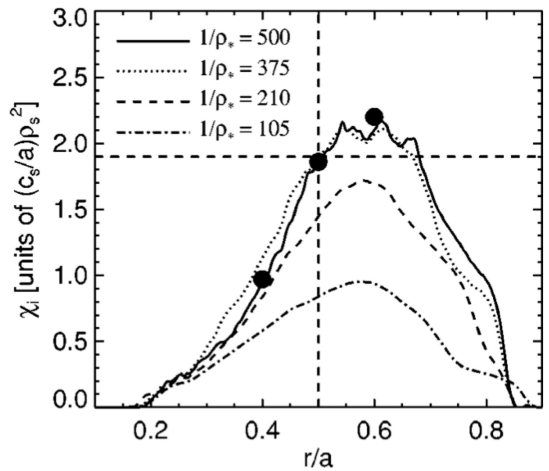
General Geometry





"Dimits Shift in Realistic Gyrokinetic Plasma-Turbulence Simulations", Mikkelsen, D.R. and Dorland, W. PRL 2008.

Local vs. Global



r/a	Global GK ($\frac{\rho^2 v_t}{L_n}$)	Local GK ($\frac{\rho^2 v_t}{L_n}$)	Local GF ($\frac{\rho^2 v_t}{L_n}$)
0.4	1.25	1.25	1.8
0.5	2.4	2.4	2.3
0.6	2.6	2.8	3.0

"The local limit of global gyrokinetic simulations", Candy, J, et al. PoP 2004"

NLPM Study Conclusions

- Nonlinear Phase Mixing introduces physically-motivated damping
- Model appears to overdamp at larger gradients in general geometry
- Unclear that we have captured original behavior of the `gryffin` code
- Like to include improved zonal flow closure by Beer & Hammett
- Overall, NLPM improves turbulent flux predictions

Tokamak: Transport

Equations from Barnes' code, `Trinity`

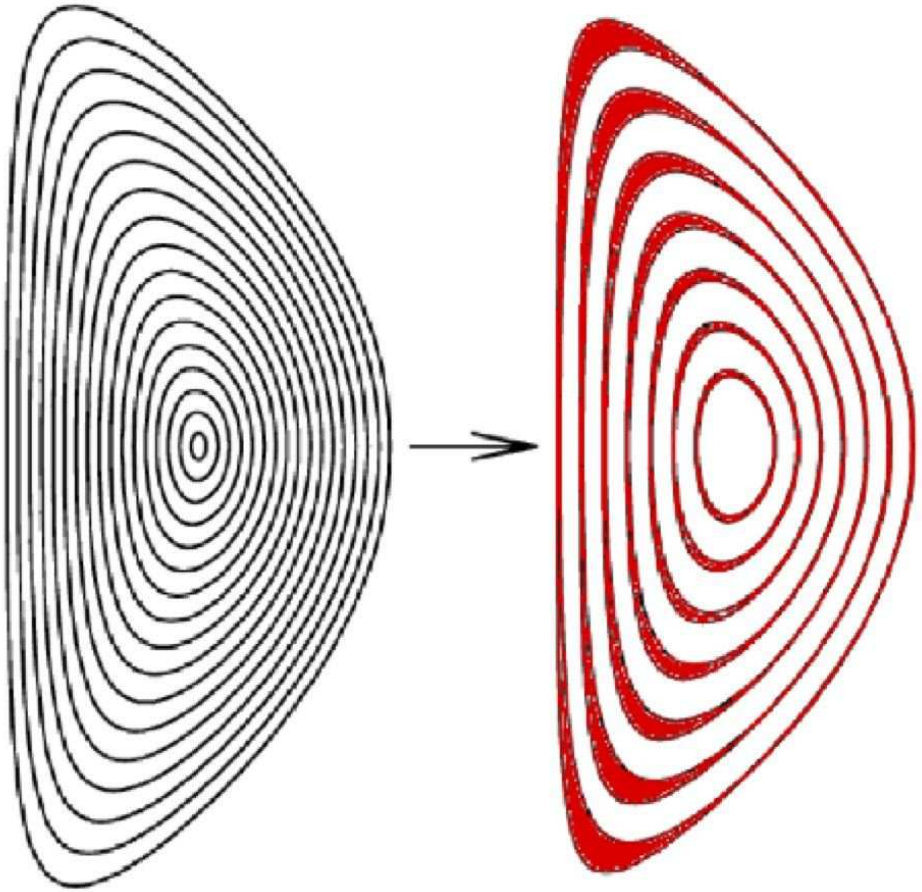
$$\frac{\partial n}{\partial \tau} = -\frac{\partial \psi}{\partial V}\frac{\partial}{\partial \psi}\left(\frac{\partial V}{\partial \psi}\langle\langle \mathbf{\Gamma}\cdot\nabla\psi\rangle\rangle\right) + C + \langle\langle S_n\rangle\rangle$$

$$\frac{\partial \langle L\rangle_t}{\partial t} = -\sum_s \frac{\partial \psi}{\partial V}\frac{\partial}{\partial \psi}\left(\frac{\partial V}{\partial \psi}\langle\langle \mathbf{\Pi}\cdot\nabla\psi\rangle\rangle\right) + \sum_s \langle\langle S_{L_s}\rangle\rangle$$

$$\frac{3}{2}\frac{\partial p}{\partial \tau} = -\frac{\partial \psi}{\partial V}\frac{\partial}{\partial \psi}\left[\frac{\partial V}{\partial \psi}\langle\langle \mathbf{Q}\cdot\nabla\psi\rangle\rangle\right] + C + \langle\langle S_p\rangle\rangle + \texttt{equil}$$

"Direct multiscale coupling of a transport code to gyrokinetic turbulence codes", M. Barnes, I. G. Abel, W. Dorland, T. Görler, G. W. Hammett, and F. Jenko Phys. Plasmas **17**, 056109 (2010)

Trinity: **Flux Surfaces**



`Trinity` and GPUs

- `Trinity` solved on CPU
- Many copies of turbulence code run at each time step
- Gyrofluid code on GPU
- Massively parallel operation!
- Requires demonstration of `gryffin` working on a GPU

GPU

- Ported simplified version of `gryffin` to GPU
  - 25x speed up
- Credible turbulence calculation on the GPU
- Multi-GPU code for single turbulence calculation
  - Slow
- Determined practical approach: One turbulence calculation/GPU

Future Work

- Use model we have derived and code we have tested to explore parameter space in solar wind conditions
- Continue validation and full updating of resurrected code `gryffin`
- Apply experience to complete the port of `gryffin` and `Trinity` to GPU

Future Work

- Use model we have derived and code we have tested to explore parameter space in solar wind conditions
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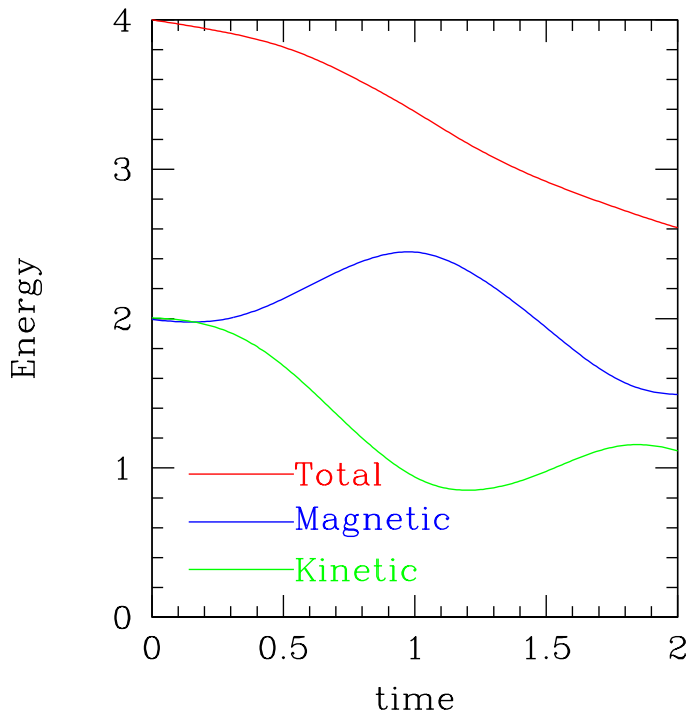
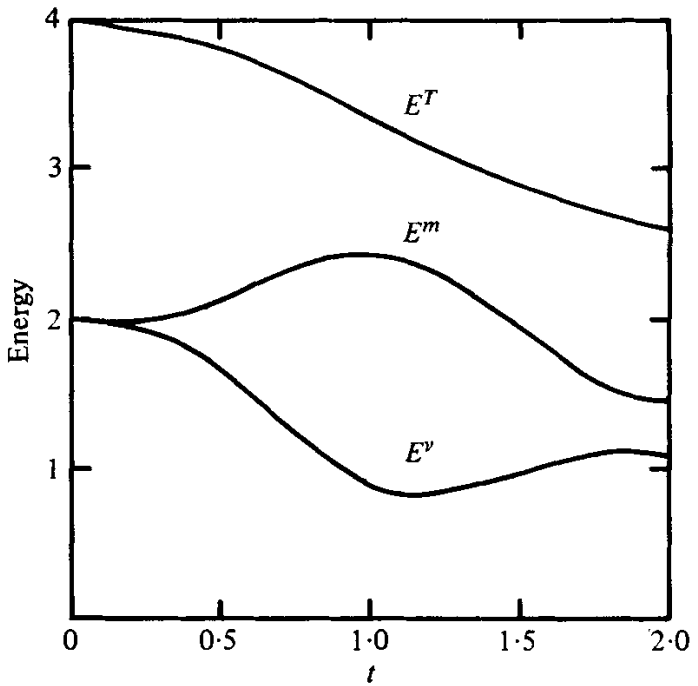
$\longrightarrow$ Find viable, new magnetic confinement configurations!

Orzsag-Tang


Energy
time
Total
Magnetic
Kinetic
0
1
2
3
4
0
0.5
1
1.5
2


Energy
t
E^T
E^m
E^v
1
2
3
4
0
0·5
1·0
1·5
2·0

Orszag-Tang

AB3/BDF2

Adams-Bashforth 3rd order/Backwards Difference Formula 2nd order

$$\frac{3}{2}u_{n+1} - 2u_n + \frac{1}{2}u_{n-1} = \frac{8\Delta t}{3}N(u_n) - \frac{7\Delta t}{3}N(u_{n-1}) + \frac{2\Delta t}{3}N(u_{n-2}) + L(u_{n+1})$$

where u is the state vector, $N()$ is the nonlinear operator, and $L()$ is the linear operator.

ITG 25x Speed Up

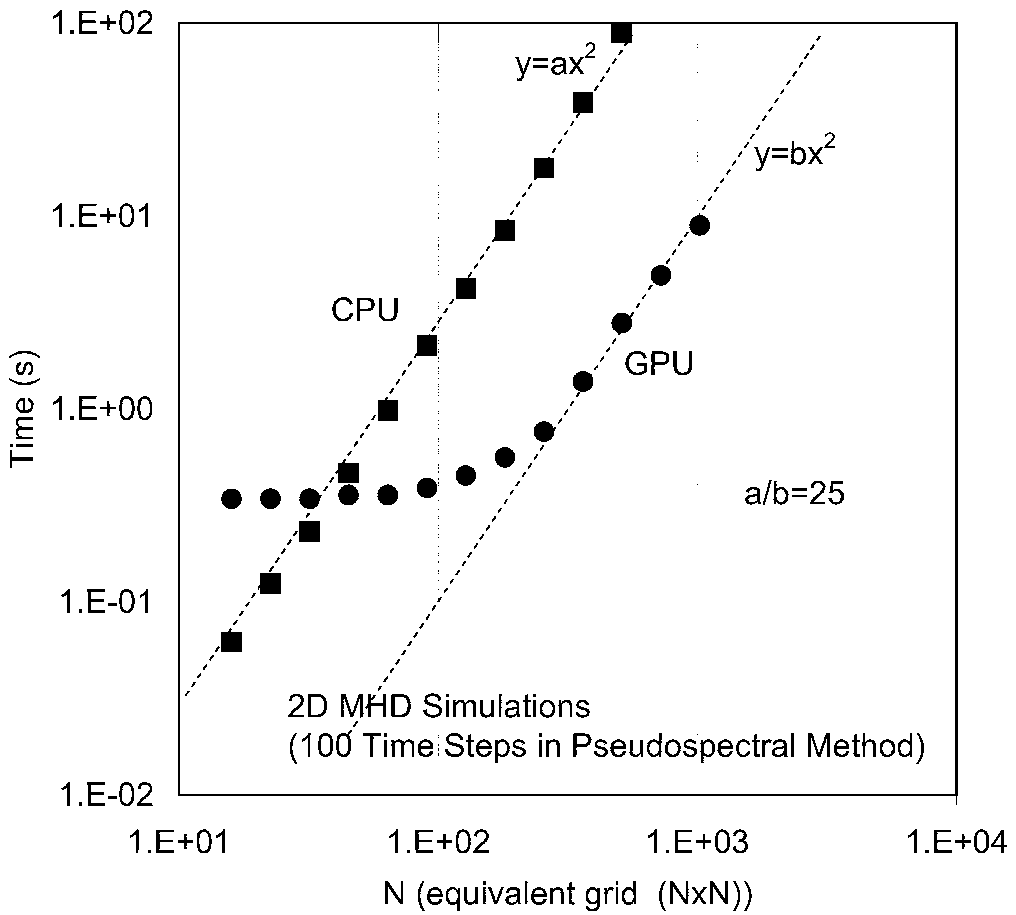


Figure 3: Dependences of the wall clock time for execution of 100 time steps in the pseudospectral method for serial CPU implementation (the squares) and for the GPU (the circles).