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Core Gyrofluid Simulations of Ion Temperature Gradient Turbulence using BOUT++

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Outline

- Verification of BOUT++ with eigenvalue solver
- Effect of gyro-averaging on ITG linear growth rate
- Comparison between various gyrofluid models
- Preliminary results of nonlinear simulations
- Summary

Governing equations for ITG benchmark

(M. Ottaviani and G. Manfredi, PoP '99)

- Vorticity equation

$$\left(\frac{\partial}{\partial t}+\mathbf{V}_E\cdot\nabla\right)\Omega=-\mathbf{V}_E\cdot\nabla n_{eq}-n\nabla_{\parallel}V_{i\parallel}+n\left(\mathbf{V}_E+\mathbf{V}_{p_i}\right)\cdot\left(\boldsymbol{\kappa}+\nabla\ln B\right)+n\mathbf{V}_{p_i}\cdot\nabla\left(\frac{n_1-\Omega}{n}\right)$$

where $\Omega=n_1-n_{pol}$: generalized vorticity, $n_{pol}=\dfrac{ne}{m_i\omega_{ci}^2}\nabla_{\perp}^2\varphi$, $n_1=n_0\dfrac{e\varphi_1}{T_e}$: adiabatic response

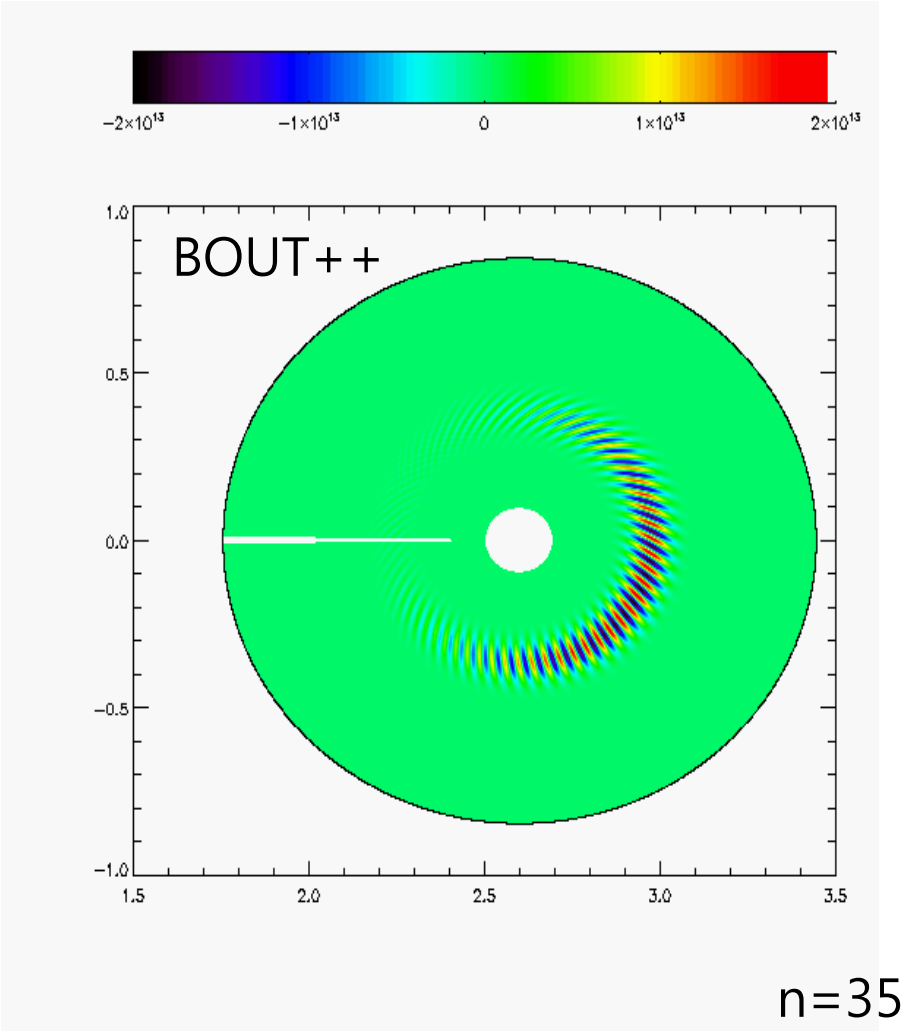
- Ion parallel velocity equation

$$\left(\frac{\partial}{\partial t}+\mathbf{V}_E\cdot\nabla\right)V_{i\parallel}=-\frac{e}{m_i}\nabla_{\parallel}\varphi-\frac{1}{m_i n}\nabla_{\parallel}p_i$$

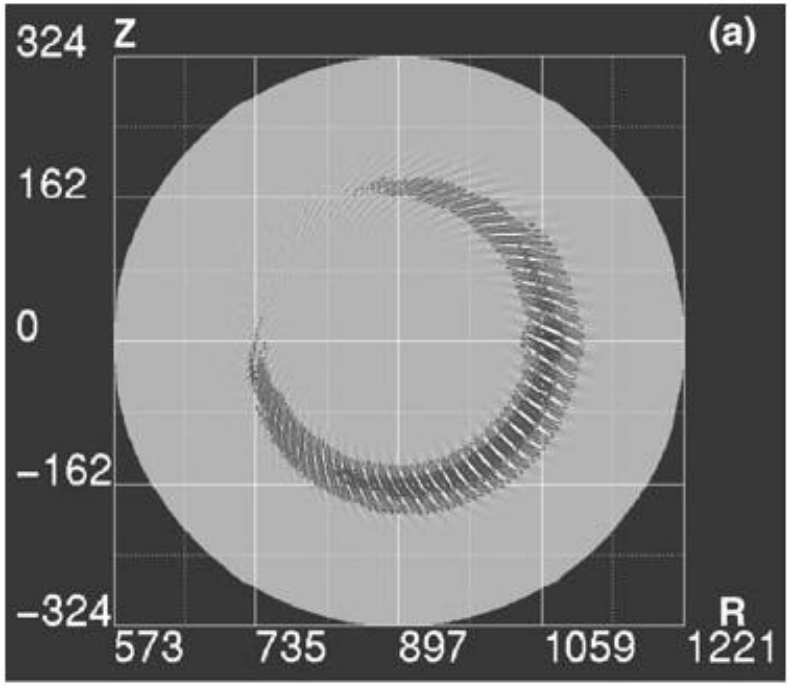
- Ion temperature equation

$$\left(\frac{\partial}{\partial t}+\mathbf{V}_E\cdot\nabla\right)T_i=-\frac{2}{3}T_i\nabla_{\parallel}V_{i\parallel}$$

Eigenmode structure for ITG



GT3D result
(Idomura et. al., NF '03)

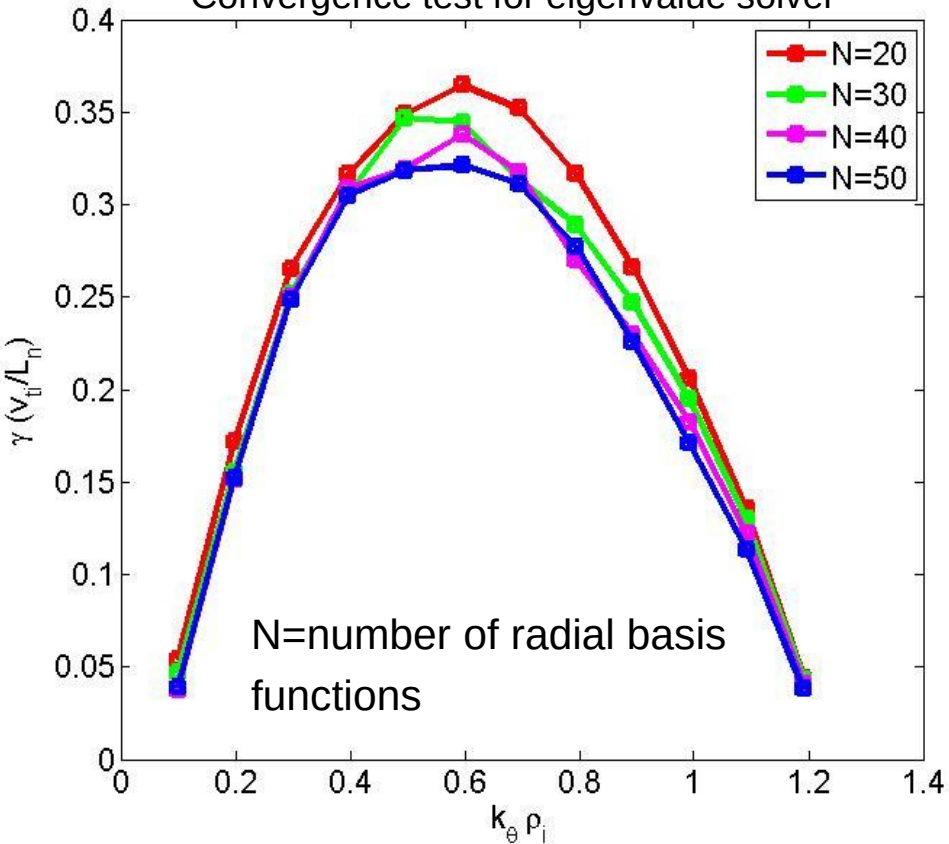


n=35, ITER-like condition

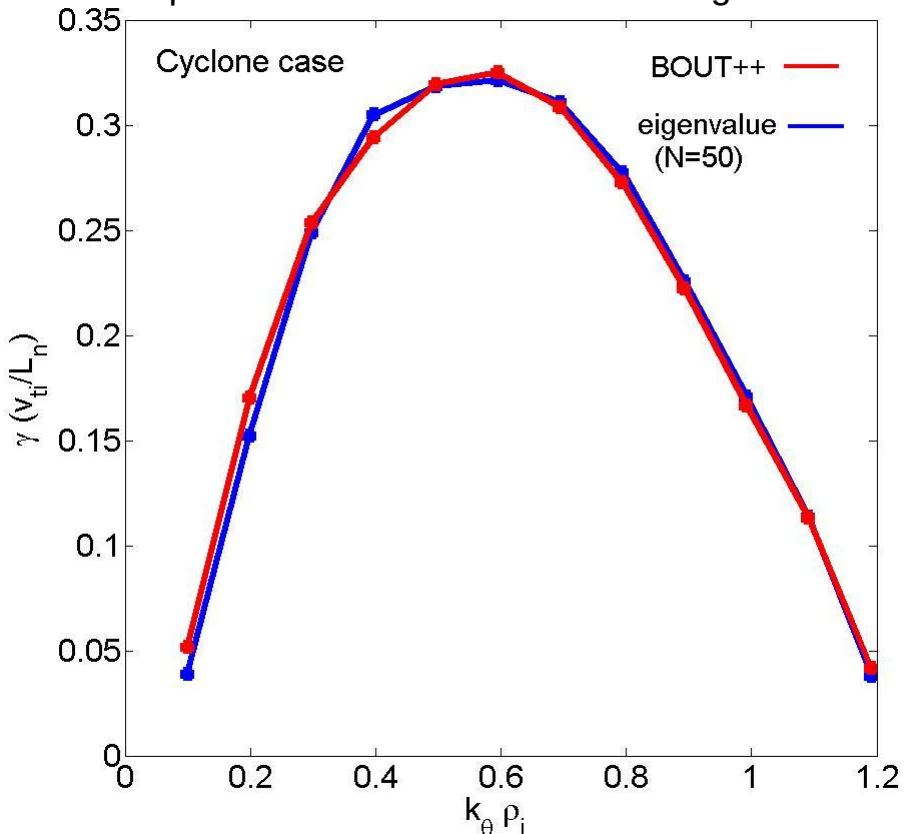
Verification of BOUT++ with an eigenvalue solver

- In eigenvalue solver, $f_{mnp} = \frac{\sqrt{2}}{J_{m+1}(\alpha_{mp})} J_m\left(\frac{\alpha_{mp}}{a} r\right) e^{i(m\theta - n\zeta)}$ $(p = 1 \sim N)$ is used as basis function.
- The fluid equations are then projected on to the set of basis functions.
- Eigenvalues are obtained by using matlab.
- BOUT++ results agree well with eigenvalues → ***BOUT++ correctly solves a given equation set***.

Convergence test for eigenvalue solver



Comparison between BOUT++ and eigenvalue solver



Effect of gyro-averaging on linear growth rate (BOUT++)

Normalized eqs.

$$\frac{\partial \tilde{\Omega}_i}{\partial \bar{t}} = -\left[\tilde{\varphi}, \frac{\hat{n}_i}{\rho_*}\right]_0 - \hat{n}_i \hat{\nabla}_\parallel \tilde{V}_{i\parallel} - \frac{2i}{\hat{T}_i}\left(\hat{n}_i \omega_d \tilde{\varphi} + \omega_d \tilde{p}_i\right) + \left[\frac{\hat{p}_i}{\rho_*}, \frac{\tilde{n}_i - \tilde{\Omega}_i}{\hat{n}_i}\right]_0$$

$$\frac{\partial \tilde{V}_{i\parallel}}{\partial \bar{t}} = -\hat{\nabla}_\parallel \tilde{\varphi} - \frac{1}{\hat{n}_i}\hat{\nabla}_\parallel \tilde{p}_i$$

$$\frac{\partial \tilde{T}_i}{\partial \bar{t}} = -\left[\tilde{\varphi}, \frac{\hat{T}_i}{\rho_*}\right]_0 - \frac{2}{3}\hat{T}_i \hat{\nabla}_\parallel \tilde{V}_{i\parallel}$$

Replace φ with gyro-averaged potential in $E \times B$ convection term.

$$\frac{\partial \tilde{\Omega}_i}{\partial \bar{t}} = -\left[\tilde{\Phi}, \frac{\hat{n}_i}{\rho_*}\right]_0 - \hat{n}_i \hat{\nabla}_\parallel \tilde{V}_{i\parallel} - \frac{2i}{\hat{T}_i}\left(\hat{n}_i \omega_d \tilde{\varphi} + \omega_d \tilde{p}_i\right) + \left[\frac{\hat{p}_i}{\rho_*}, \frac{\tilde{n}_i - \tilde{\Omega}_i}{\hat{n}_i}\right]_0$$

$$\frac{\partial \tilde{V}_{i\parallel}}{\partial \bar{t}} = -\hat{\nabla}_\parallel \tilde{\varphi} - \frac{1}{\hat{n}_i}\hat{\nabla}_\parallel \tilde{p}_i$$

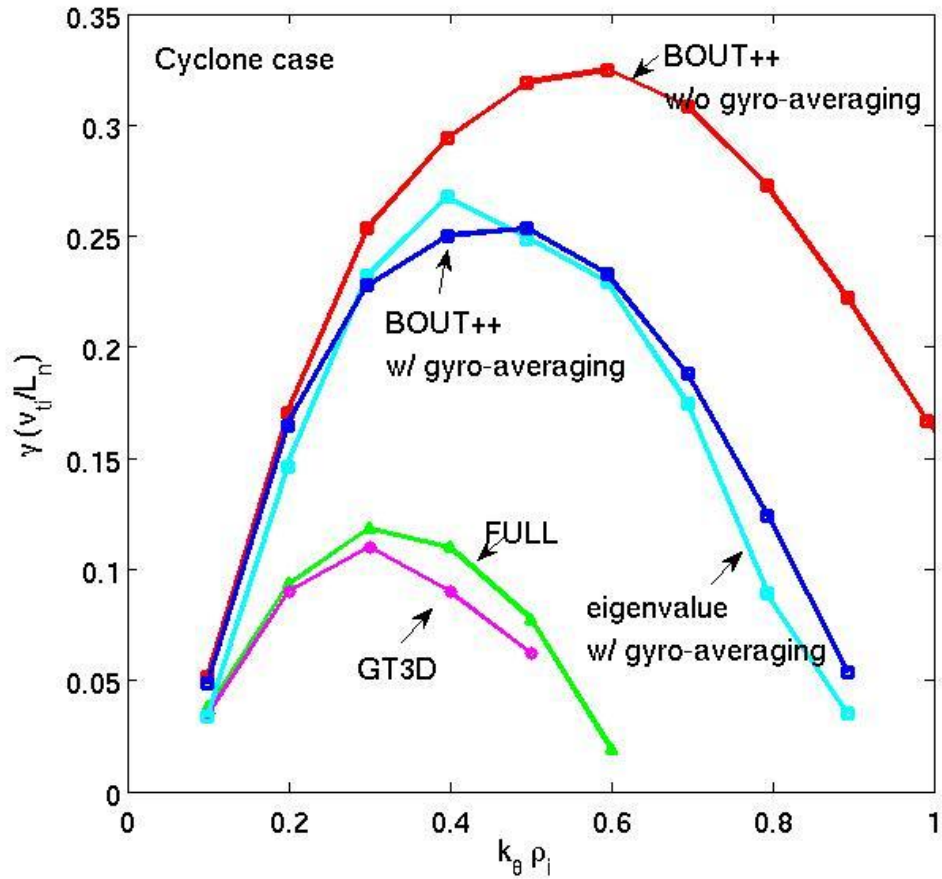
$$\frac{\partial \tilde{T}_i}{\partial \bar{t}} = -\left[\tilde{\Phi}, \frac{\hat{T}_i}{\rho_*}\right]_0 - \frac{2}{3}\hat{T}_i \hat{\nabla}_\parallel \tilde{V}_{i\parallel}$$

where $\tilde{\Phi} = \Gamma_0^{1/2}\tilde{\varphi} \cong \left(1 + \frac{b}{2}\right)^{-1}\tilde{\varphi}$ is gyro-averaged potential,

$$i\omega_d = \left(v_t^2 / \omega_c\right)\mathbf{b} \times \nabla \ln B \cdot \nabla, \quad b = -\rho_i^2 \nabla_\perp^2$$

- Gyro-averaged potential is obtained by solving following equations.

$$\left(\frac{\hat{n}_i}{\hat{T}_e} - \hat{n}_i \tilde{\nabla}_\perp^2\right)\tilde{\varphi} = \tilde{\Omega}_i \rightarrow \tilde{\varphi} \Rightarrow \left(1 - \frac{1}{2}\tilde{\nabla}_\perp^2\right)\tilde{\Phi} = \tilde{\varphi} \rightarrow \tilde{\Phi}$$



Effect of Landau damping and gyro-averaging on linear growth rate (results by eigenvalue solver)

- Case I : Landau damping (LD)

$$-i\omega\tilde{\Omega}_i = -\left[\tilde{\varphi}, \frac{\hat{n}_i}{\rho_*}\right]_0 - \hat{n}_i\hat{\nabla}_\parallel\tilde{V}_{i\parallel} - \frac{2i}{\hat{T}_i}\left(\hat{n}_i\omega_d\tilde{\varphi} + \omega_d\tilde{p}_i\right) + \left[\frac{\hat{p}_i}{\rho_*}, \frac{\tilde{n}_i - \tilde{\Omega}_i}{\hat{n}_i}\right]_0$$

$$-i\omega\tilde{V}_{i\parallel} = -\hat{\nabla}_\parallel\tilde{\varphi} - \frac{1}{\hat{n}_i}\hat{\nabla}_\parallel\tilde{p}_i$$

$$-i\omega\tilde{T}_i = -\left[\tilde{\varphi}, \frac{\hat{T}_i}{\rho_*}\right]_0 - \frac{2}{3}\hat{T}_i\hat{\nabla}_\parallel\tilde{V}_{i\parallel} - \sqrt{\frac{8}{\pi}\hat{T}_i}\left|\hat{\nabla}_\parallel\right|\tilde{T}_i$$

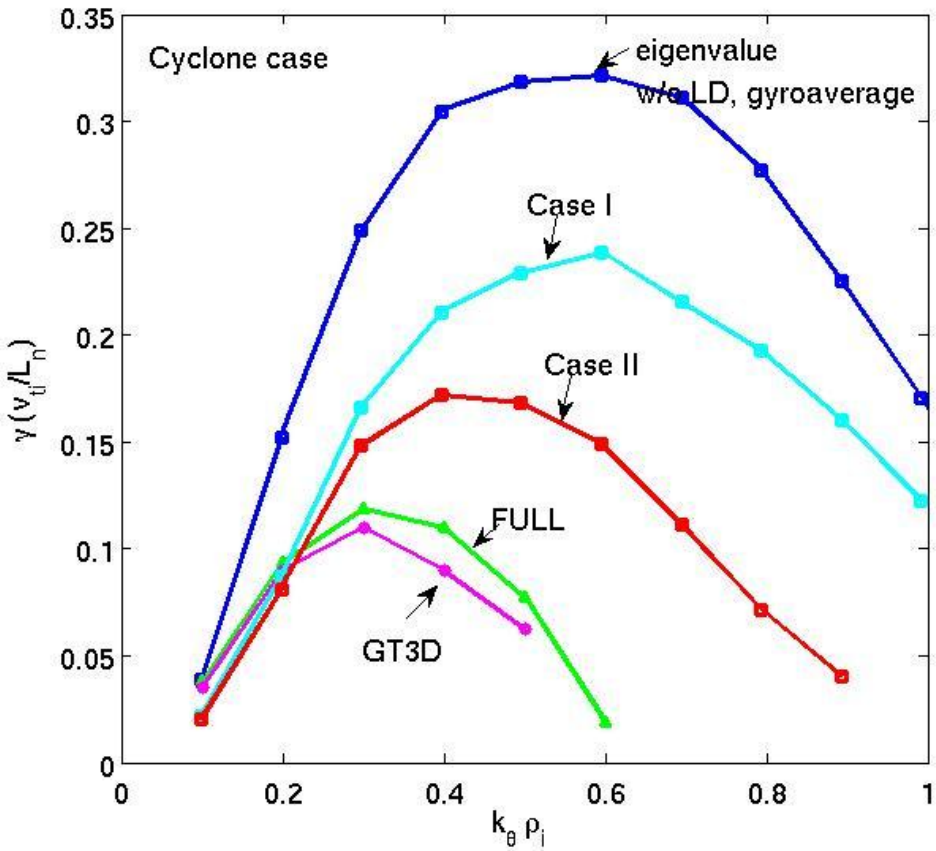
- Case II : LD + gyro-averaging

$$-i\omega\tilde{\Omega}_i = -\left[\tilde{\Phi}, \frac{\hat{n}_i}{\rho_*}\right]_0 - \hat{n}_i\hat{\nabla}_\parallel\tilde{V}_{i\parallel} - \frac{2i}{\hat{T}_i}\left(\hat{n}_i\omega_d\tilde{\varphi} + \omega_d\tilde{p}_i\right) + \left[\frac{\hat{p}_i}{\rho_*}, \frac{\tilde{n}_i - \tilde{\Omega}_i}{\hat{n}_i}\right]_0$$

$$-i\omega\tilde{V}_{i\parallel} = -\hat{\nabla}_\parallel\tilde{\varphi} - \frac{1}{\hat{n}_i}\hat{\nabla}_\parallel\tilde{p}_i$$

$$-i\omega\tilde{T}_i = -\left[\tilde{\Phi}, \frac{\hat{T}_i}{\rho_*}\right]_0 - \frac{2}{3}\hat{T}_i\hat{\nabla}_\parallel\tilde{V}_{i\parallel} - \sqrt{\frac{8}{\pi}\hat{T}_i}\left|\hat{\nabla}_\parallel\right|\tilde{T}_i$$

Hammett-Perkins closure



TRB model

(X. Garbet et. al., PoP '01)

- Vorticity equation

$$\left(\frac{\partial}{\partial t}+\mathbf{V}_E\cdot\nabla\right)\Omega=-\mathbf{V}_E\cdot\nabla n_{eq}-n\nabla_{\parallel}V_{i\parallel}+n\left(\mathbf{V}_E+\mathbf{V}_{p_i}\right)\cdot\left(\boldsymbol{\kappa}+\nabla\ln B\right)+n\mathbf{V}_{p_i}\cdot\nabla\left(\frac{n_1-\Omega}{n}\right)$$

where $\Omega=n_1-n_{pol}$: generalized vorticity, $n_{pol}=\dfrac{ne}{m_i\omega_{ci}^2}\nabla_{\perp}^2\varphi$, $n_1=n_0\dfrac{e\varphi_1}{T_e}$: adiabatic response

- Ion parallel velocity equation

$$\left(\frac{\partial}{\partial t}+\mathbf{V}_E\cdot\nabla\right)V_{i\parallel}=-\frac{e}{m_i}\nabla_{\parallel}\varphi-\frac{1}{m_i n}\nabla_{\parallel}p_i$$

- Ion temperature equation

$$\left(\frac{\partial}{\partial t}+\mathbf{V}_E\cdot\nabla\right)T_i=-\frac{2}{3}T_i\nabla_{\parallel}V_{i\parallel}+\boxed{\frac{2}{3}T_i\left[-n\nabla_{\parallel}V_{i\parallel}+n\left(\mathbf{V}_E+\mathbf{V}_{p_i}\right)\cdot\left(\boldsymbol{\kappa}+\nabla\ln B\right)\right]}$$

Beer model

(M. A. Beer and G. W. Hammett, PoP '96)

- **Guiding center density equation**

$$\frac{\partial \tilde{n}_i}{\partial t}+\mathbf{V}_{\tilde{\Phi}}\cdot\nabla\tilde{n}_i+B\nabla_\parallel\frac{n_0\tilde{V}_{i\parallel}}{B}+\frac{n_0}{2T_{i0}}\left[\hat{\nabla}_\perp^2\mathbf{V}_{\tilde{\Phi}}\right]\cdot\nabla\tilde{T}_{i\perp}-\frac{en_0}{T_{i0}}\left(1+\frac{1}{2}\eta_i\hat{\nabla}_\perp^2\right)i\omega_*\tilde{\Phi}+\frac{en_0}{T_{i0}}\left(2+\frac{1}{2}\hat{\nabla}_\perp^2\right)i\omega_d\tilde{\Phi}+\frac{1}{T_{i0}}i\omega_d\left(\tilde{p}_{i\parallel}+\tilde{p}_{i\perp}\right)=0$$

- **Guiding center parallel velocity equation**

$$\frac{\partial \tilde{V}_{i\parallel}}{\partial t}+\mathbf{V}_{\tilde{\Phi}}\cdot\nabla\tilde{V}_{i\parallel}+\frac{e}{m_i}\nabla_\parallel\tilde{\Phi}+\frac{B}{m_in_0}\nabla_\parallel\frac{\tilde{p}_{i\parallel}}{B}+\boxed{\frac{1}{2}\left[\hat{\nabla}_\perp^2\mathbf{V}_{\tilde{\Phi}}\right]\cdot\nabla\frac{\tilde{q}_{i\perp}}{T_{i0}}}+\left(\frac{\tilde{T}_{i\perp}}{m_i}+\frac{e}{2m_i}\hat{\nabla}_\perp^2\tilde{\Phi}\right)\nabla_\parallel\ln B+\boxed{\frac{1}{p_{i0}}i\omega_d\left(\tilde{q}_{i\parallel}+\tilde{q}_{i\perp}\right)}+4i\omega_d\tilde{V}_{i\parallel}=0$$

- **Guiding center parallel & perpendicular pressure equation**

$$\frac{\partial \tilde{p}_{i\parallel}}{\partial t}+\mathbf{V}_{\tilde{\Phi}}\cdot\nabla\tilde{p}_{i\parallel}+B\nabla_\parallel\frac{1}{B}\boxed{\left(\tilde{q}_{i\parallel}}+p_{i0}\tilde{V}_{i\parallel}\right)+2p_{i0}B\nabla_\parallel\frac{\tilde{V}_{i\parallel}}{B}+\frac{n_0}{2}\left[\hat{\nabla}_\perp^2\mathbf{V}_{\tilde{\Phi}}\right]\cdot\nabla\tilde{T}_{i\perp}+2\tilde{q}_{i\perp}\nabla_\parallel\ln B$$
$$-en_0\left(1+\eta_i+\frac{1}{2}\eta_i\hat{\nabla}_\perp^2\right)i\omega_*\tilde{\Phi}+en_0\left(4+\frac{1}{2}\hat{\nabla}_\perp^2\right)i\omega_d\tilde{\Phi}+4i\omega_d\tilde{p}_{i\parallel}+\boxed{n_0i\omega_d\left(3\tilde{T}_{i\parallel}+\tilde{T}_{i\perp}\right)+2n_0\left|\omega_d\right|\left(\nu_1\tilde{T}_{i\parallel}+\nu_2\tilde{T}_{i\perp}\right)}=0$$

$$\frac{\partial \tilde{p}_{i\perp}}{\partial t}+\mathbf{V}_{\tilde{\Phi}}\cdot\nabla\tilde{p}_{i\perp}+B^2\nabla_\parallel\frac{1}{B^2}\boxed{\left(\tilde{q}_{i\perp}}+p_{i0}\tilde{V}_{i\parallel}\right)+\frac{1}{2}\left[\hat{\nabla}_\perp^2\mathbf{V}_{\tilde{\Phi}}\right]\cdot\nabla\tilde{p}_{i\perp}+n_0\left[\hat{\hat{\nabla}}_\perp^2\mathbf{V}_{\tilde{\Phi}}\right]\cdot\nabla\tilde{T}_{i\perp}$$
$$-en_0\left[1+\frac{1}{2}\hat{\nabla}_\perp^2+\eta_i\left(1+\frac{1}{2}\hat{\nabla}_\perp^2+\hat{\hat{\nabla}}_\perp^2\right)\right]i\omega_*\tilde{\Phi}+en_0\left(3+\frac{3}{2}\hat{\nabla}_\perp^2+\hat{\hat{\nabla}}_\perp^2\right)i\omega_d\tilde{\Phi}+3i\omega_d\tilde{p}_{i\perp}+\boxed{n_0i\omega_d\left(\tilde{T}_{i\parallel}+2\tilde{T}_{i\perp}\right)+2n_0\left|\omega_d\right|\left(\nu_3\tilde{T}_{i\parallel}+\nu_4\tilde{T}_{i\perp}\right)}=0$$

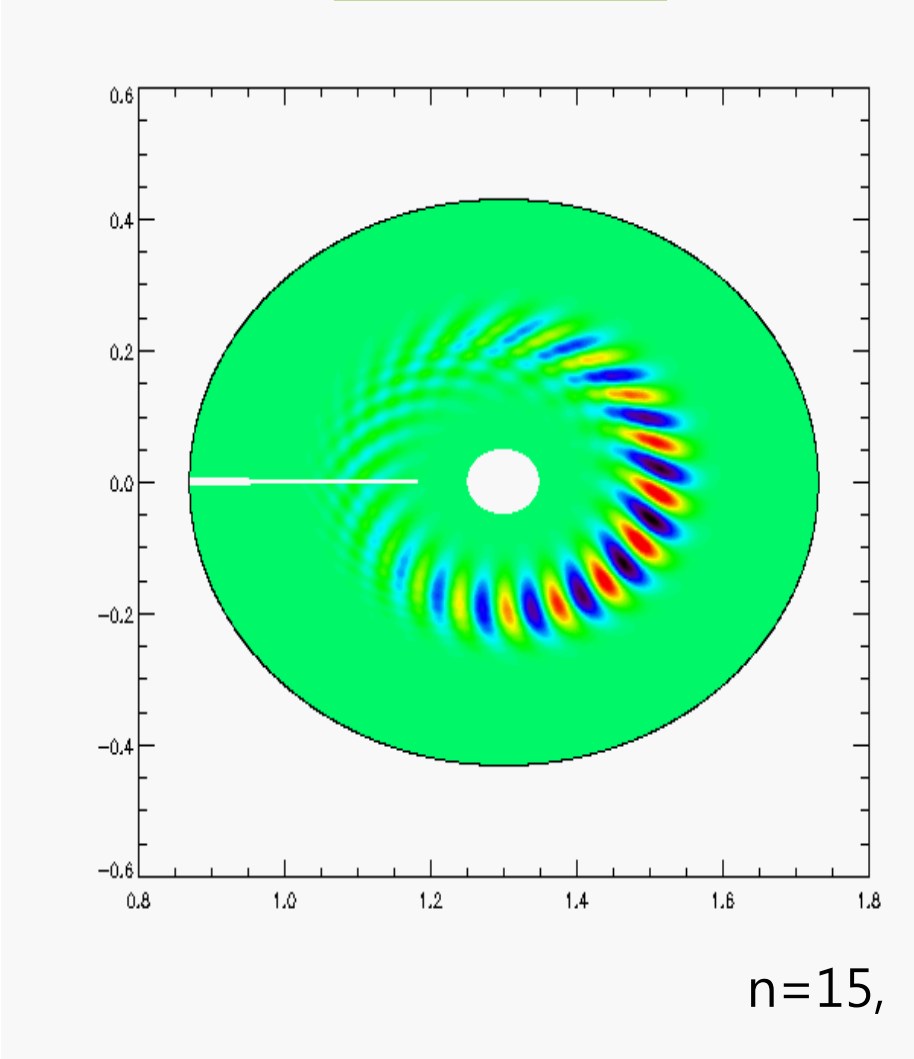
($\boxed{}$: Landau damping, $\boxed{}$: toroidal closure→ not implemented yet)

- **Quasi-neutrality**

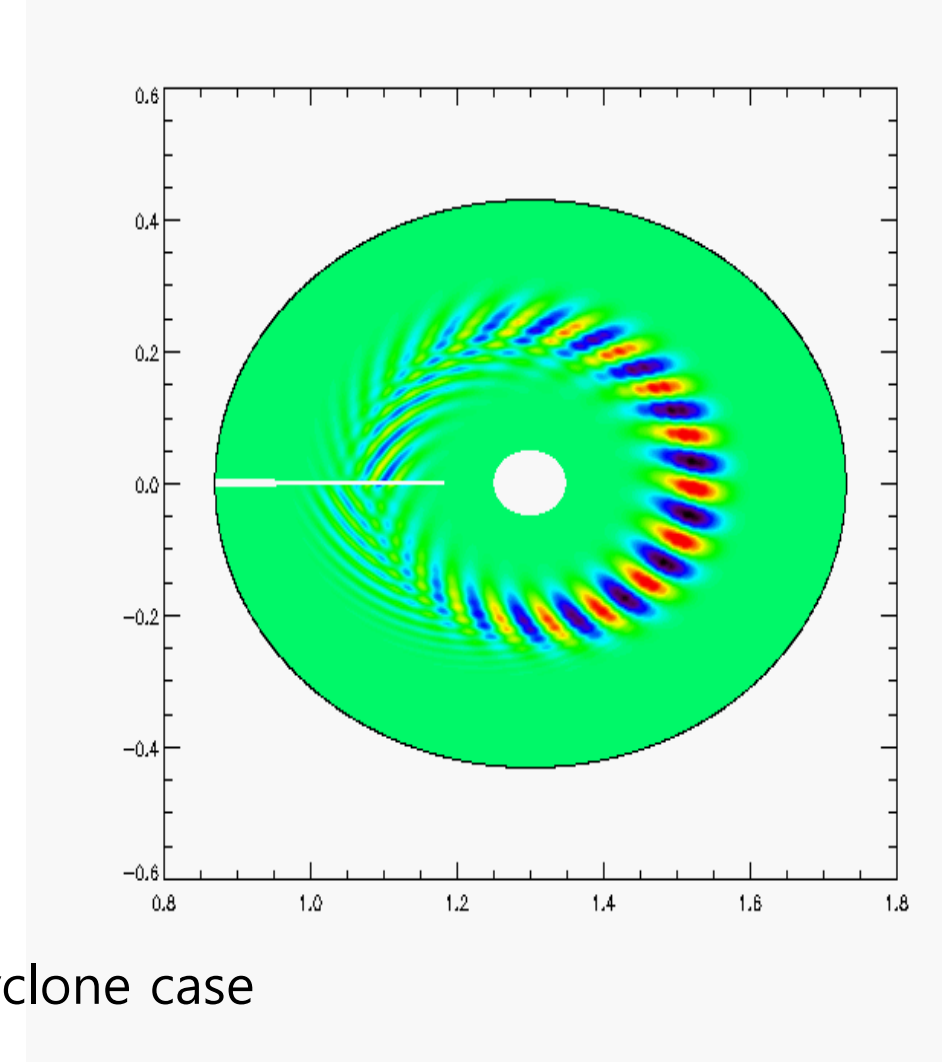
$$\frac{n_0}{T_{e0}}\tilde{\varphi}+\frac{n_0}{T_{i0}}\left(1-\Gamma_0\right)\tilde{\varphi}=\Gamma_0^{1/2}\tilde{n}_i+\frac{n_0}{T_{i0}}b\frac{\partial\Gamma_0^{1/2}}{\partial b}\tilde{T}_i \quad \text{where } \tilde{\Phi}=\Gamma_0^{1/2}\tilde{\varphi},\ \frac{1}{2}\hat{\nabla}_\perp^2\tilde{\Phi}=\left(b\frac{\partial\Gamma_0^{1/2}}{\partial b}\right)\tilde{\varphi},\ \hat{\hat{\nabla}}_\perp^2\tilde{\Phi}=b\frac{\partial^2}{\partial b^2}\left(b\Gamma_0^{1/2}\right)\tilde{\varphi},\ \Gamma_0^{1/2}\cong\frac{1}{1+b/2}$$

Eigenmode structures for TRB and Beer models

TRB model

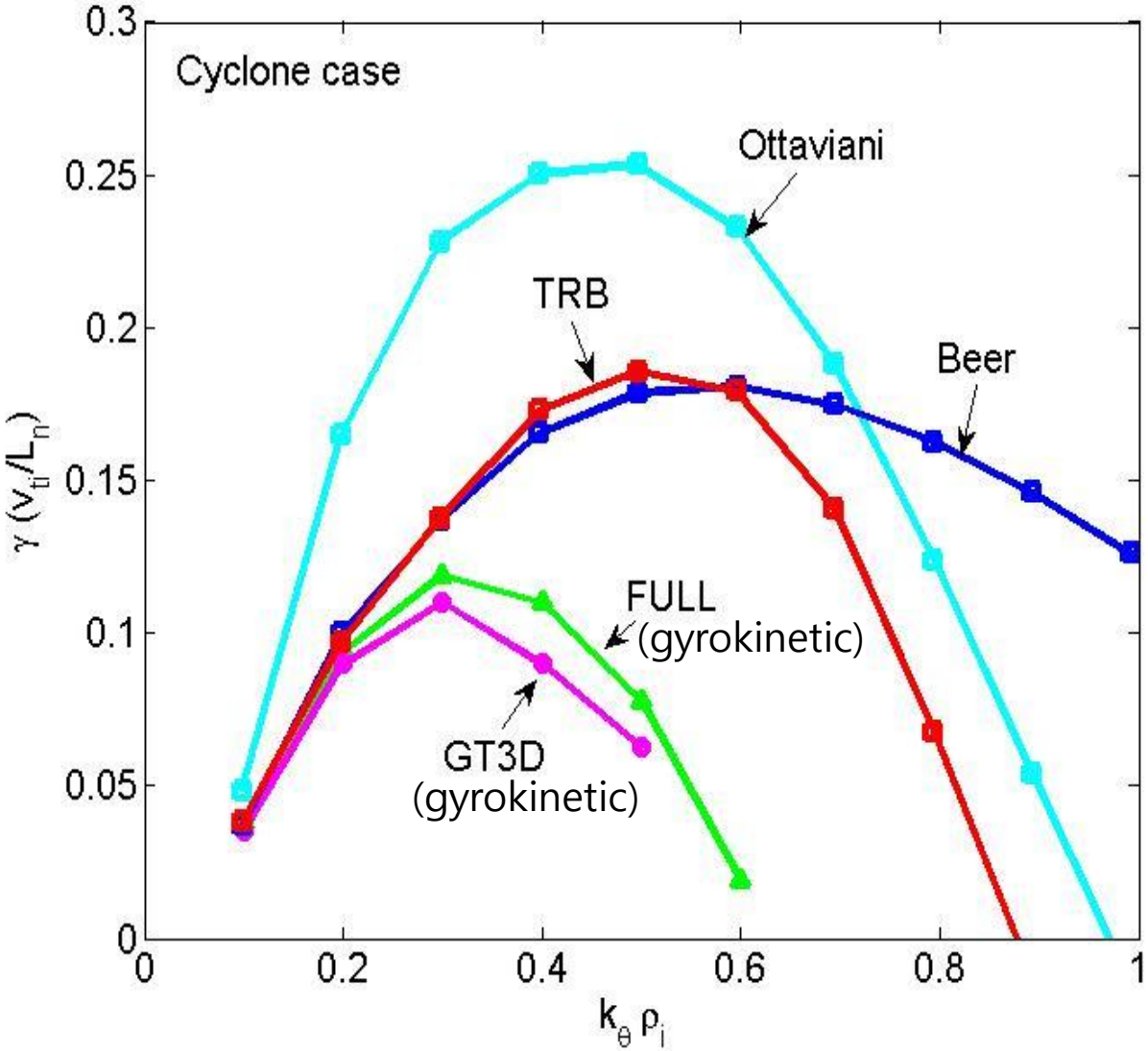


Beer model



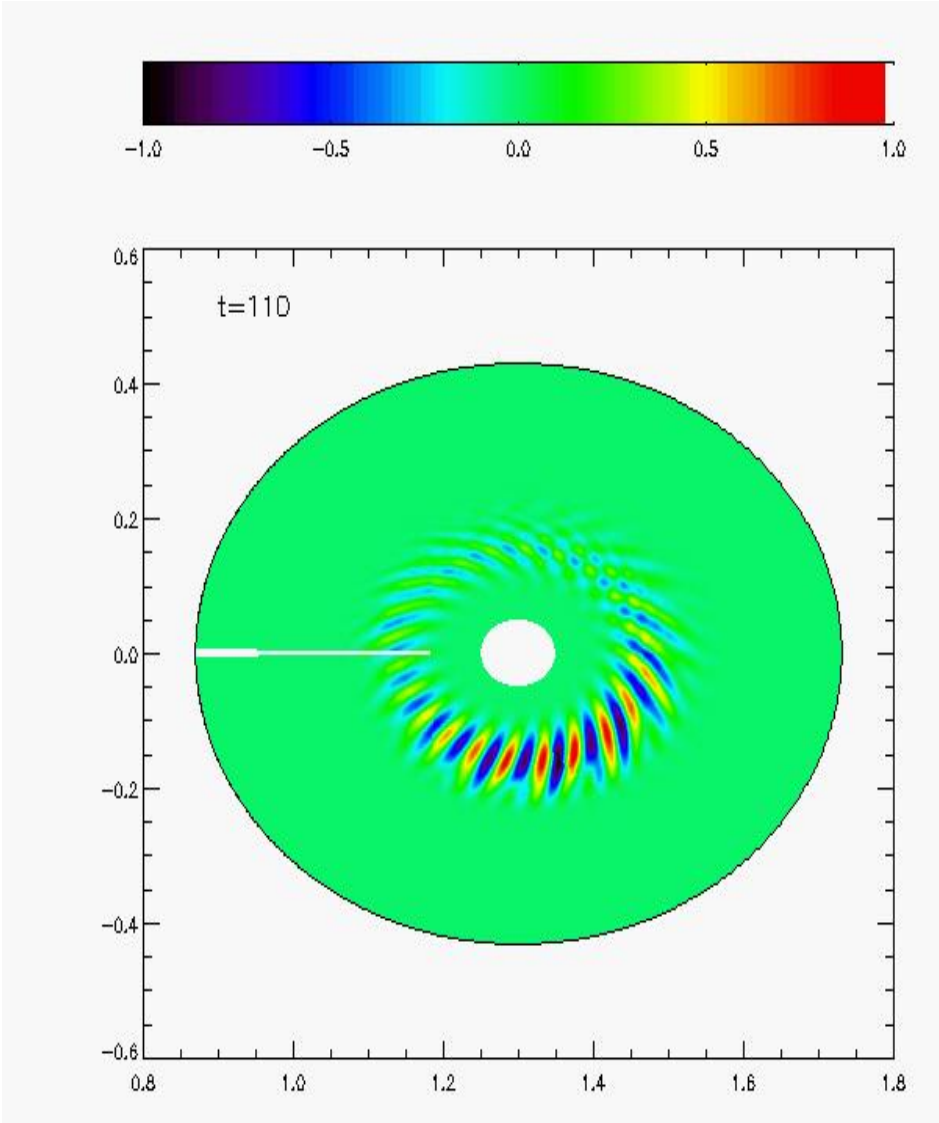
n=15, cyclone case

Comparison between models

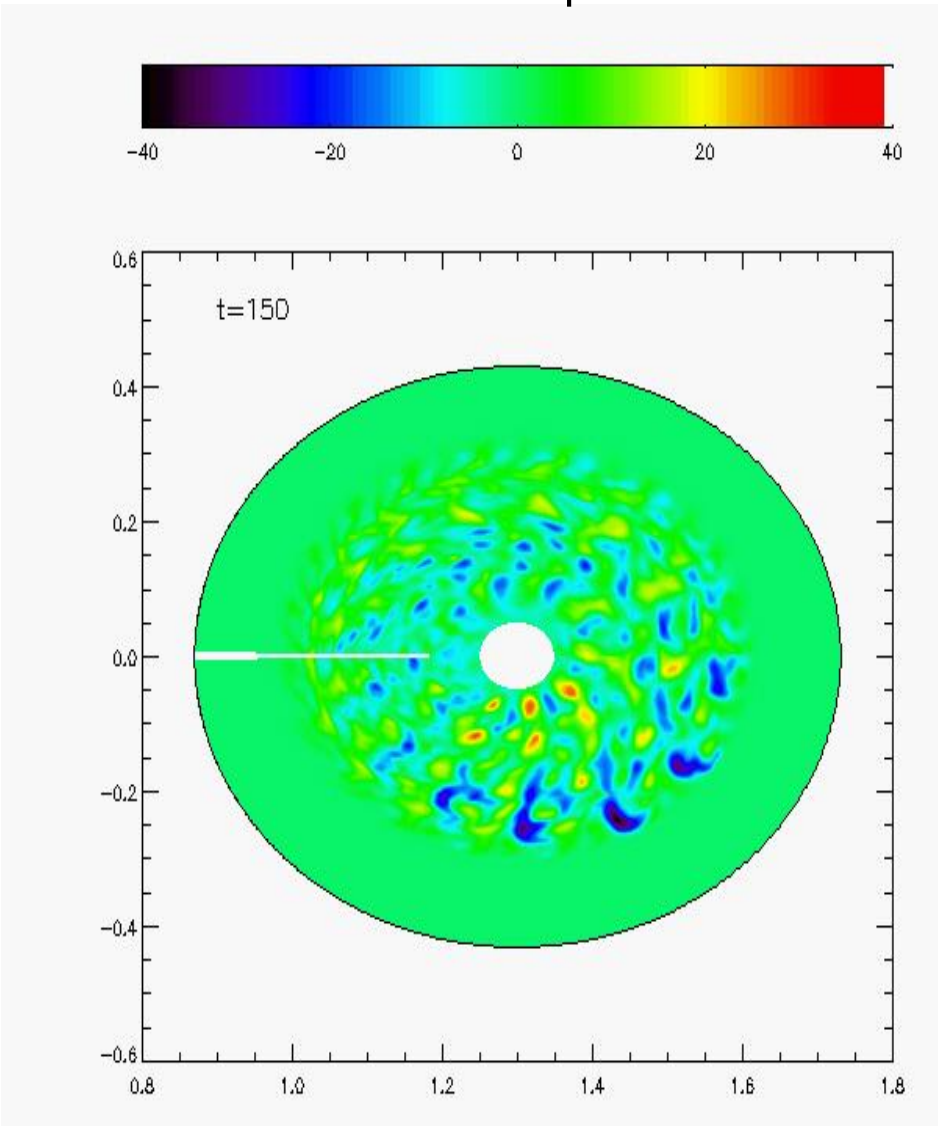


Nonlinear simulations

Linear phase

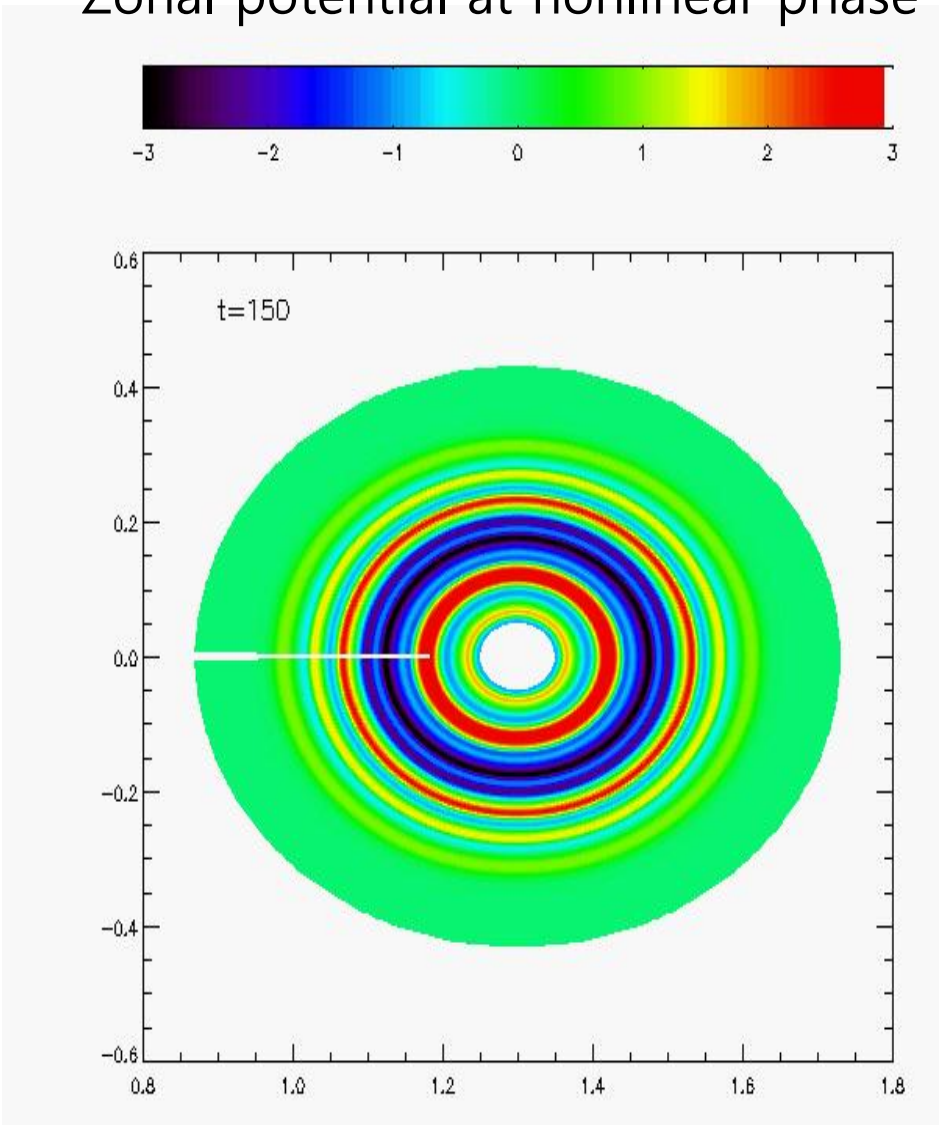


Nonlinear phase

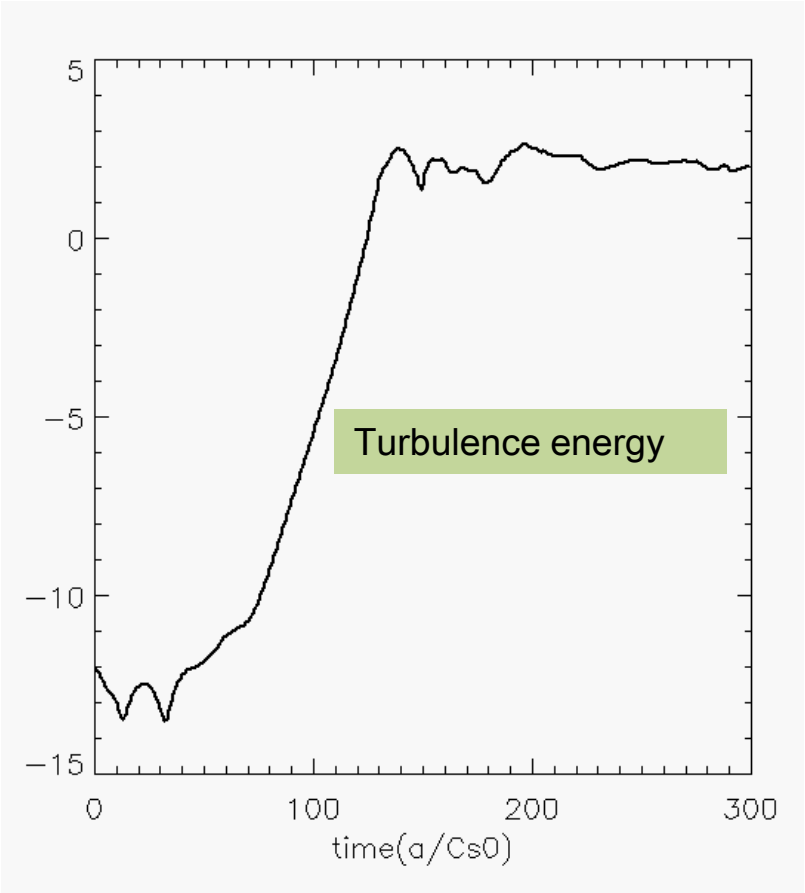


Nonlinear simulations

Zonal potential at nonlinear phase



- ITG mode growth is saturated by nonlinear interactions and zonal flows are produced.



Summary

- **Code development and verification for ITG simulation :**
  - Core gyrofluid modules developed using BOUT++ framework
  - Verification done using newly developed eigenvalue solver
  - Gyro-averaging effect investigated
  - Linear benchmark exercises among various models done (w/o Landau damping)
  - Nonlinear saturation and zonal flow generation observed

- **On-going works include:**
  - Implementation of Landau damping operator (see A.M. Dimits, et.al., GP8.00114)
  - Internal transport barrier formation study in reversed-shear plasmas: effects of non-resonant modes
  - Study for external-intrinsic torque interaction