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Influence of Equilibrium Shear Flow on Peeling-Ballooning Instability

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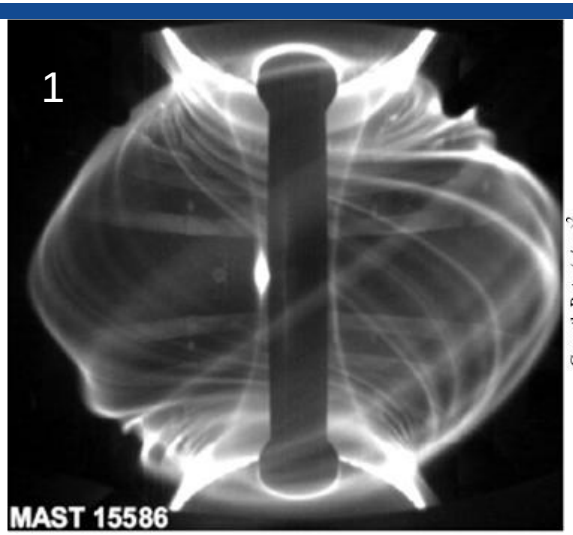
Abstract

The influence of equilibrium shear flow on peeling-ballooning modes is studied with the BOUT++ code. A set of reduced MHD equations is modified by adding an equilibrium shear flow. Flow shear has a stabilizing effect on peeling-ballooning instability, but its strength depends on toroidal mode number n. Modes with intermediate mode number n change from most unstable to most stable due to the existence of sufficient large flow shear, while low n and high n modes remain unstable. As a result, a feedback mechanism for ELM crash is proposed.

Outlines

- Research Motivation
- Simulation Model
- Simulation Results
  - Ideal
  - Ideal with diamagnetic terms
  - Non-ideal
- Discussion and Summary

Research Motivation



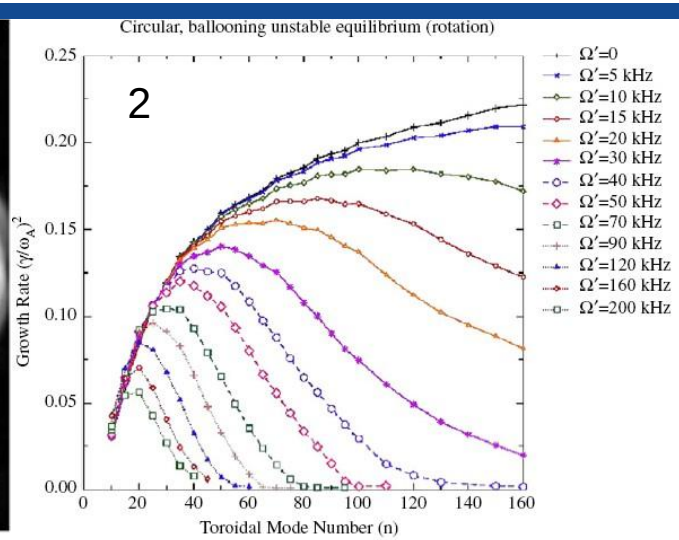
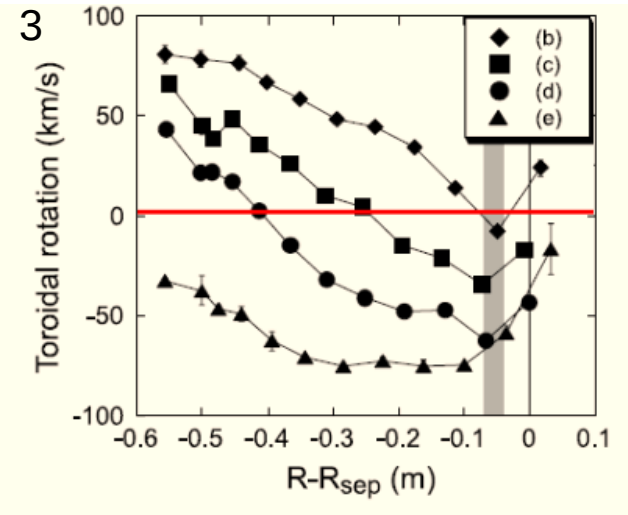


Fig1. ELM image from MAST (Scannell .R *Plasma Phys. Control. Fusion* ***49 1431***)

Fig2. Ideal MHD simulation about shear flow influence on peeling-ballooning mode (H.R.Wilson, *Plasma Phys. Control. Fusion* **48** (2006) A71–A84)



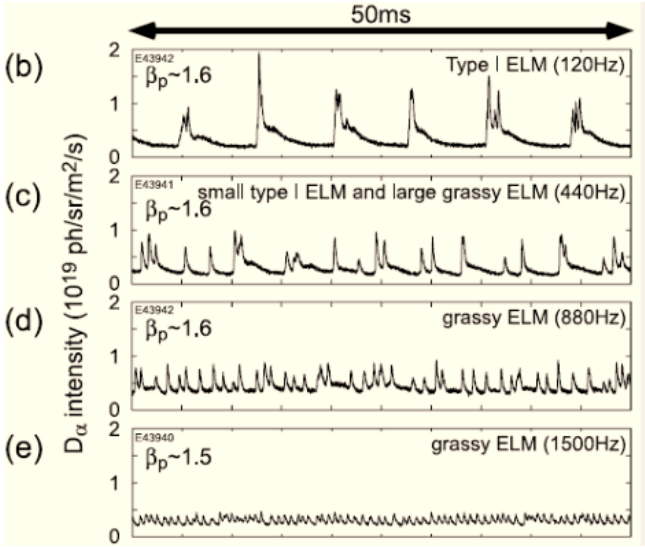


Fig3. Experiment shows rotation frequency can change ELM frequency significantly. (N.Oyama, Nucl. Fusion **45** (2005) 871–881)

Simulation Model: **Equations and definitions**

- 3-Fields Reduced MHD Equations with equilibrium flow

$$\begin{cases} \frac{dP}{dt} + \mathbf{V}_1 \cdot P_0 = 0 \\ \frac{d\varpi}{dt} + \mathbf{V}_1 \cdot \nabla \varpi_0 = B_0^2 \nabla_{\parallel} J_{\parallel} + 2\mathbf{b_0} \times \kappa \cdot \nabla P_1 \\ \frac{\partial A_{\parallel}}{\partial t} = -\partial_{\parallel}\phi - \tilde{\mathbf{b}} \cdot \nabla \Phi - \eta J_{\parallel} \end{cases}$$

- Definitions

$$\frac{d}{dt} = \frac{\partial}{\partial t} + (\mathbf{V}_{EP0} + \mathbf{V}_{EV0}) \cdot \nabla \qquad \Phi = \Phi_{dia0} + \Phi_{EV0}$$

$$\varpi = \frac{\rho_0}{B_0}(\nabla_{\perp}^2 \phi + \frac{1}{n_0 Z_i e}\nabla_{\perp}^2 P) \qquad \varpi_0 = \frac{\rho_0}{B_0}\nabla_{\perp}^2 \Phi_{V0}$$

$$\mathbf{V}_{EP0} = \frac{\mathbf{b}_0 \times \nabla \Phi_{dia0}}{B_0} \qquad \mathbf{V}_{EV0} = \frac{\mathbf{b}_0 \times \nabla \Phi_{V0}}{B_0}$$

Ion force balance

$$-\frac{1}{Zen_i}\nabla P_{i0} - \nabla \Phi_{dia0} - \nabla \Phi_{V0} + (\boxed{\mathbf{V}_{dia0} + \mathbf{V}_{EP0}} + \mathbf{V}_{EV0}) \times \mathbf{B} = 0$$

Net flow

$$\Phi_{dia0} = -\frac{1}{Zen_i}P_{i0}$$

Balance each other, but V_{EP0} is convection flow

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Simulation Model: **Net flow and diamagnetic effects**

Total convection flow

Kelvin-Helmholtz term

$$\frac{\partial \varpi}{\partial t} + (\mathbf{V}_{EP0} + \mathbf{V}_{EV0}) \cdot \nabla \varpi + \mathbf{V}_1 \cdot \nabla \varpi_0 = B_0^2 \nabla_{\parallel} J_{\parallel} + 2\mathbf{b_0} \times \kappa \cdot \nabla P_1$$

Diamagnetic convection flow

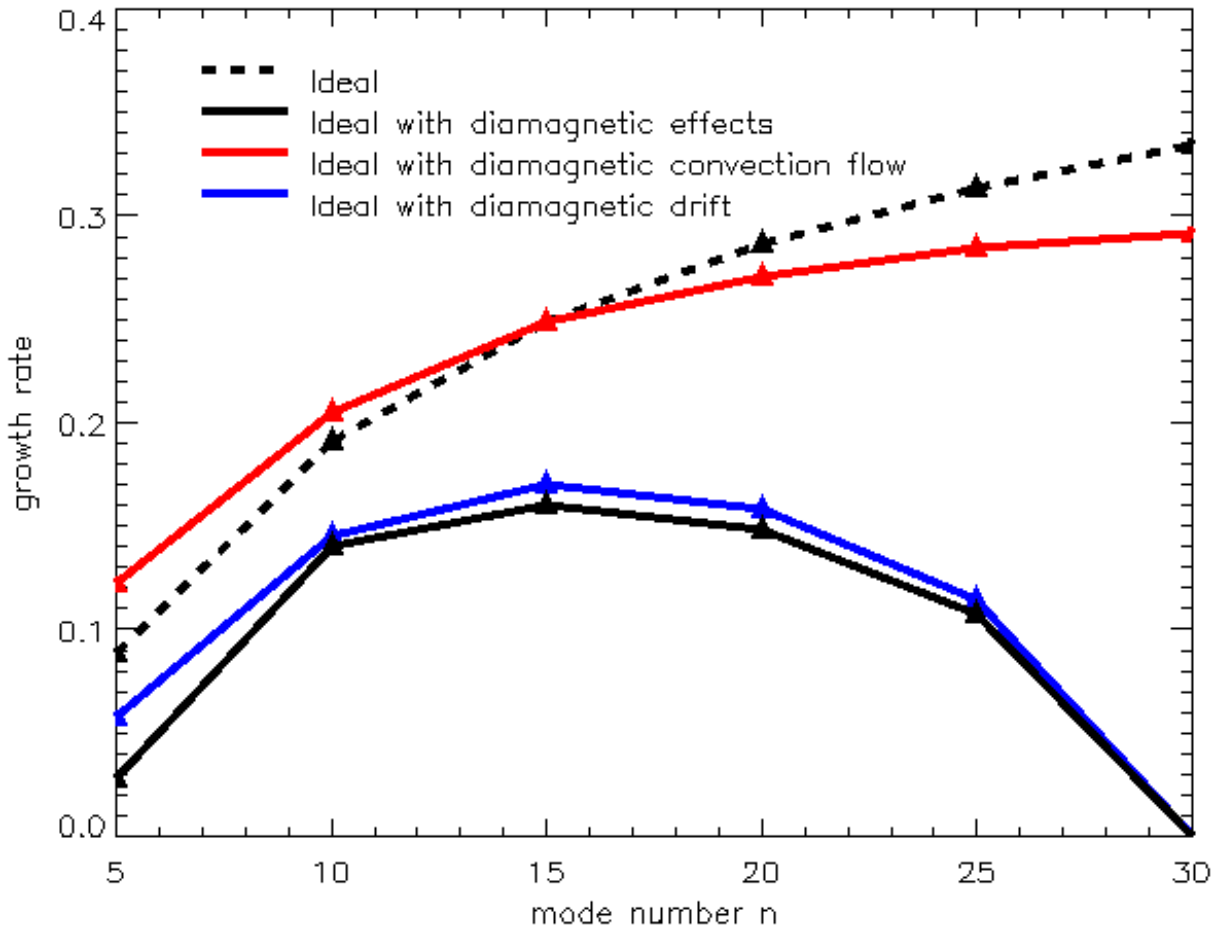
Net flow

Diamagnetic drift

$$\varpi = \frac{\rho_0}{B_0}(\nabla_\perp^2 \phi + \frac{1}{n_0 Z_i e} \nabla_\perp^2 P)$$

- **Diamagnetic effects:**
  - **Diamagnetic convection flow: E×B** flow that balances diamagnetic flow, is determined by pressure profile, introduces negative electric fi Φ_{dia0} ;
  - **Diamagnetic drift:** inversely depends on density;
- **Net flow:** perpendicular component of toroidal rotation, modeled by a simple function via Φ_{V0} , flexible;
- **Kelvin-Helmholtz term:** curl of net flow, can be switched off;
- **Total convection flow:** flow shear effects come from this total convection flow rather than the net flow.

Simulation Model: **Diamagnetic drift plays the dominant role rather than the diamagnetic convection flow**



- The two elements of diamagnetic effects have different influence on peeling-ballooning mode.

Fig. Diamagnetic drift is dominant, while balanced convection flow shows the same influence on mode growth rate, i.e. destabilizing low n modes and stabilizing high n modes.

Simulation Model: **Net flow expression**

- Consider the toroidal rotation $\mathbf{V} = R^2\Omega(\psi)\nabla\zeta$, by taking the ordering $k_{\parallel}/k_{\perp} \ll 1$, terms associated with $V_{\parallel}$ are removed.
- Perpendicular component is related with radial elctric field and if assuming Φ_{V0} is a flux function, its relation with toroidal rotation frequency is $\Phi'_{V_0}(\psi) = \Omega(\psi)$

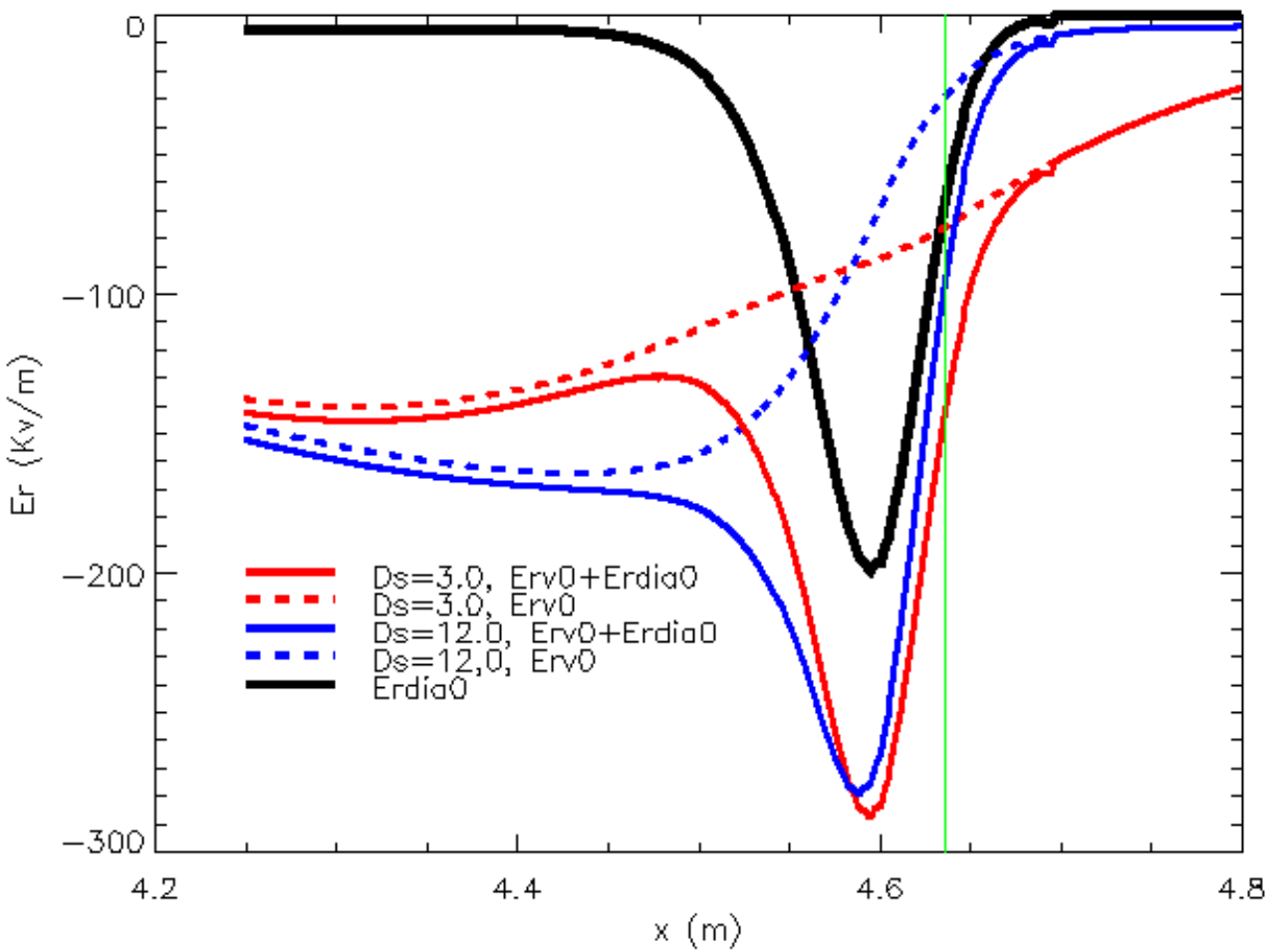
$$\mathbf{V}_{0\perp} = \frac{\mathbf{b}_0 \times \nabla\psi}{B_0}\frac{d\Phi_{V0}}{d\psi}$$

- Net flow profile is determined by a simple function

$$\frac{d\Phi_{V0}(\psi)}{d\psi} = D_0[1 - \tanh(D_s(x - x_0))] + C$$

where $x = (\psi - \psi_{axis})/(\psi_{sep} - \psi_{axis})$ is the normalized radial coordinate, D_s is the shear parameter D_0 flow magnitude parameter and x_0 determines flow location. Net flow direction is also changeable.

Simulation Model: **Net flow in co-direction to diamagnetic convection flow**

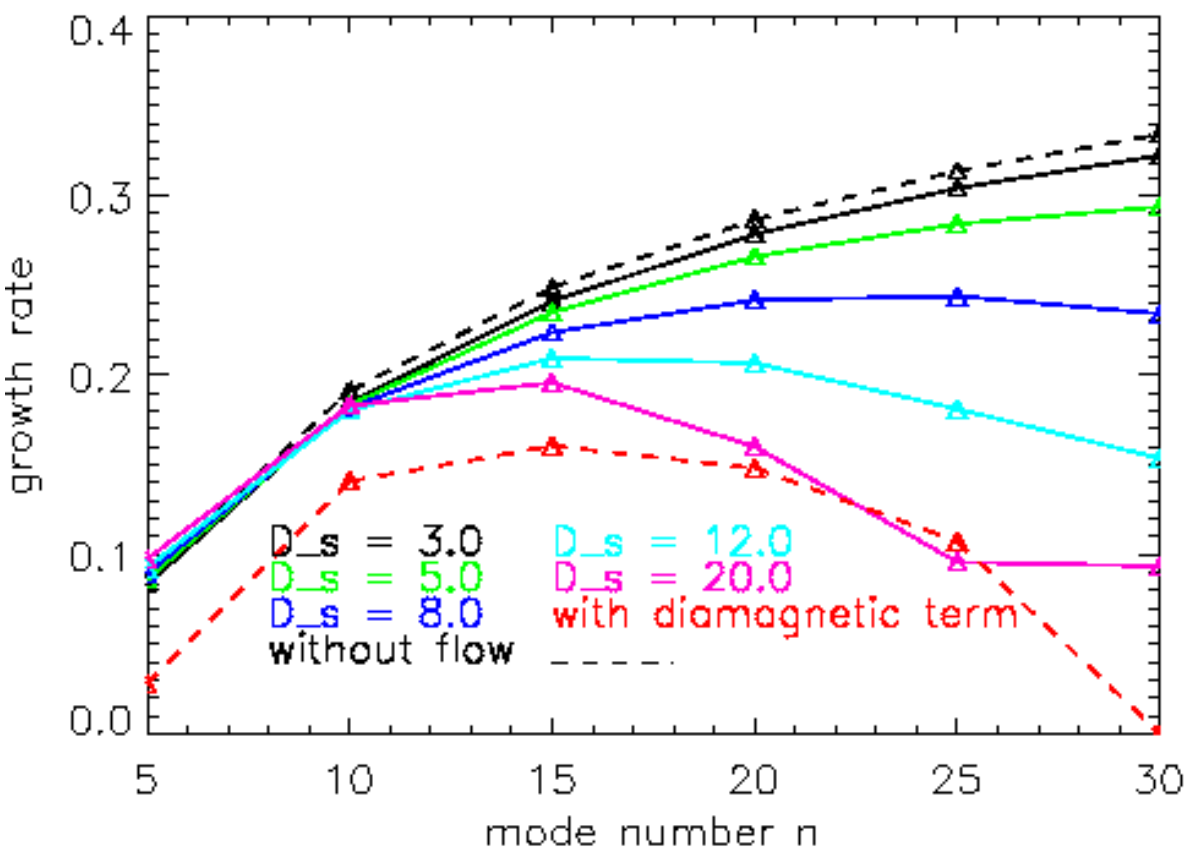


- Diamagnetic electric field is dominant at the pedestal in our simulation, which means the balanced convection flow is larger than the net flow we add.
- Since it is total convection flow that influence peeling-ballooning mode, new parameters are needed to describe the flow shear instead of Ds.

Fig. Black solid line is diamagnetic electric field, which shows reversed shear profile and is dominant at the at pedestal in our simulation. Red and blue dash lines are net flow electric field for Ds=3.0 and 12.0. Red and blue solid lines are the total electric field . Green line shows SOL boundary.

Simulation Results:

Flow shear stabilizes high n modes in ideal MHD without diamagnetic effect



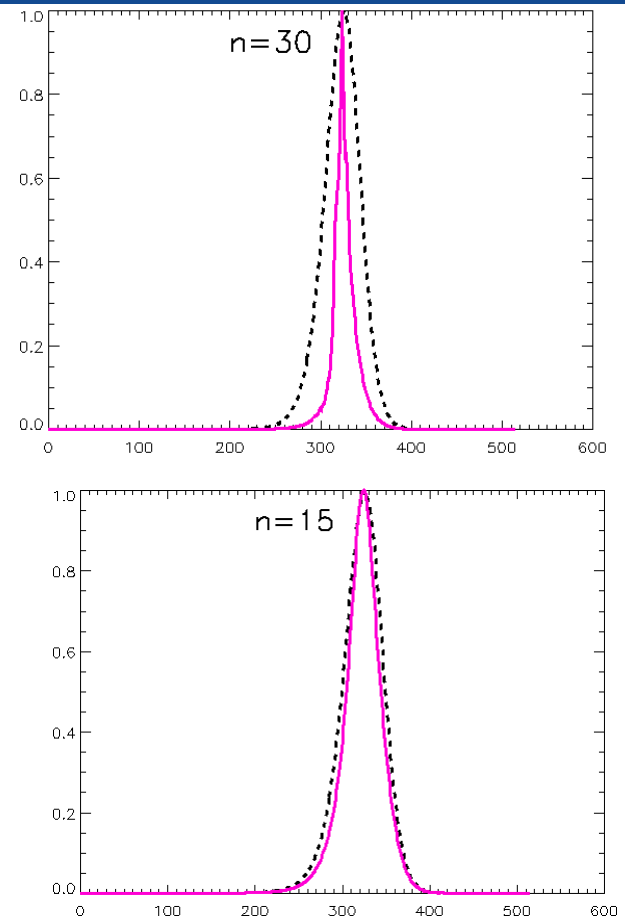
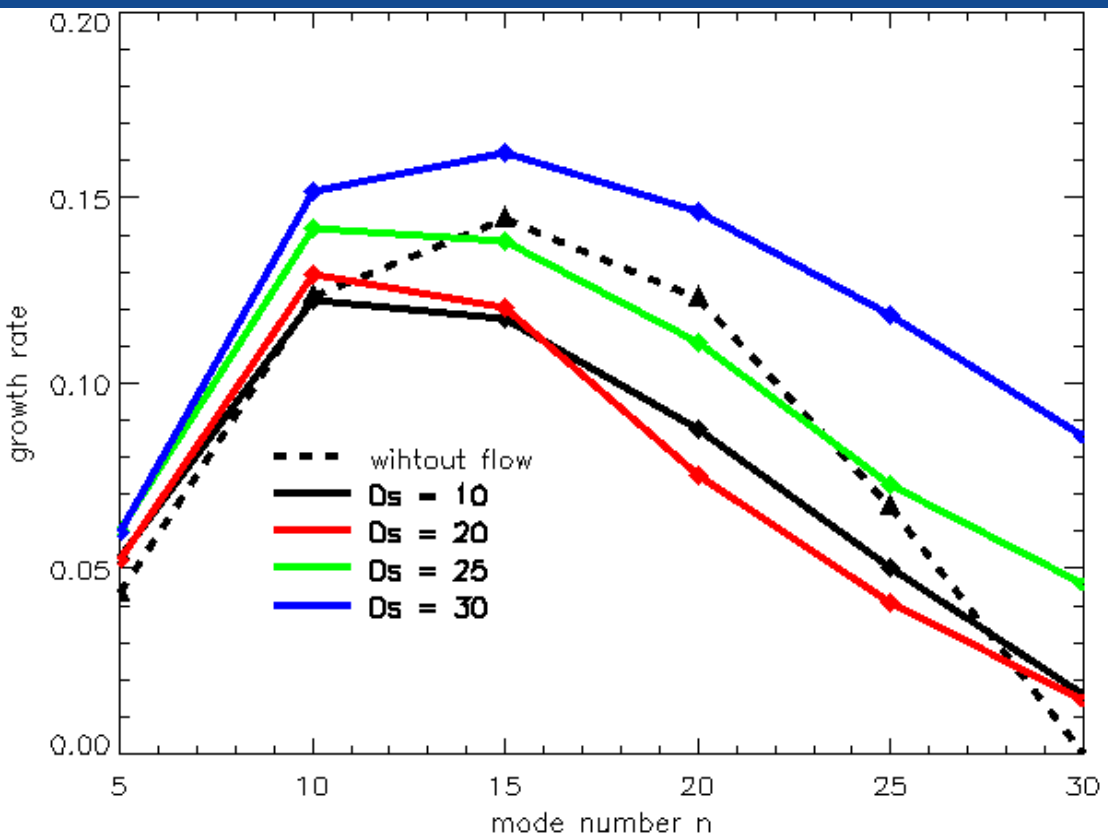
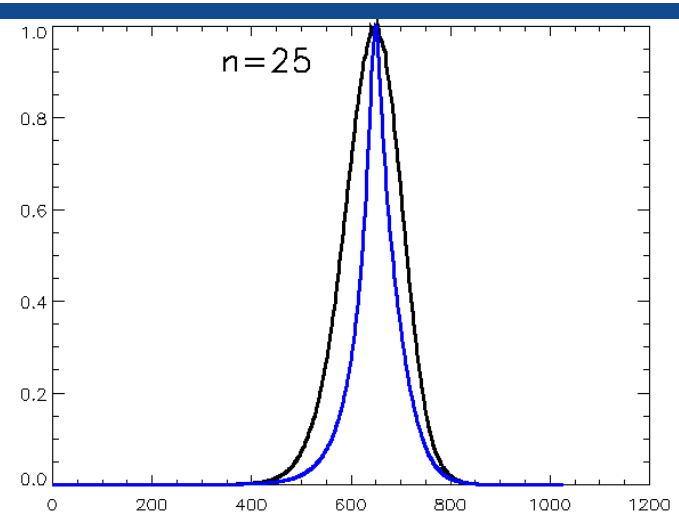


Fig. Result for ideal case without diamagnetic term. Red dash line describes the case with diamagnetic term but without flow. Kelvin-Helmholtz term is kept. (D_0=130)

Simulation Results: With diamagnetic effects, flow shear becomes destabilizing for high n modes





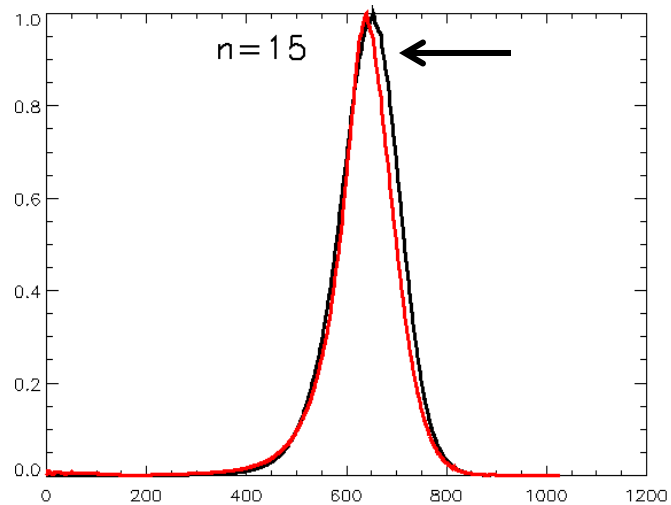
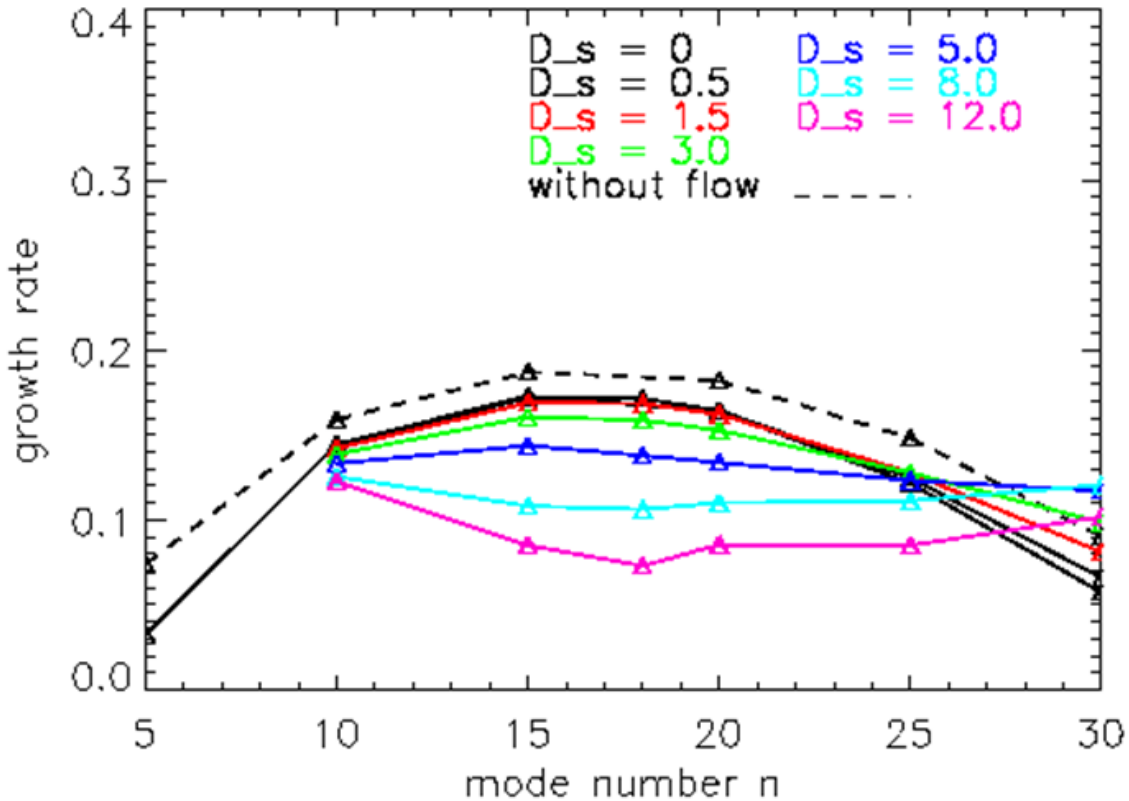
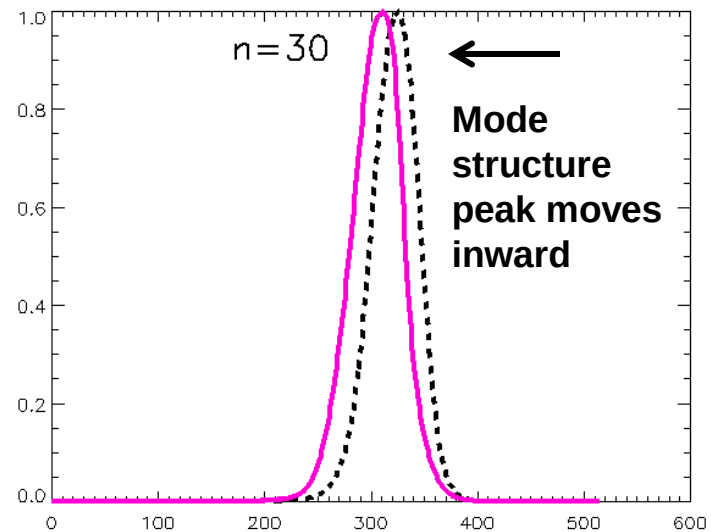


Fig. Mode growth rate for ideal case with diamagnetic effects. The Right side two figures show the mode structure. Since diamagnetic effects are dominant in our simulation, the overall result doesn't show significant change, i.e. the most unstable mode still has intermediate mode number. The mode structure for high n mode becomes much narrower while intermediate to low n mode do not show dramatic change in mode structure.

Simulation Results: With diamagnetic effects and resistivity, flow shear stabilizes intermediate n modes





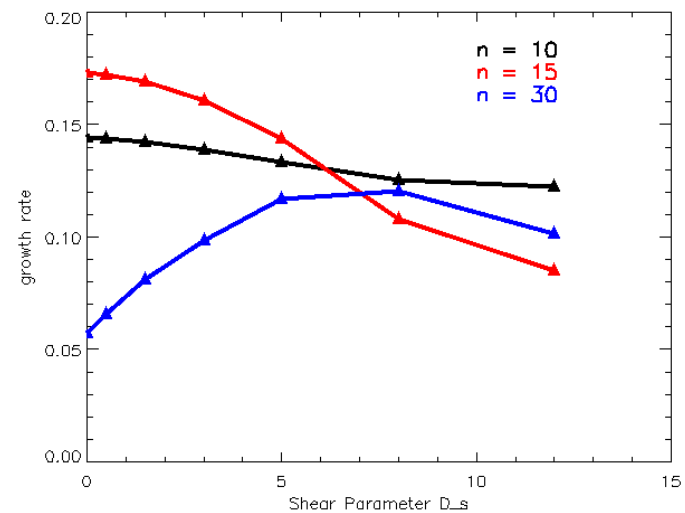


Fig. Non-ideal cases with diamagnetic effects, resistivity and hyper-resistivity, flow shear is stabilizing for mode with intermediate mode number(n=15~20) , while for low n and high n mode, at least no such strong stabilizing effect .

Simulation Model: **Net flow in counter-direction to diamagnetic convection flow**

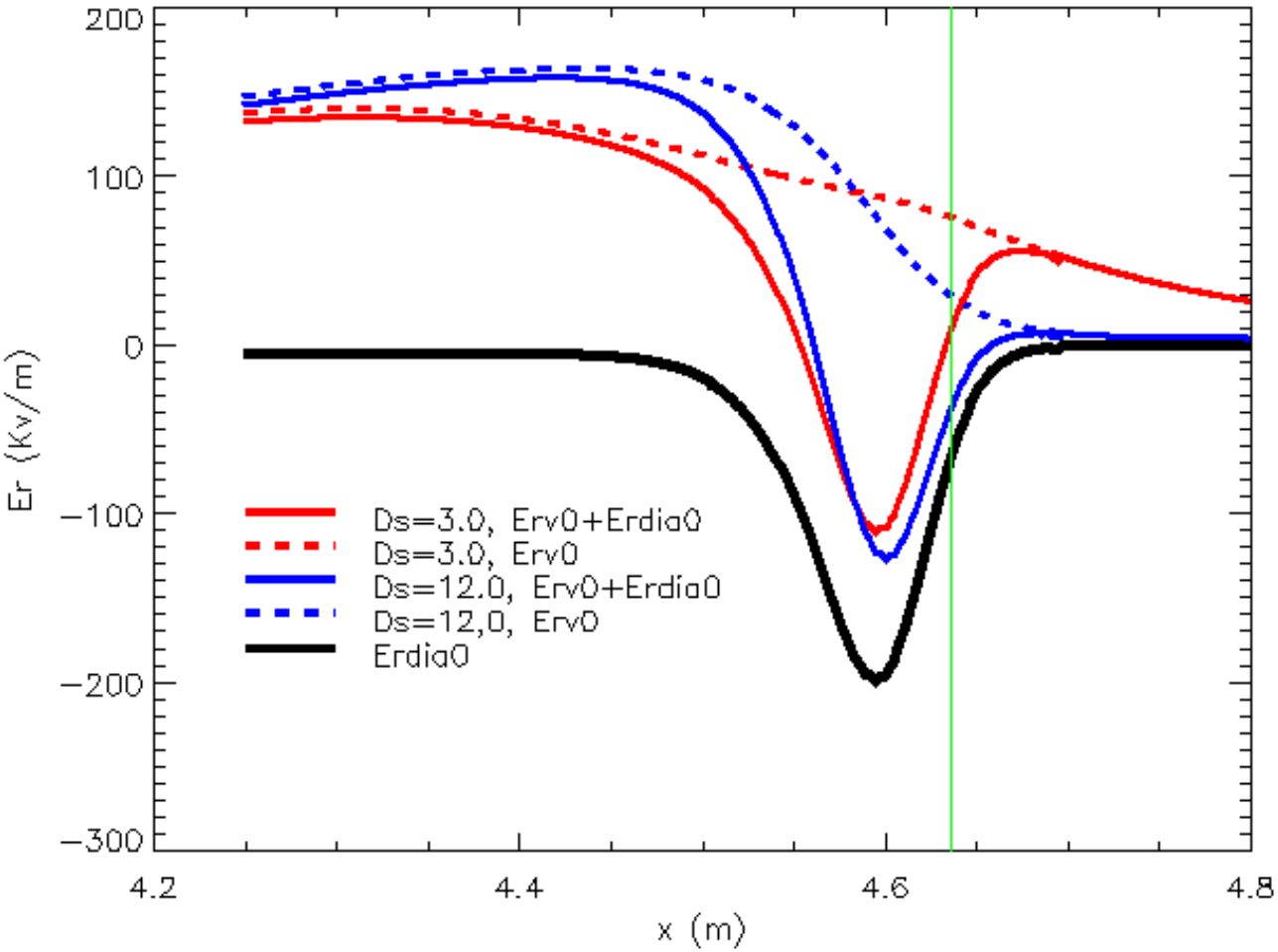
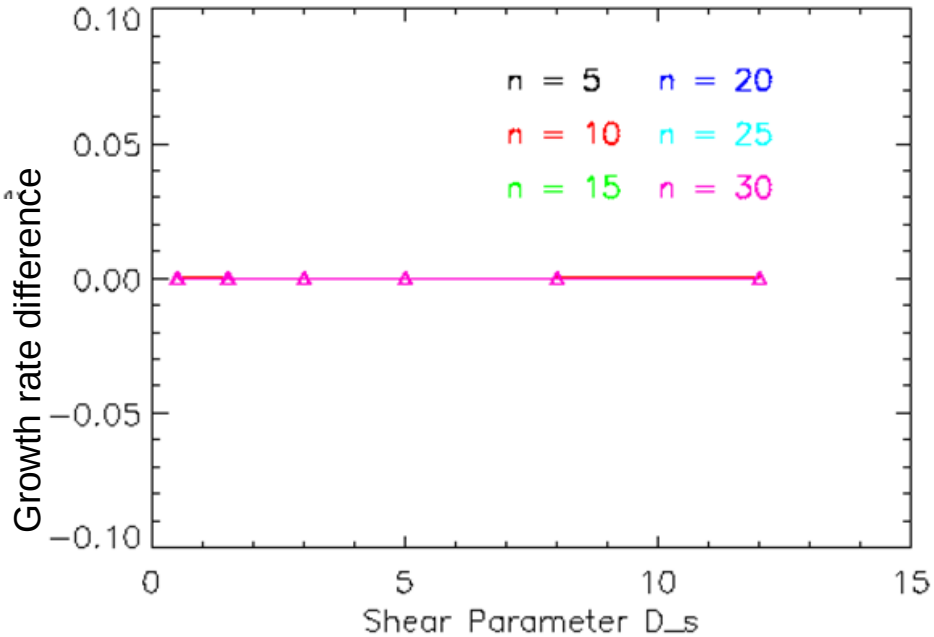


Fig. All lines have the same meaning as in left figure, but the direction of net flow is changed, so the direction of net flow electric field is positive in this figure.

- At inner region, electric field is governed by net flow, however this region is not import for peeling-ballooning modes because these modes are highly localized at pedestal;
- Even for positive net flow electric field, pedestal still has a negative electric field, and this means at the pedestal, the direction of total convection flow is not changed unless a extremely large net flow is applied;
- Because the shear of diamagnetic electric field is zero at its largest point, the shear of total convection flow is governed by net flow.

Simulation Results: I. Flow shear effect is symmetric in flow direction for ideal MHD; II. Kelvin-Helmholtz term increases growth rate significantly.



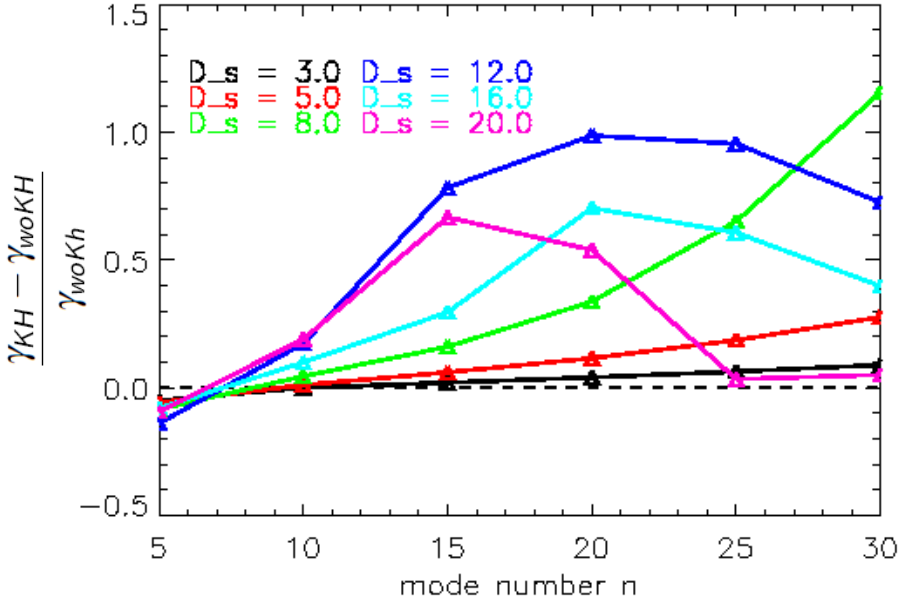
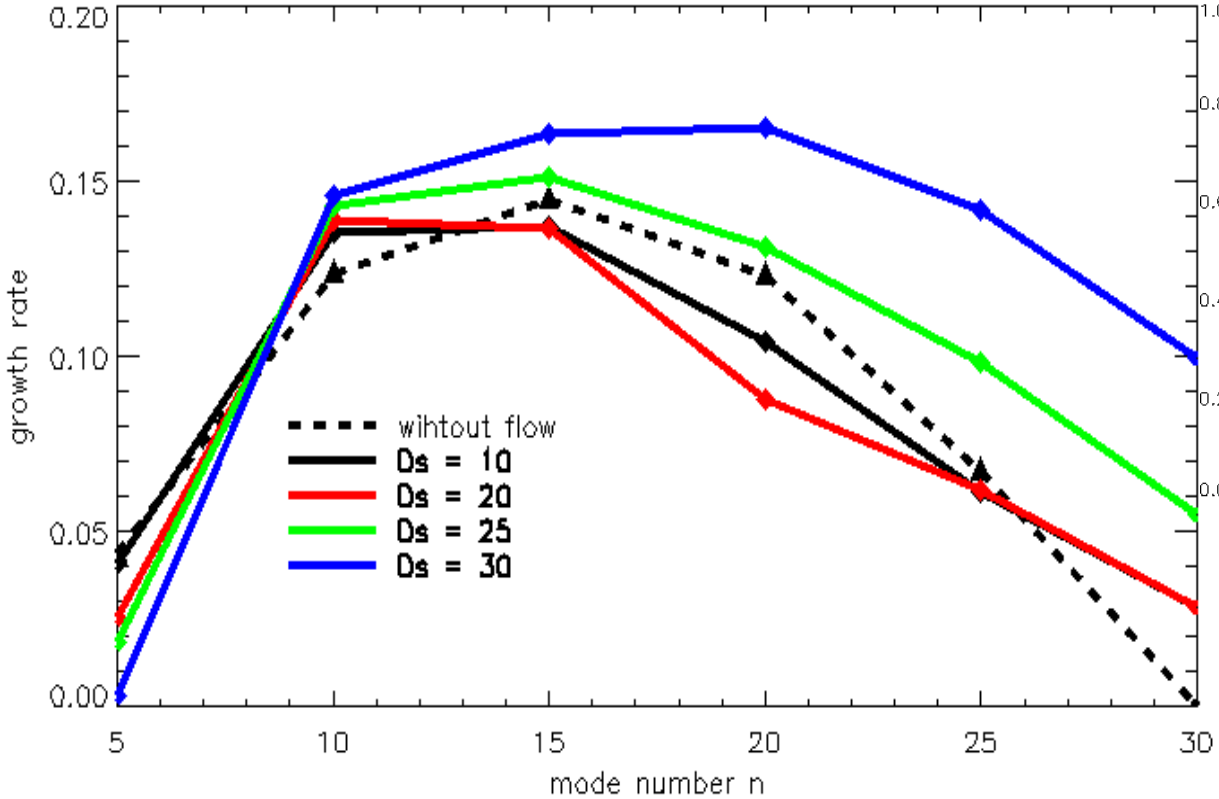
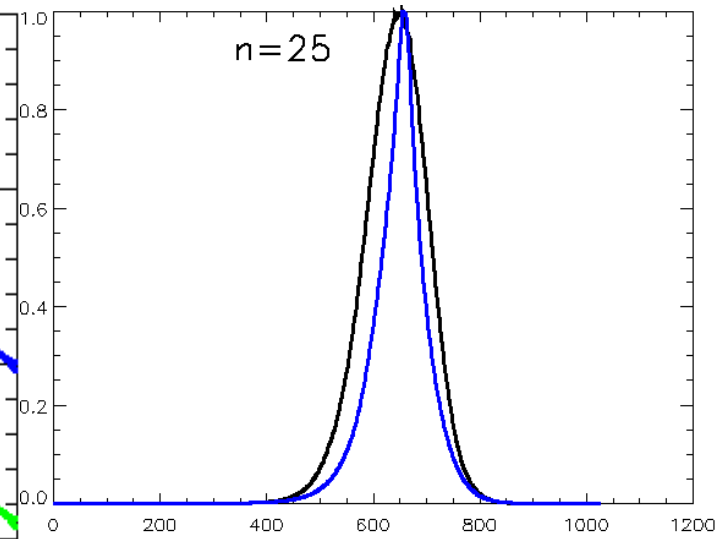


Fig. Growth rate difference between two opposite net flow directions. For ideal case, flow direction doesn't change mode growth rate, so we can for ideal case, we have **flow direction symmetry**.

Fig. Relative change in growth rate due to Kelvin-Helmholtz term. Kelvin-Helmholtz term is destabilizing for n=10~30. When flow shear increases, the destabilizing effect is mainly on intermediate n modes.

Simulation Results: **Net flow direction doesn't show much influence on mode growth rate.**





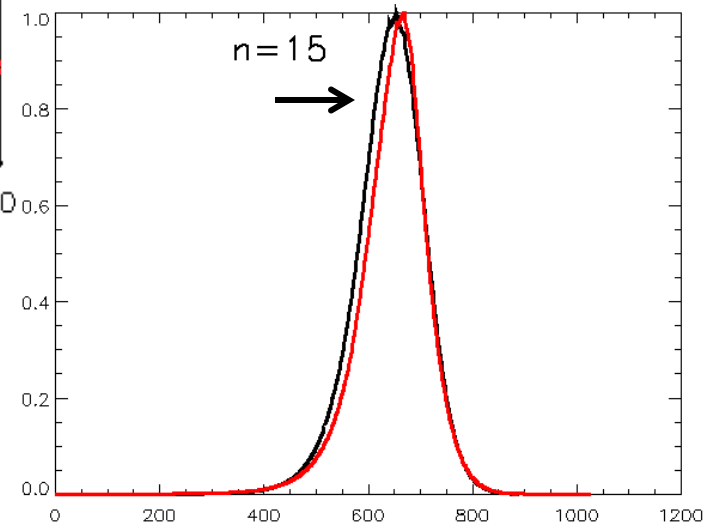
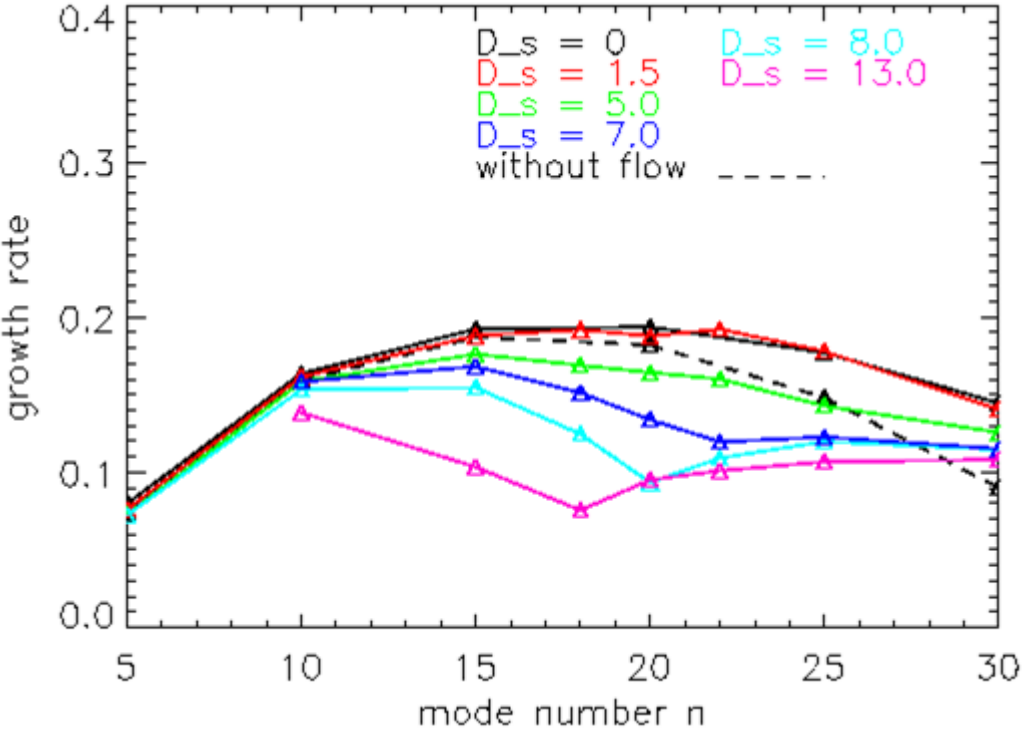
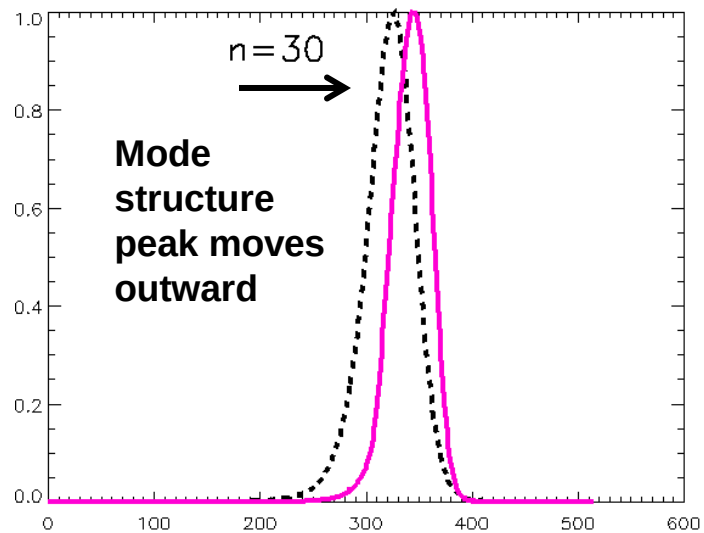


Fig. The change of net flow direction doesn't have much influence on high n mode, but for low n mode, we see their growth rate decreases when shear parameter increases.

Simulation Results: The change of net flow direction leads to opposite move of mode structure





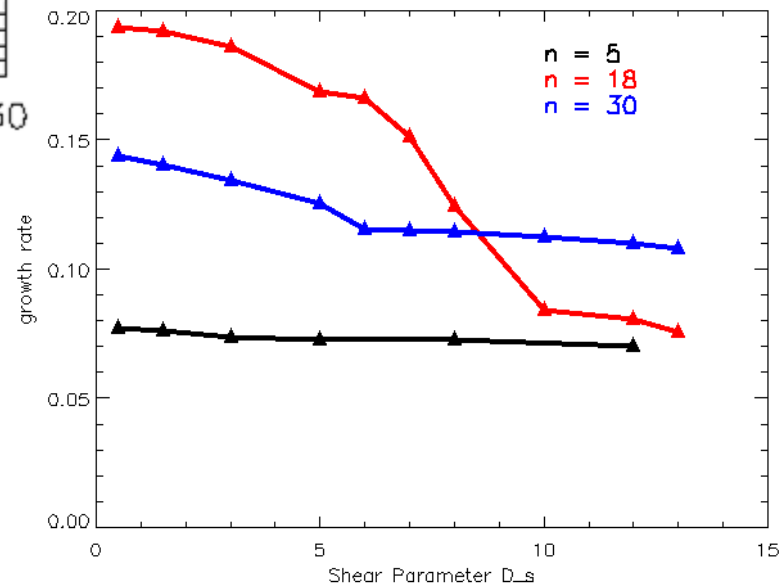
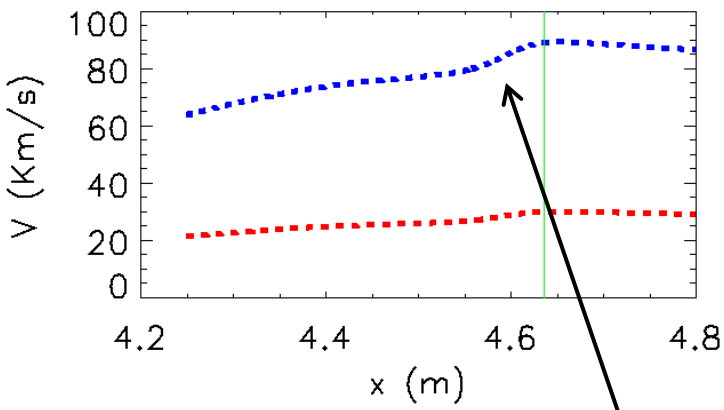


Fig. These two figure shows clearly that in non-ideal simulations, flow shear has strongest stabilizing effect on modes with intermediate mode number(n=15~20).

Simulation Model: When $\Omega(\psi) = \frac{d\Phi_{V0}}{d\psi} = const$, flow velocity is still sheared because of the non-uniform B field

- A constant convection flow should only change the mode frequency by Doppler shift, so in our simulation without diamagnetic effects, we should see the mode growth rate doesn't change for these flat net flow profile except very small difference caused by the non-uniform magnetic field.



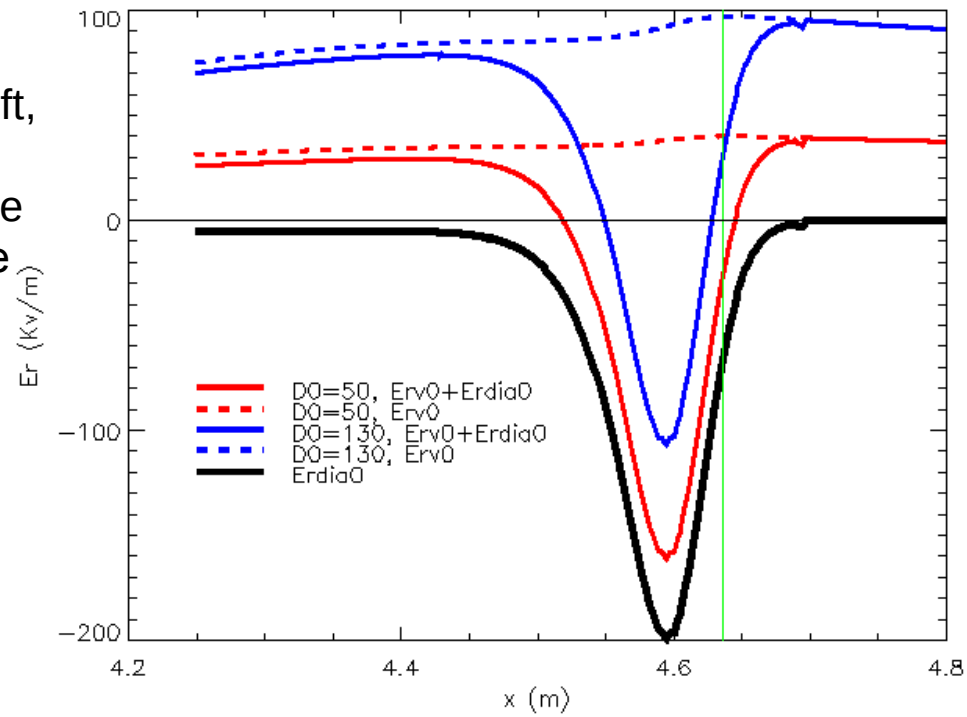


Fig. Set D_s=0 and change the value of D_0.

$$V_{0\perp} = \frac{RB_p}{B}\frac{d\Phi_{V0}}{d\psi} \Rightarrow \frac{\partial V_{0\perp}}{\partial r} = \boxed{\frac{\partial}{\partial r}\left(\frac{RB_p}{B}\right)}\frac{d\Phi_{V0}}{d\psi} + \frac{RB_p}{B}\frac{d\psi}{dr}\frac{d^2\Phi_{V0}}{d\psi^2}$$

Geometry induced shear

Vanish if Ds=0

Simulation Results: **Flat net flow shows little influence on growth rate in ideal MHD, which is consistent with Doppler shift.**

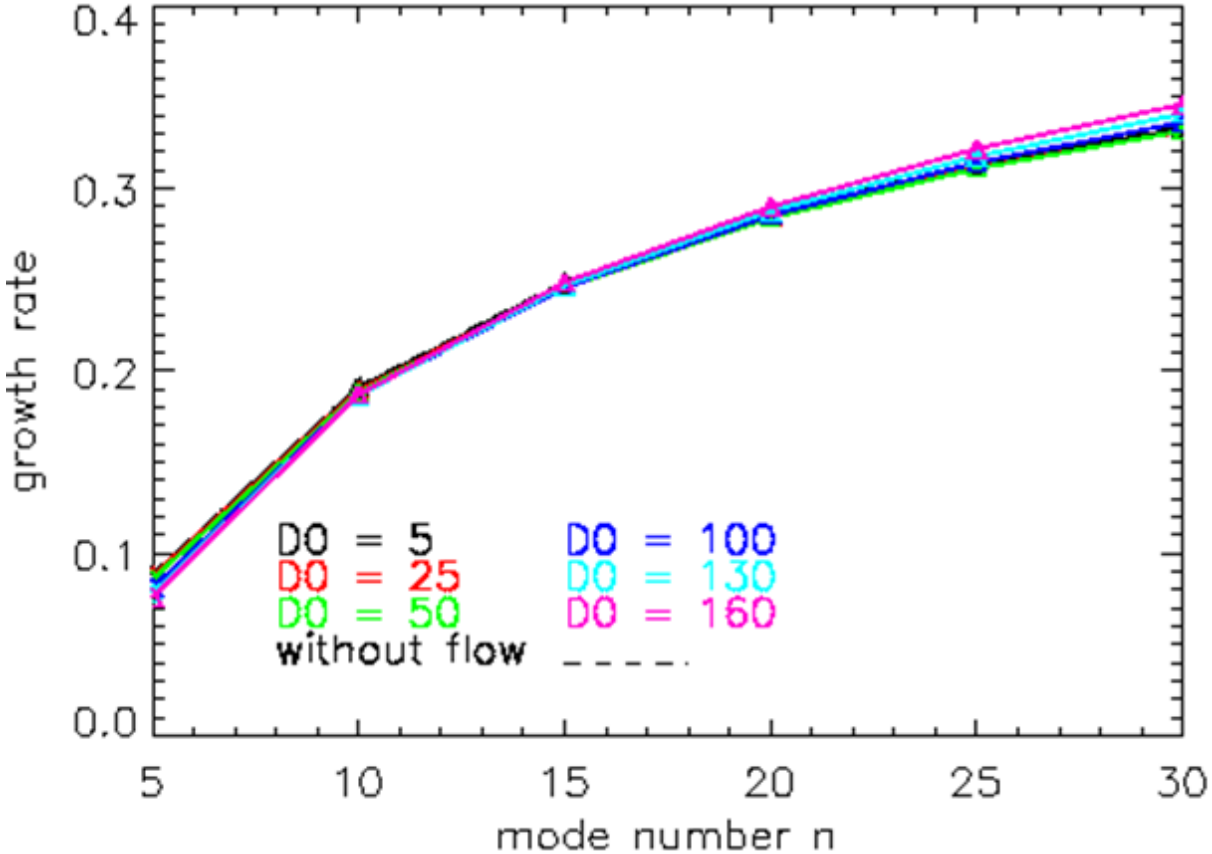


Fig. For ideal case, when D_s=0, mode growth rates do not change much as flow amplitude increases dramatically. The small difference is caused by the non-uniform nature of magnetic field. This shows good agreement with our expectation

Simulation Results: Flow shear from non-uniform B field destabilizes high n modes

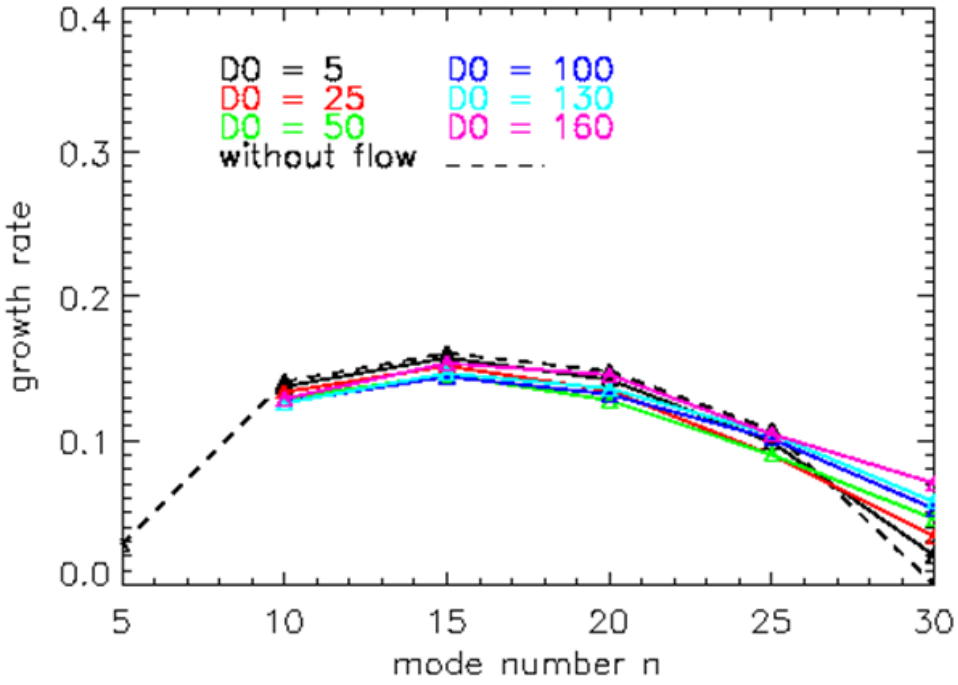


Fig. For negative net flow electric field.

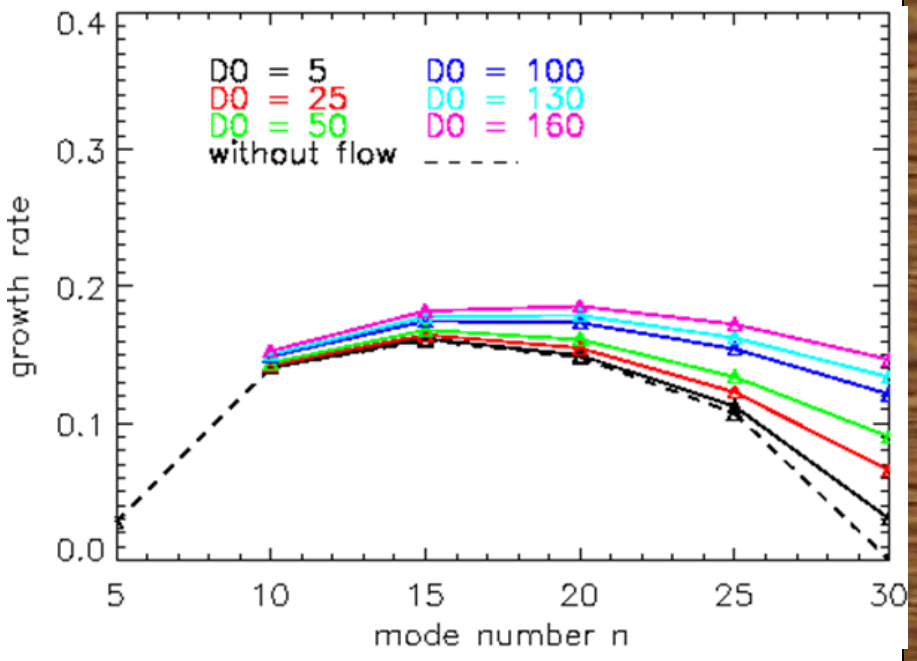


Fig. For positive net flow electric field.

- Diamagnetic effects amplifies the weak non-uniform property of the flat flow;

Simulation Results: **The influence of net flow is strongest when the largest flow shear locates at the position of largest pressure gradient.**

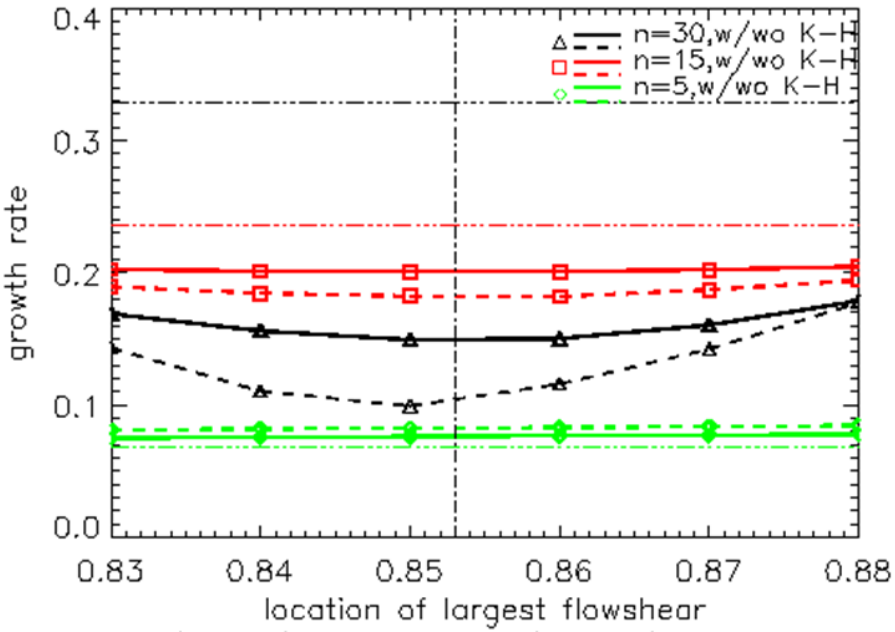


Fig. Since peeling-ballooning mode is highly localized in radial direction, the location of largest flow shear can influence the mode growth rate and structure as in the figure. The vertical black dot line shows the position of largest pressure gradient. Horizontal dot lines shows the growth rates without flow for n=5, 15 and 30.

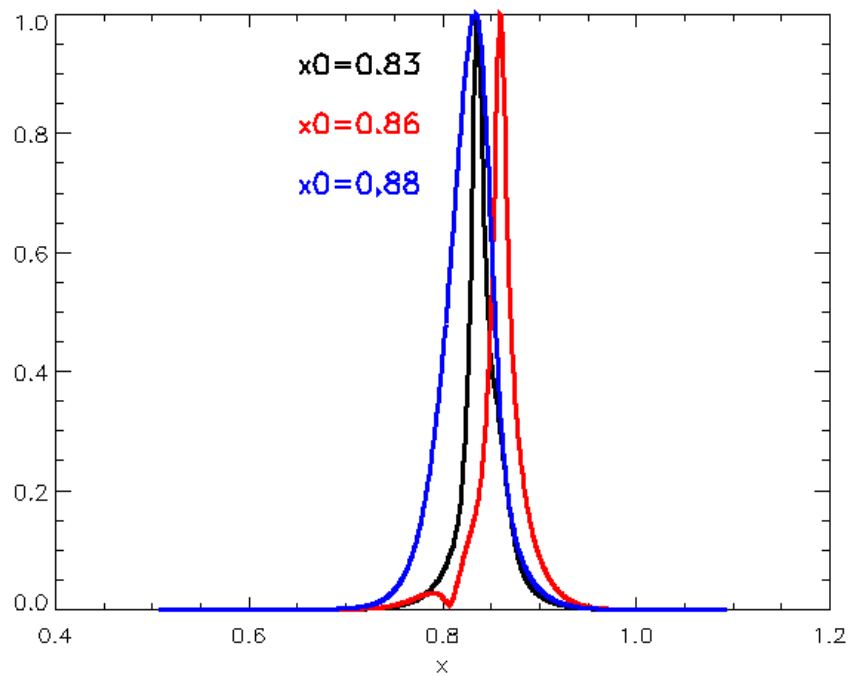
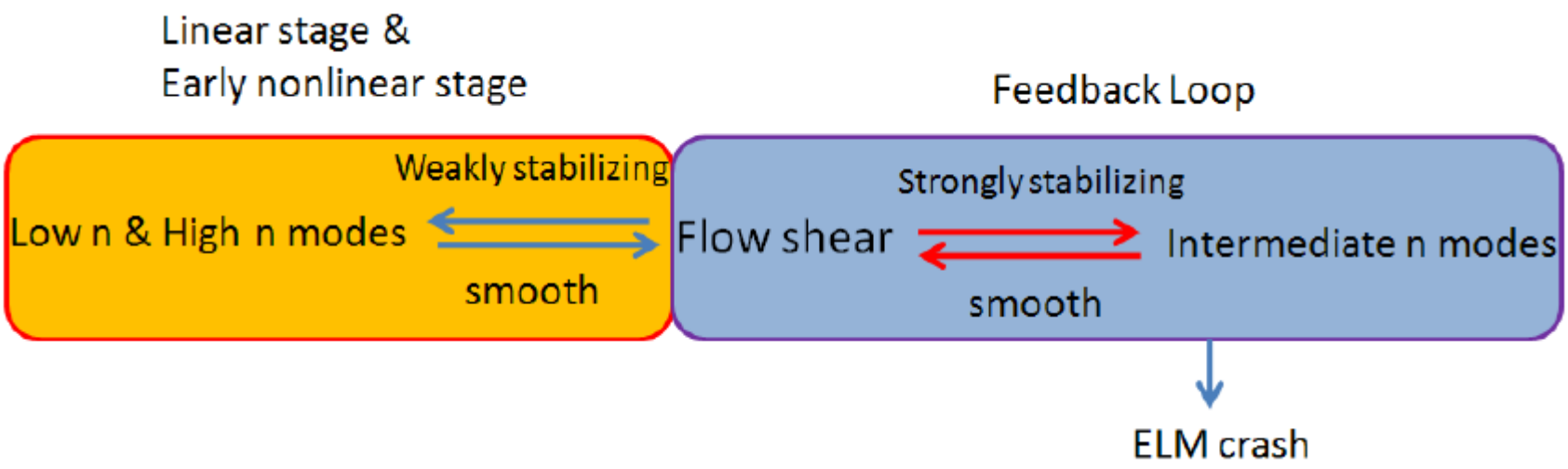


Fig. Mode structure for n=30 and Ds=30, the move of flow location moves the mode structure. When the flow moves out from the largest pressure gradient region, its influence disappear.

Discussion: **Flow governed feedback mechanism for ELM**



One feedback mechanism for ELM crash:

1. Low n and high n modes grow and smooth flow profile at the early nonlinear stage
2. Intermediate n modes become unstable due to the decrease of flow shear;
3. The growing of intermediate n modes accelerate the smoothing process of flow profile and leads to itself become more and more unstable;
4. This accelerate process results in some crash event like ELM.

Discussion: **Flow direction symmetry**

- Flow direction symmetry:
  - ➢Without diamagnetic effects: total convection flow is net flow and flow direction is symmetry;
  - ➢Diamagnetic electric field specifies a direction thus the destroys the flow direction symmetry ;
  - ➢Diamagnetic term also makes the two perpendicular direction become different for peeling-ballooning mode as showed in the figure below

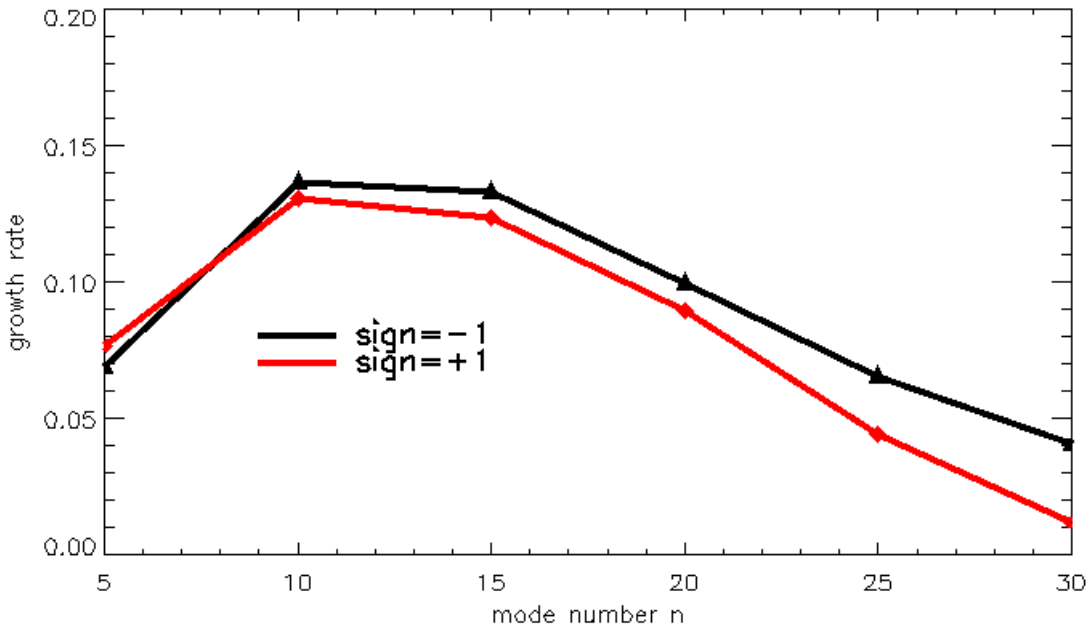


Fig. Turn off balanced convection flow and Kelvin-Helmholtz term while keep diamagnetic term, thus net flow equals total convection flow. If diamagnetic term doesn't change flow direction symmetry, mode growth rate should not change when net flow direction is changed as in ideal case. But this figure shows clearly that mode growth rate changes as flow direction changes.

Summary

- Ideal case
  - Flow shear has strong stabilizing effect on high n mode and is destabilizing for low n modes for ideal MHD;
  - Flow direction is symmetry;
  - Kelvin-Helmholtz term is destabilizing and the effects depends on mode number and shear;
- Diamagnetic effects break flow direction symmetry for peeling-ballooning mode;
- Non-ideal case
  - Diamagnetic term is dominant which makes the influence of flow shear on peeling-ballooning modes much weaker compared with ideal case;
  - Flow shear can lower growth rate of intermediate n modes but not stabilizing high n and low n modes;
  - High n modes get much narrower mode structure for large shear and structure peak moves by flow shear.

Summary

- The influence of flow shear on peeling-ballooning mode has tight relation with other physics effects like diamagnetic effects and resistivity, so more accurate physics model for edge plasmas like Gyrofluid are needed.

- Further issues:
  - Diamagnetic term is reverse proportional to density, so if net flow is related with density, it can influence will be much larger;
  - Is net flow must be **E**X**B** flow?
  - Keep parallel flow component;
  - Nonlinear simulation