中国科学技术大学
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Simulation study of the evolution of toroidally symmetric parallel current during ELM burst based on BOUT++

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- ☐ Background
- ☐ Numerical implementation
- ☐ Evolution & impact of $\langle J_{\parallel} \rangle$
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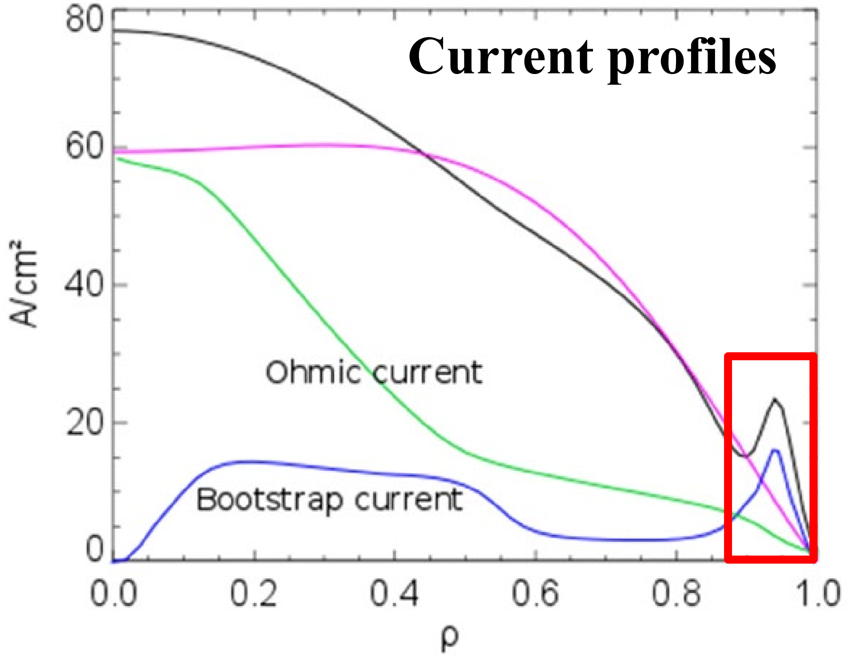
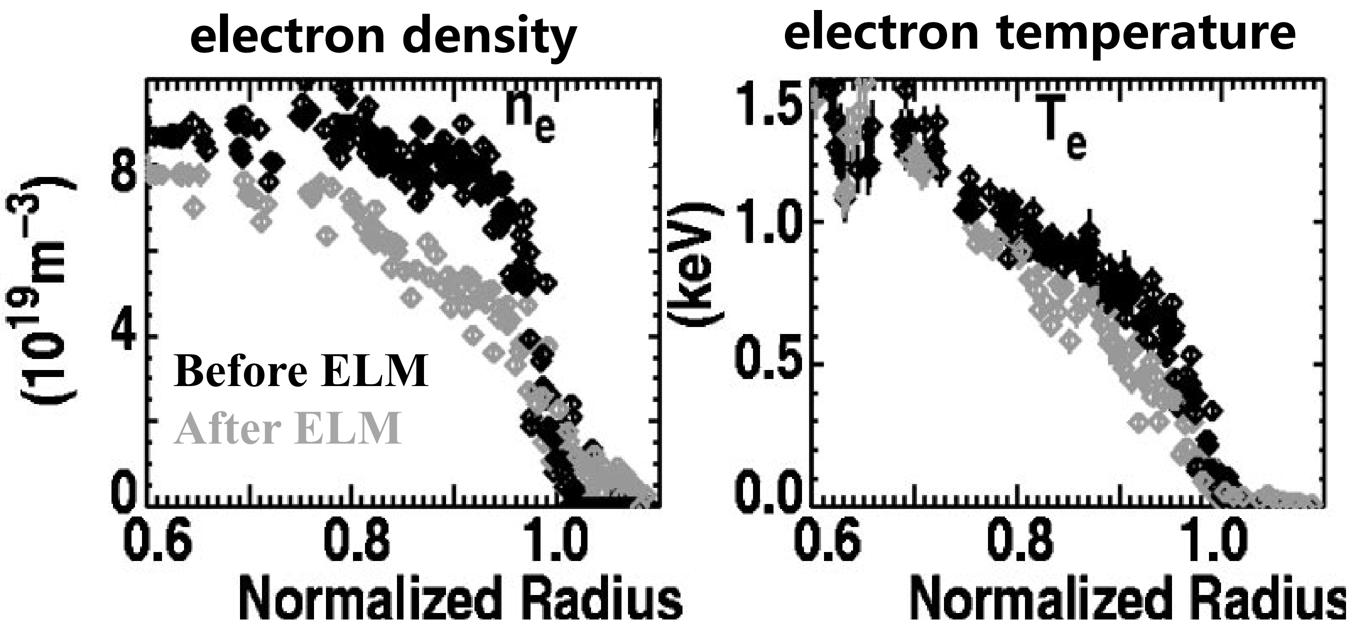
Background

- ➢ $J_{\parallel}$ in pedestal mainly determined by n, T_i, T_e.
- ➢ n, T profiles varies significantly during ELM.
- ➢ $J_{\parallel}$ is expected to have significant variation during ELM.

Li, Ren et al, PPCF 2013



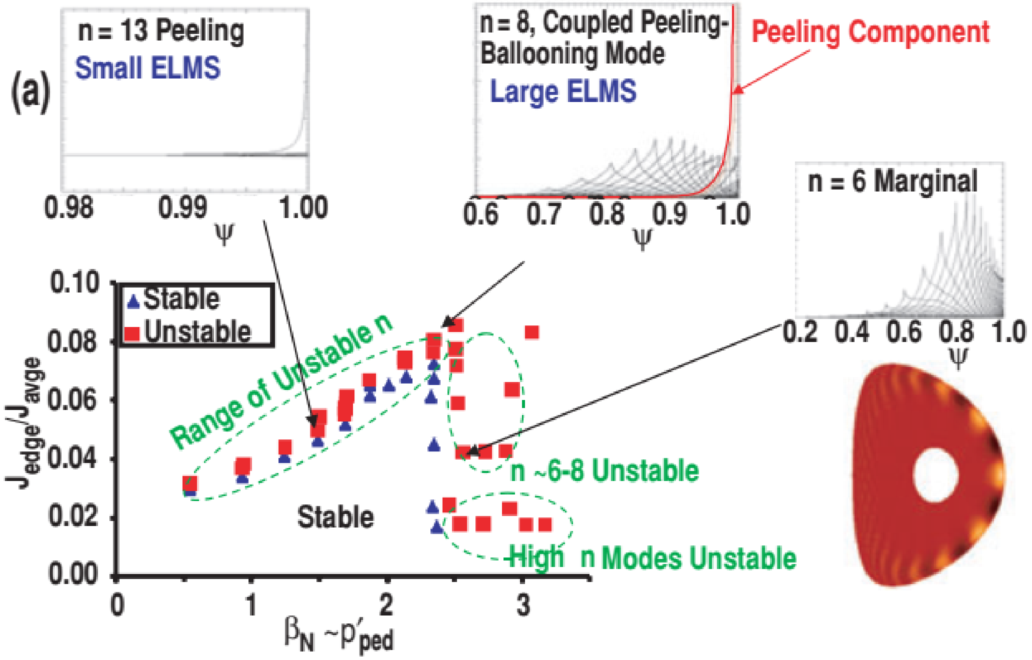
$$J_{BS} \propto \nabla P_e, \nabla T_e, \nabla T_i$$



Wade M R, Burrell K H et al, PoP 2005

Background

- ➢ PB instability is one of physical mechanisms of ELM explosion.
- ➢ $J_{\|}$ is source of peeling instability.
- ➢ Evolution of $J_{\|}$ may also cause remarkable impact on ELM evolution & turbulence transport.

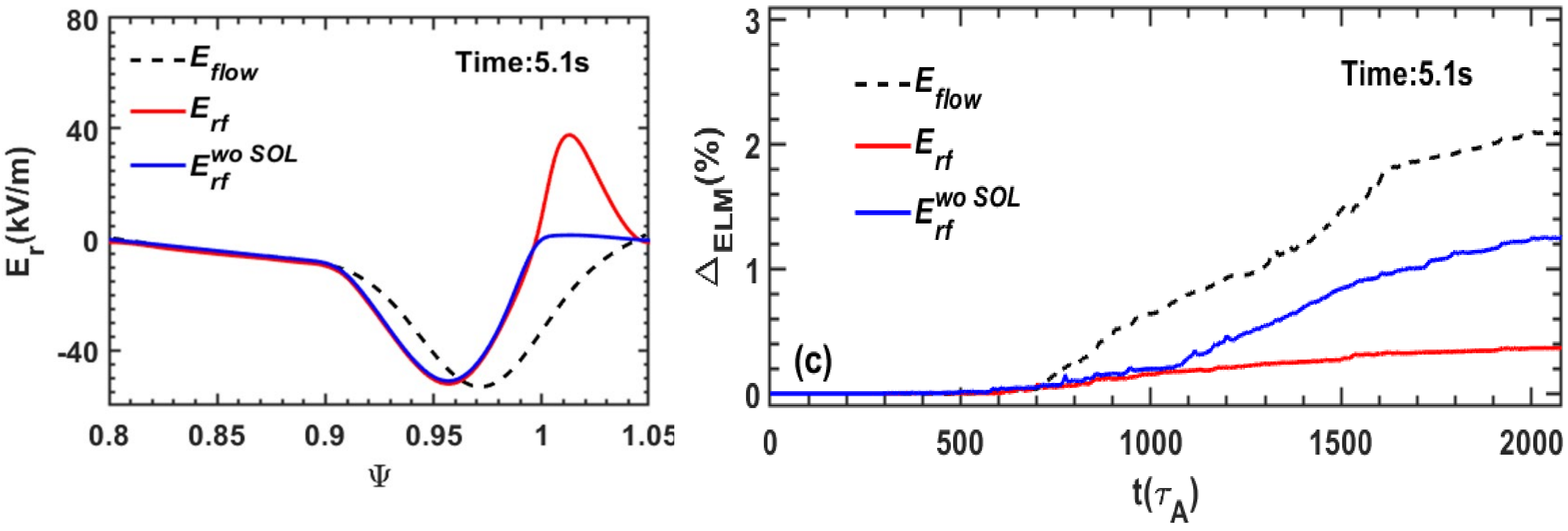


P.B. Snyder et al, NF 2011

Background

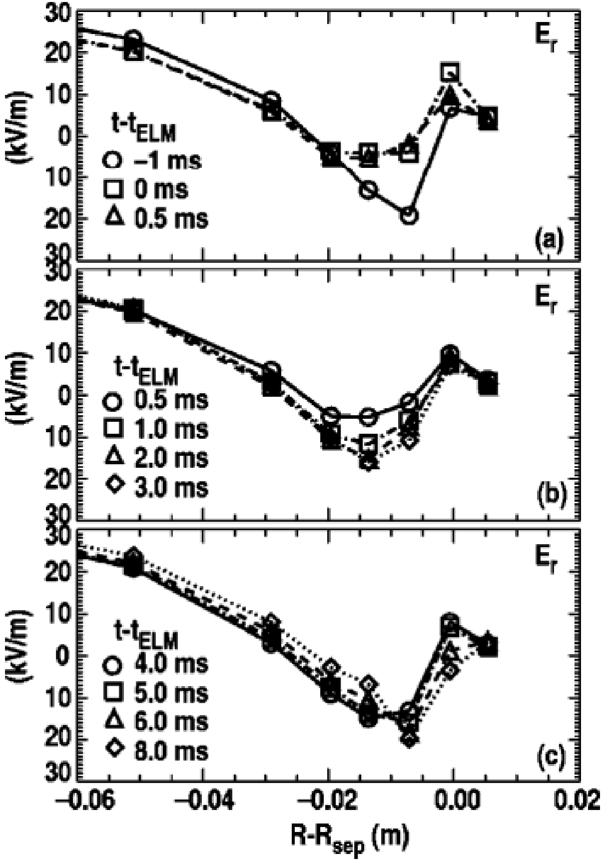
- ➢ **ELM size can be remarkably impacted by E_r**
- ➢ **Significant change of E_r is observed during ELM burst**

Li YL, Xia TY et al., NF 2022



ELM size simulated by BOUT++ with different E_r profile

E_r during ELM



Wade M R, Burrell K H et al, PoP 2005

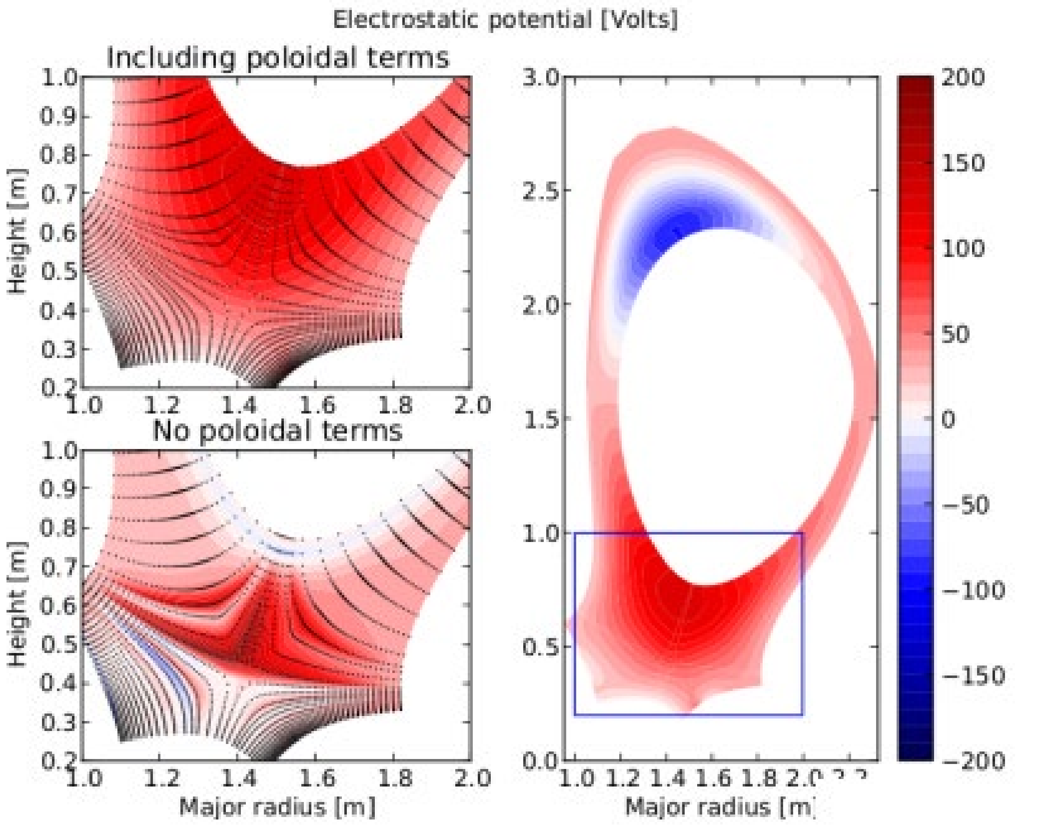
Contents

Numerical implementation

- **Due to $k_\perp \gg k_\parallel$ (flute perturbation), poloidal derivation is ignored in solving Laplace equation for n≠0 components in BOUT++ simulation.**
- **It is Inappropriate for n=0 component.**

$$\nabla_\perp = \nabla - \nabla_\parallel = e^x \left(\partial_x - \frac{g_{yx}}{g_{yy}}\partial_y\right) + e^z \left(\partial_z - \frac{g_{yz}}{g_{yy}}\partial_y\right)$$

$$\begin{aligned}\nabla_\perp^2 \simeq \quad & \left(g^{xx}\partial_x^2\right) + \left(\frac{1}{J}\left[\partial_x\left\{Jg^{xx}\right\} + \partial_y\left\{Jg^{yx}\right\} + \partial_z\left\{Jg^{zx}\right\}\right]\partial_x\right) \\ & + \left(g^{zz}\partial_z^2\right) + \left(\frac{1}{J}\left[\partial_x\left\{Jg^{xz}\right\} + \partial_y\left\{Jg^{yz}\right\} + \partial_z\left\{Jg^{zz}\right\}\right]\partial_z\right) \\ & + 2\left(g^{xz}\partial_x\partial_z\right) \\ = & \left(g^{xx}\partial_x^2\right) + G^x\partial_x + \left(g^{zz}\partial_z^2\right) + G^z\partial_z + 2\left(g^{xz}\partial_x\partial_z\right)\end{aligned}$$



BOUT++ 6filed-2fluid equation

$$
\begin{aligned}
\frac{\partial}{\partial t}\varpi &= -\frac{1}{B_0}\hat{b}\times\nabla_\perp\Phi\cdot\nabla\varpi + B_0^2\nabla_\parallel\left(\frac{J_\parallel}{B_0}\right) + 2\hat{b}\times\vec{\kappa}\cdot\nabla p_i \\
&\quad -\frac{1}{2\Omega_i}\left[\frac{1}{B_0}\hat{b}\times\nabla P_i\cdot\nabla(\nabla_\perp^2\Phi) - Z_i e B_0\hat{b}\times\nabla n_i\cdot\nabla\left(\frac{\nabla_\perp\Phi}{B_0}\right)^2\right] \\
&\quad +\frac{1}{2\Omega_i}\left[\frac{1}{B_0}\hat{b}\times\nabla\Phi\cdot\nabla(\nabla_\perp^2 P_i) - \nabla_\perp^2\left(\frac{1}{B_0}\hat{b}\times\nabla\Phi\cdot\nabla P_i\right)\right] + \mu_{\parallel i}\nabla_{\parallel 0}^2\varpi \\
\frac{\partial}{\partial t}n_i &= -\frac{1}{B_0}\hat{b}\times\nabla_\perp\Phi\cdot\nabla n_i - \frac{2n_i}{B_0}\hat{b}\times\vec{\kappa}\cdot\nabla\Phi - \frac{2}{Z_i e B_0}\hat{b}\times\vec{\kappa}\cdot\nabla P_i \\
&\quad -n_i B_0\nabla_\parallel\left(\frac{V_{\parallel i}}{B_0}\right) \\
\frac{\partial}{\partial t}V_{\parallel i} &= \frac{1}{B_0}\hat{b}\times\nabla_\perp\Phi\cdot\nabla V_{\parallel i} - \frac{1}{m_i n_{i0}}\hat{b}\cdot\nabla P \\
\frac{\partial}{\partial t}A_\parallel &= -\nabla_\parallel\phi + \frac{\eta}{\mu_0}\nabla_\perp^2 A_\parallel + \frac{1}{e n_{e0} B_0}\nabla_\parallel P_e + \frac{0.71 k_B}{e B_0}\nabla_\parallel T_e - \frac{\eta_H}{\mu_0}\nabla_\perp^4 A_\parallel \\
\frac{\partial}{\partial t}T_i &= -\frac{1}{B_0}\hat{b}\times\nabla_\perp\Phi\cdot\nabla T_i \\
&\quad -\frac{2}{3}T_i\left[\left(\frac{2}{B_0}\hat{b}\times\vec{\kappa}\right)\cdot\left(\nabla\Phi + \frac{1}{Z_i e n_{i0}}\nabla P_i + \frac{5}{2}\frac{k_B}{Z_i e}\nabla T_i\right) + B_0\nabla_\parallel\left(\frac{V_{\parallel i}}{B_0}\right)\right] \\
&\quad +\frac{2}{3 b_{i0} k_B}\nabla_{\parallel 0}\left(\kappa_{\parallel i}\nabla_{\parallel 0}T_i\right) + \frac{2}{3 n_{i1} k_B}\nabla_\perp\cdot\left(\kappa_{\perp i}\nabla_\perp T_i\right) + \frac{2m_e}{m_i}\frac{Z_i}{\tau_e}\left(T_e - T_i\right) \\
\frac{\partial}{\partial t}T_e &= -\frac{1}{B_0}\hat{b}\times\nabla_\perp\Phi\cdot\nabla T_e \\
&\quad -\frac{2}{3}T_e\left[\left(\frac{2}{B_0}\hat{b}\times\vec{\kappa}\right)\cdot\left(\nabla\Phi - \frac{1}{e n_{e0}}\nabla P_e - \frac{5}{2}\frac{k_B}{e}\nabla T_e\right) + B_0\nabla_\parallel\left(\frac{V_{\parallel e}}{B_0}\right)\right] \\
&\quad +0.71\frac{2T_e}{3 e n_{e0}}B_0\nabla_\parallel\left(\frac{J_\parallel}{B_0}\right) + \frac{2}{3 n_{e0} k_B}\nabla_{\parallel 0}\left(\kappa_{\parallel e}\nabla_{\parallel 0}T_e\right) + \frac{2}{3 n_{e0} k_B}\nabla_\perp\cdot\left(\kappa_{\perp e}\nabla_\perp T_e\right) \\
&\quad -\frac{2m_e}{m_i}\frac{1}{\tau_e}\left(T_e - T_i\right) + \frac{2}{3 n_{e0} k_B}\eta_\parallel J_\parallel^2
\end{aligned}
$$

$$
\begin{aligned}
\varpi &= n_{i0}\frac{m_i}{B_0}\left(\nabla_\perp^2\phi + \frac{1}{n_{i0}}\nabla_\perp\phi\cdot\nabla_\perp n_{i0} + \frac{1}{n_{i0} Z_i e}\nabla_\perp^2 p_{i1}\right), \\
J_\parallel &= J_{\parallel 0} - \frac{1}{\mu_0}B_0\nabla_\perp^2\psi, \\
V_{\parallel e} &= V_{\parallel i} + \frac{1}{\mu_0 Z_i e n_i}\nabla_\perp^2 A_\parallel.
\end{aligned}
$$

Compressible terms	**Parallel velocity terms**
Gyro-viscosity	**Energy exchange**
Electron Hall	**Thermal force**
Energy flux	**Thermal conduction**

[1] X. Q. Xu, et al, PoP 7, 1951 (2000); [2] X.Q. Xu et al., 2008, Commun. Comput. Phys. 4, 949. [3] T.Y. Xia et al., 2013, Nucl. Fusion 53 073009. [4] T. Y. Xia et al., 2015, Nucl. Fusion 55, 113030.

Numerical implementation

- Solving n=0 component in BOUT++ six-field two-fluid simulation (Xia et al NF 2015)
  - Spectral method (Fourier expansion) is used to separately solve each component.
  - Original solver *InvertLaplace* is used for n≠0 components.
  - 2 dimensional iterative solver *LaplaceXY* is used for n=0 component.
- Evolution of n=0 $J_\parallel$ ($\langle J_\parallel \rangle$) and E_r ($\langle E_r \rangle$) can be included.

$\langle\ \rangle$:toroidal average

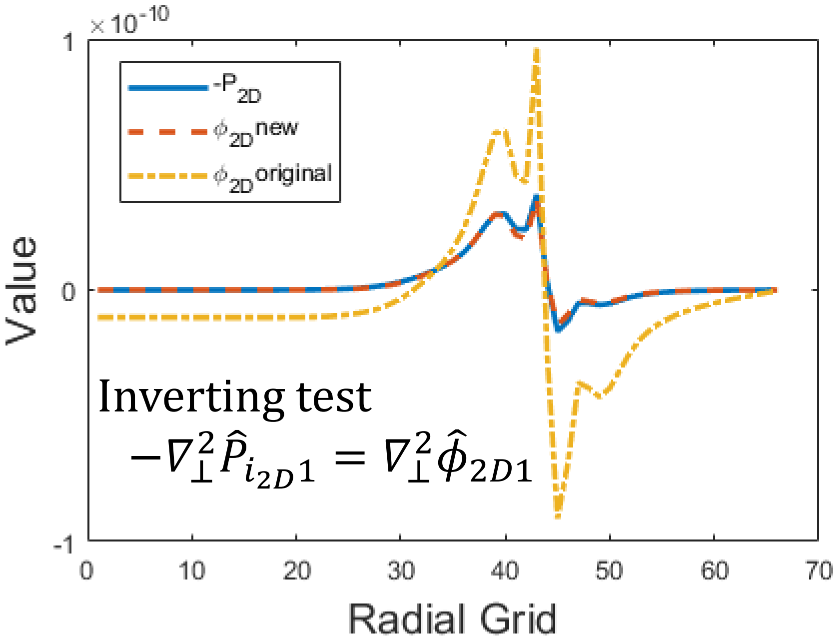
$$\frac{\partial}{\partial t}\langle\psi\rangle = -\frac{1}{B_0}\langle\nabla_\parallel\phi\rangle + \ldots$$

$$\langle J_{\parallel 1}\rangle = -\frac{1}{\mu_0}B_0\nabla_\perp^2\langle\psi\rangle$$

$$\varpi = \frac{m_i}{B_0}\left(\nabla_\perp\cdot(n_0\nabla_\perp\phi)+\frac{1}{e}\nabla_\perp^2 P_i\right)$$

$$\varpi \to \begin{cases} \langle\varpi\rangle - \dfrac{m_i}{eB_0}\nabla_\perp^2\langle P_i\rangle \xrightarrow{LaplaceXY} \langle\phi\rangle \\ \tilde{\varpi} - \dfrac{m_i}{eB_0}\nabla_\perp^2\tilde{P}_i \xrightarrow{InvertLaplace} \tilde{\phi} \end{cases}$$

Seto, Xu et al, PoP 2019



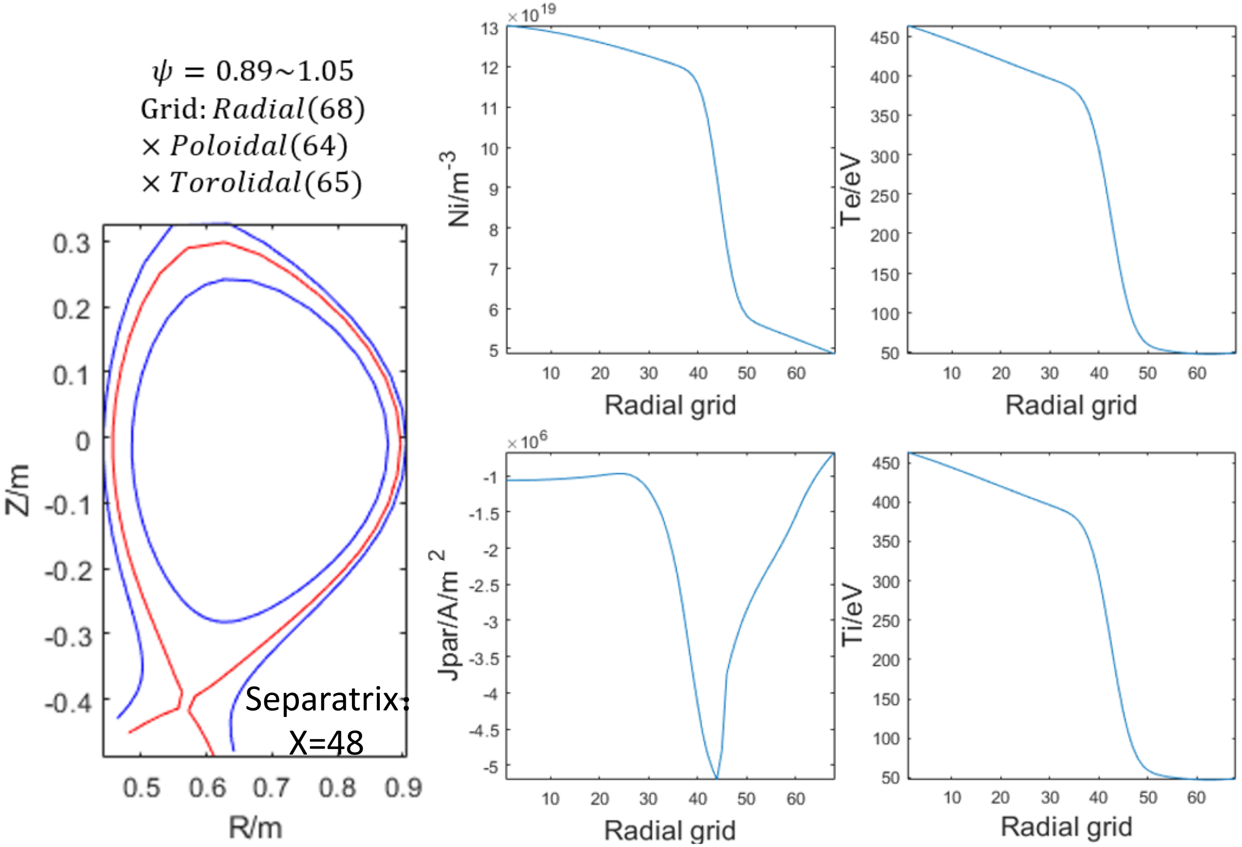
Inverting test

$$-\nabla_\perp^2\hat{P}_{i_{2D}1} = \nabla_\perp^2\hat{\phi}_{2D1}$$

Contents

Test for strong B device

- **Preliminary simulation are carried out on equilibrium with strong magnetic field (~ 4 T in pedestal) & current (~ 10^6) & pressure (~ 15 kPa).**



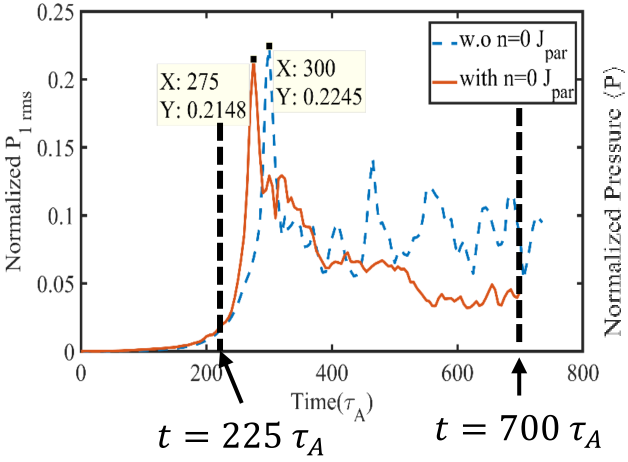
$\psi = 0.89 \sim 1.05$
Grid: $Radial(68)$
$\times\ Poloidal(64)$
$\times\ Torolidal(65)$

Separatrix: X=48

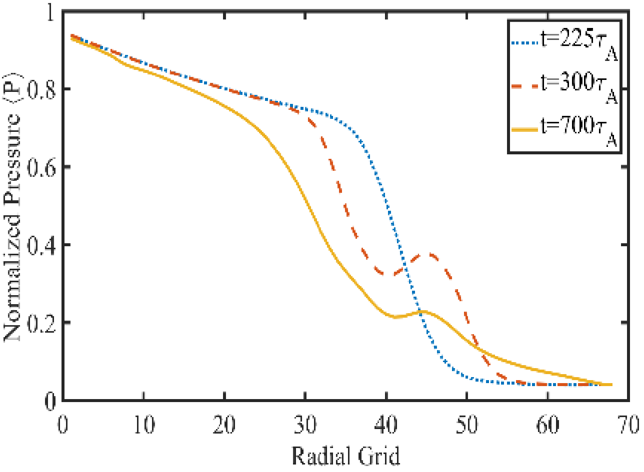
Test for strong B device

- ➢ **Linear phase**: $\langle J_\parallel \rangle$ has little variation, making little effect in linear phase.
- ➢ **Nonlinear phase**: $\langle J_\parallel \rangle$ changes significantly, affects nonlinear phase evolution of perturbation.

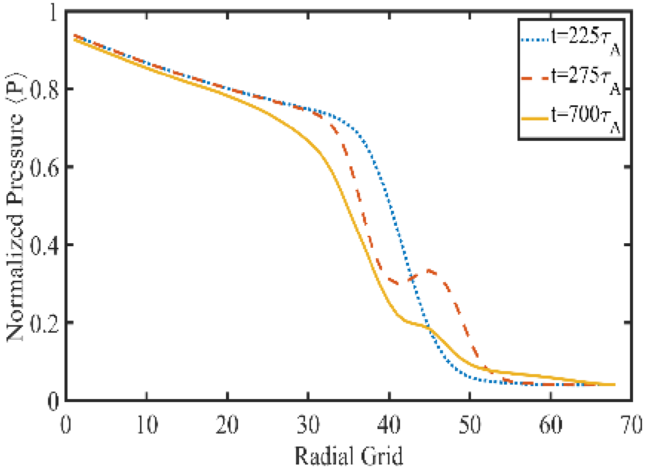
P_{rms} on outer mid plane @$(\nabla P_0)_{max}$



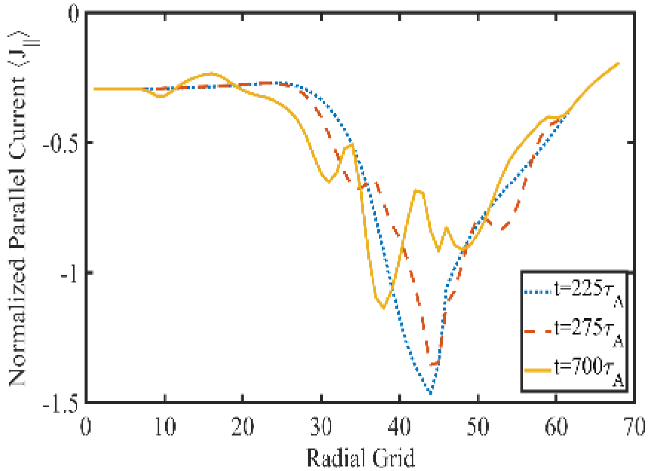
Pressure(w.o. $\langle J_\parallel \rangle$)



Pressure(with $\langle J_\parallel \rangle$)



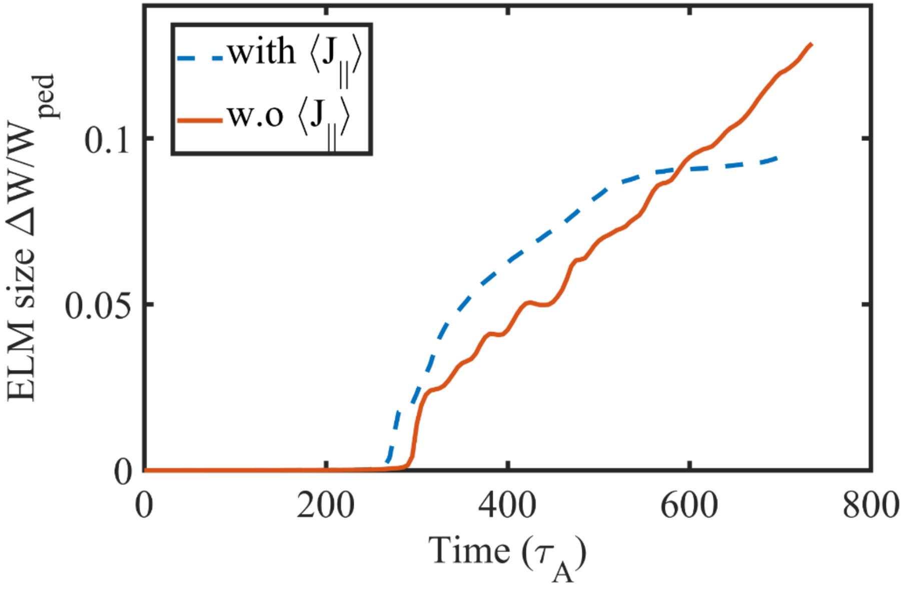
$\langle J_\parallel \rangle$



Test for strong B device

- **With $\langle J_{\parallel} \rangle$ evolution, ELM size significantly decreased and comes to saturation.**

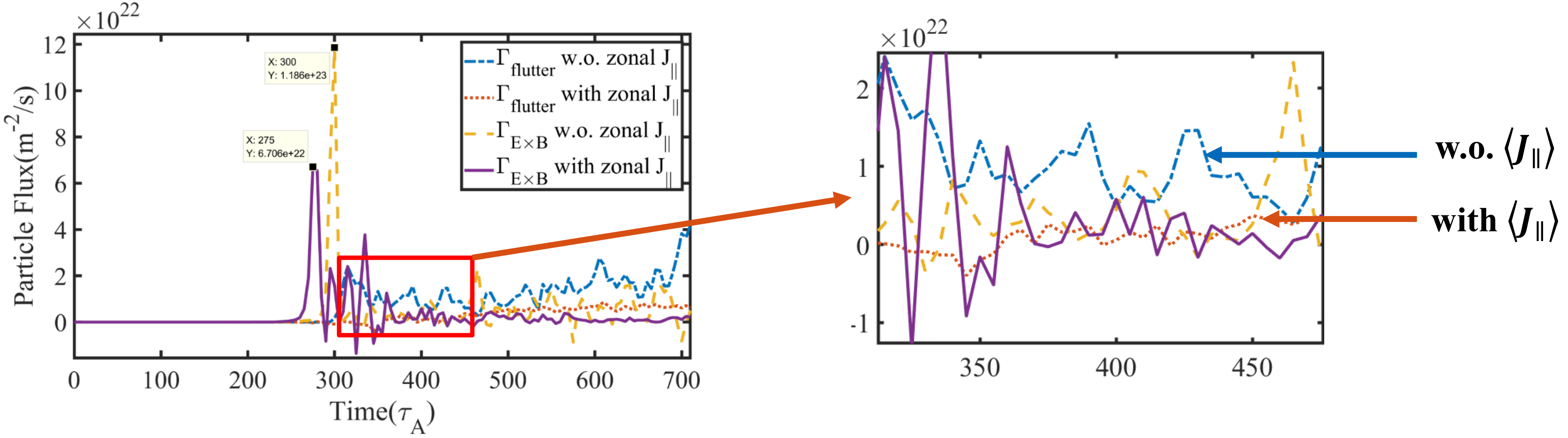
$$\Delta^{3D}_{ELM}(t) = \frac{3/2\int_{\psi_{in}}^{\psi_{out}} d\psi \oiint J d\theta d\zeta (P_0 - \langle P(t) \rangle_{\zeta})}{3/2\, P_{ped} V_{plasma}}$$


ELM size (energy loss)

Test for strong B device

- ➢ **Impact on turbulence transport : In nonlinear phase, magnetic flutter flux becomes much lower with $\langle J_{\|} \rangle$ evolution.**

Particle flux on outer mid plane
@$(\nabla P_0)_{max}$



w.o. $\langle J_{\|} \rangle$

with $\langle J_{\|} \rangle$

Test for strong B device

- ➢ $\langle J_\parallel\rangle$ is always nearly canceled with diamagnetic current in leading order.
- ➢ Without $\langle J_\parallel\rangle$, diamagnetic current will be mainly canceled by B flutter current, making it unreasonably high.

$$\nabla\cdot\boldsymbol{J}=0\Longrightarrow\nabla\cdot(enZ_i\boldsymbol{V}_{pi})=-\nabla\cdot\left(\boldsymbol{J}_\parallel+\frac{\boldsymbol{b}\times\nabla P}{B}\right)$$

$$\nabla\cdot\boldsymbol{J}_\parallel=B\nabla_\parallel\left(\frac{J_\parallel}{B}\right)\cong B_0\nabla_\parallel\left(\frac{J_\parallel}{B_0}\right)$$

Mainly canceled by B flutter

$$\nabla_\parallel=\boldsymbol{b}_0\cdot\nabla+\boldsymbol{b}_1\cdot\nabla=\nabla_\parallel^0+\nabla_\parallel^1$$

$$\underbrace{eZ_i\nabla\cdot(\langle n\boldsymbol{V}_{pi}\rangle)}_{\text{Polarization current}}=-B_0\left(\underbrace{\nabla_\parallel^0\left\langle\frac{J_{\parallel 1}}{B_0}\right\rangle+\langle\nabla_\parallel^1\rangle}_{\text{Disappear w.o. (n=0)}}\left(\frac{J_{\parallel 0}}{B_0}\right)+\underbrace{\left\langle\nabla_\parallel^1\left(\frac{J_{\parallel 1}}{B_0}\right)\right\rangle}_{\text{B flutter current}}\right)-\underbrace{\nabla\cdot\left(\frac{\boldsymbol{b}\times\nabla\langle P\rangle}{B}\right)}_{\text{Diamagnetic current}}$$

Test for strong B device

- **Magnetic flutter flux is expected to be small compared with the drift flux, and used to be ignored in continuity eq. in 6f-2fluid code.**
- **For further investigation into the impact of $\langle J_{\parallel} \rangle$ on magnetic flutter flux and particle transport, magnetic flutter terms are added in continuity eq..**

Expression of transport coefficient

$$\begin{cases} D_{\perp} = \dfrac{\langle N_i V_{E\times B}^{x} \rangle + \boxed{\left\langle \dfrac{J_{\parallel}}{Z_i e} b_1^{x} \right\rangle}}{(\nabla N_i)^{x}} \\ \chi_{j\perp} = \dfrac{\langle T_j V_{E\times B}^{x} \rangle + \langle q_{j\parallel} b_1^{x} \rangle / N_j}{(\nabla T_j)^{x}} \end{cases}$$

E×B drift flux

Magnetic flutter flx

Zhang, Chen et al, PoP 2019

Without B flutter (original code)

$$\frac{\partial n_i}{\partial t} = -\nabla \cdot [n_i(\boldsymbol{V}_E + \boldsymbol{V}_{di} + \boldsymbol{V}_{\parallel i})]$$

With B flutter

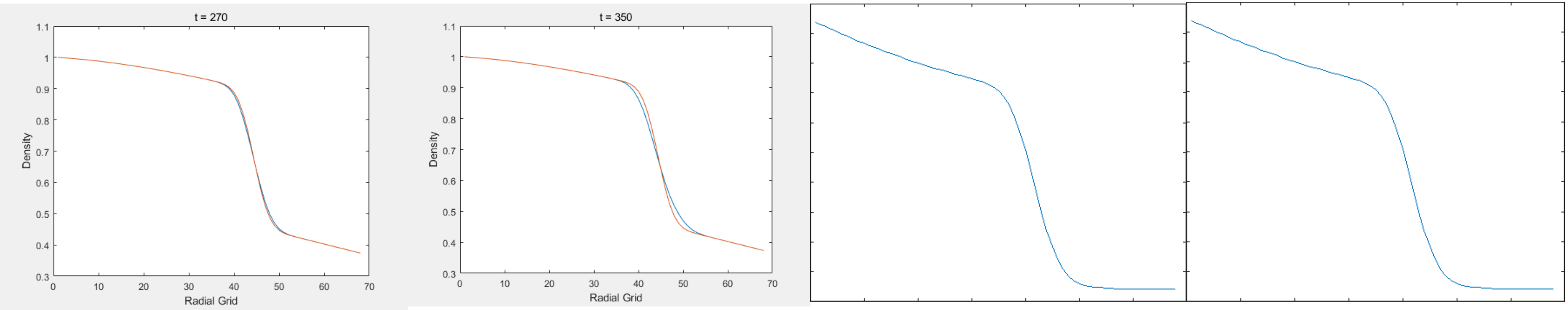
$$\frac{\partial n_i}{\partial t} = -\nabla \cdot \left[n_i(\boldsymbol{V}_E + \boldsymbol{V}_{de} + \boldsymbol{V}_{\parallel i}) - \frac{\boldsymbol{J}_{\parallel}}{Z_i e} \right]$$

Test for strong B device

- Evolution of n profiles are shown.
  - **Without $\langle J_{\parallel} \rangle$**, B flutter flux is overestimated and will cause unreasonable variation of the n profile.
  - **With $\langle J_{\parallel} \rangle$**, B flutter flux become ignorable.

Without $\langle J_{\parallel} \rangle$ **With $\langle J_{\parallel} \rangle$**

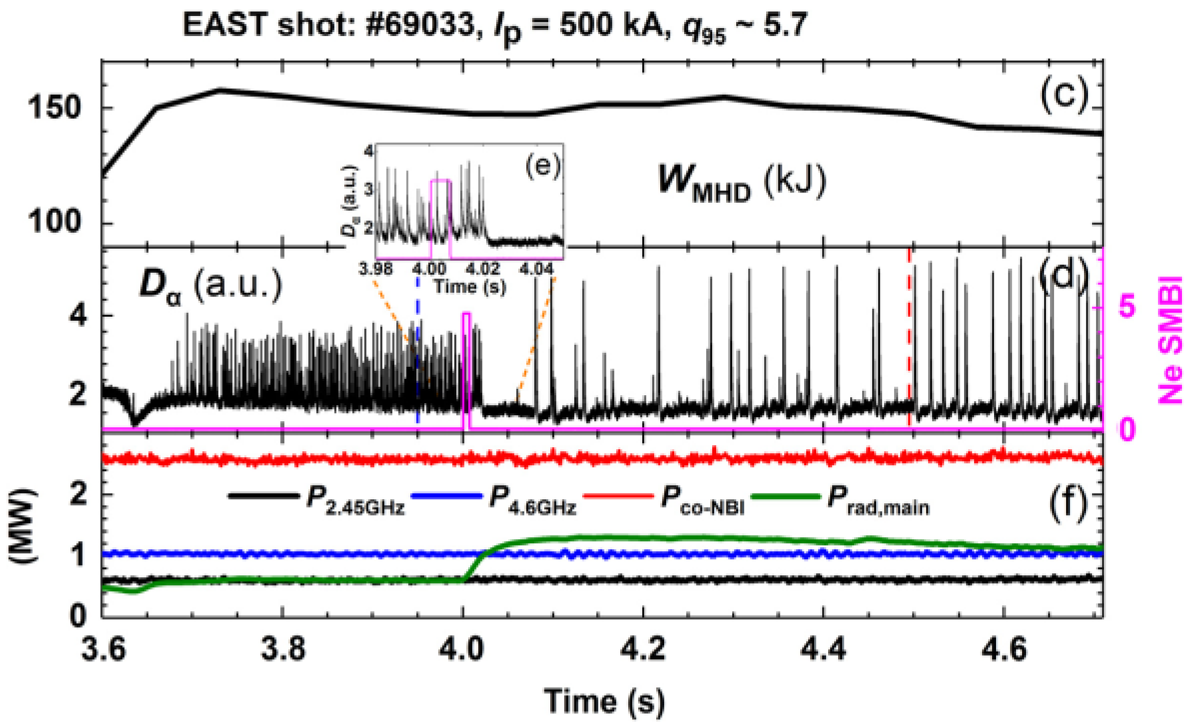
w.o. B flutter **with B flutter** **w.o. B flutter** **with B flutter**



Contents

Test for EAST

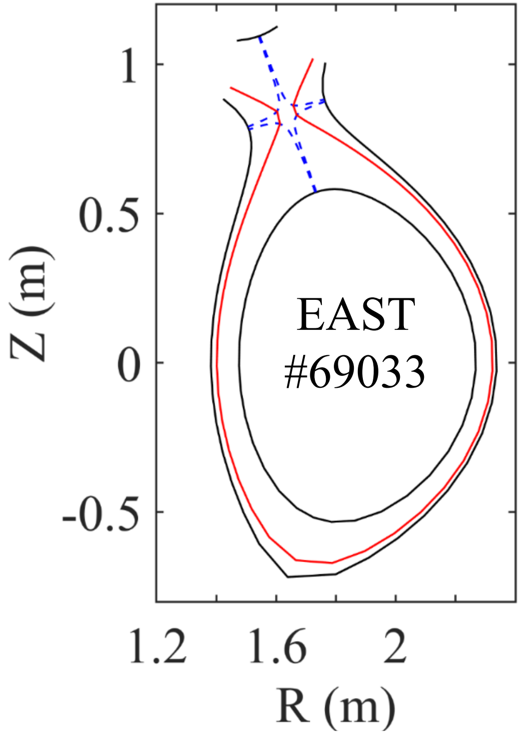
- **Further simulation on EAST #69033 discharge.**
  - **Type-III ELM (3.9 s).**
  - **Weaker B (~ 2 T)**
  - **Lower current (~ 10^5 A·m^{-2})**
  - **Lower pressure (~ 3 kPa)**



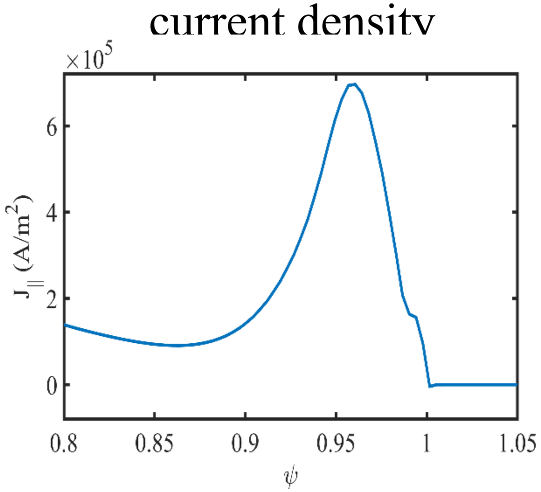
Lin, Xu et al, Phys. Lett. A 2022

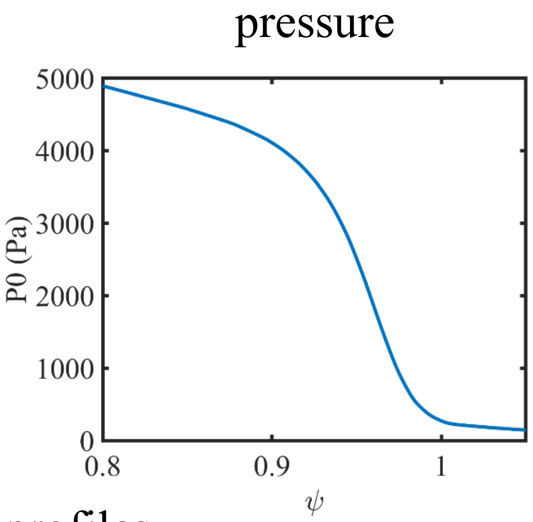
Test for EAST

➢ Initial profile: EAST#69033 (3.9 s)

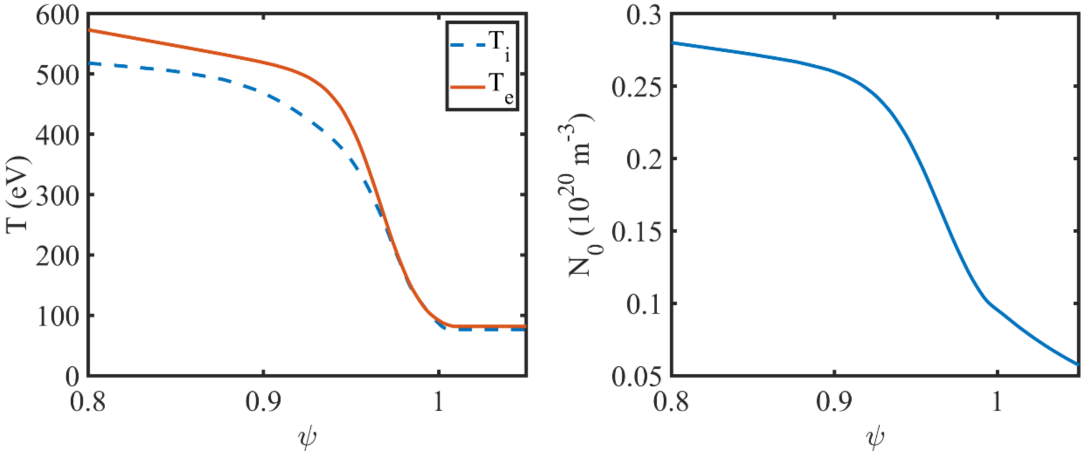


- region： ψ=0.8~1.08
- grid： radial(**68**)
  ×poloidal(64)
  ×toroidal(64)


current density

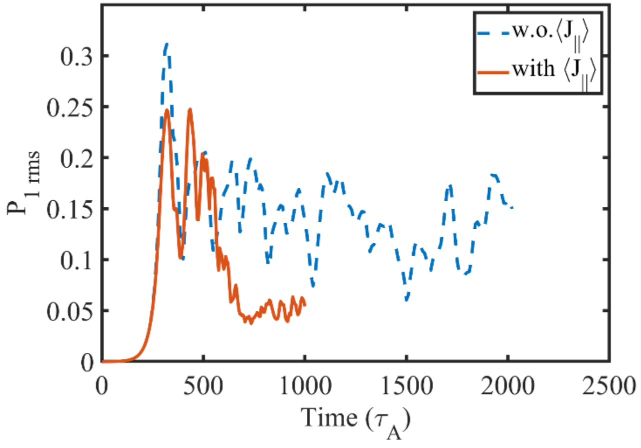

pressure

n, T profiles

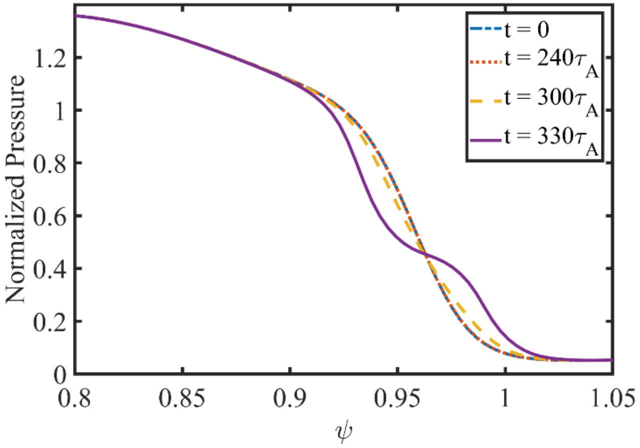


Test for EAST

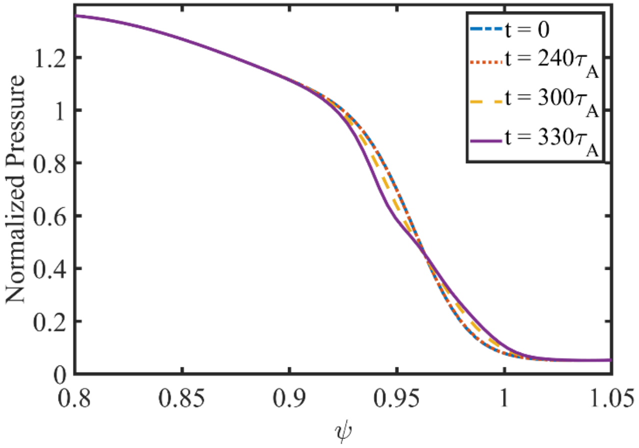
- **Evolution of profiles are similar to those of strong B cases.:**
  - **Little effect in linear phase**
  - **Significant impact on perturbation evolution in nonlinear phase**



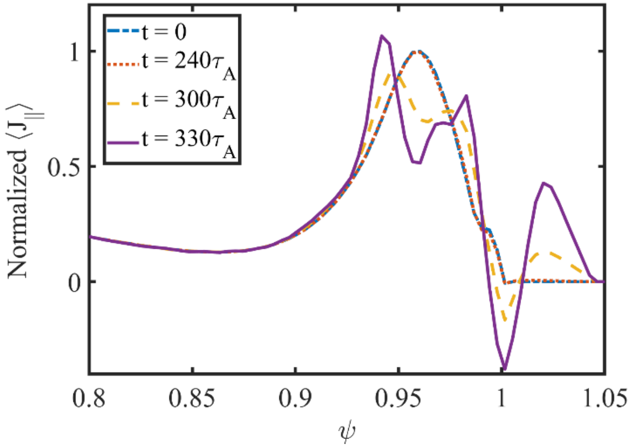
P_{rms} on outer mid plane $@(\nabla P_0)_{max}$



Pressure(w.o. $\langle J_\parallel \rangle$)



Pressure(with $\langle J_\parallel \rangle$)

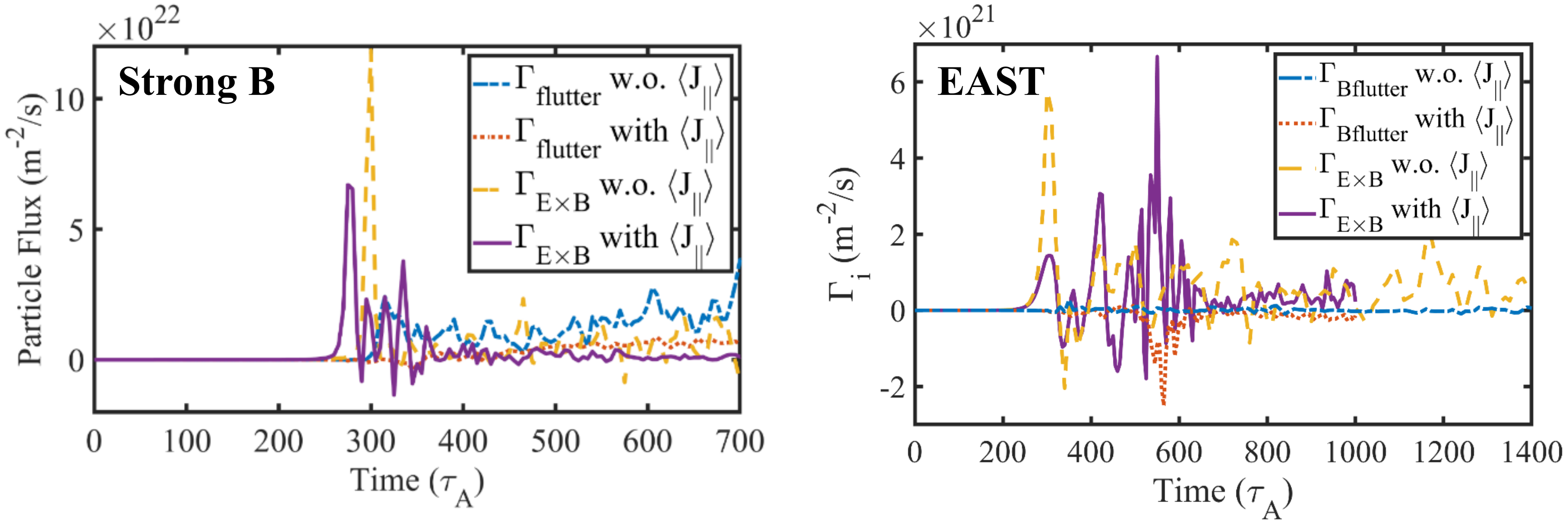


$\langle J_\parallel \rangle$

Test for EAST

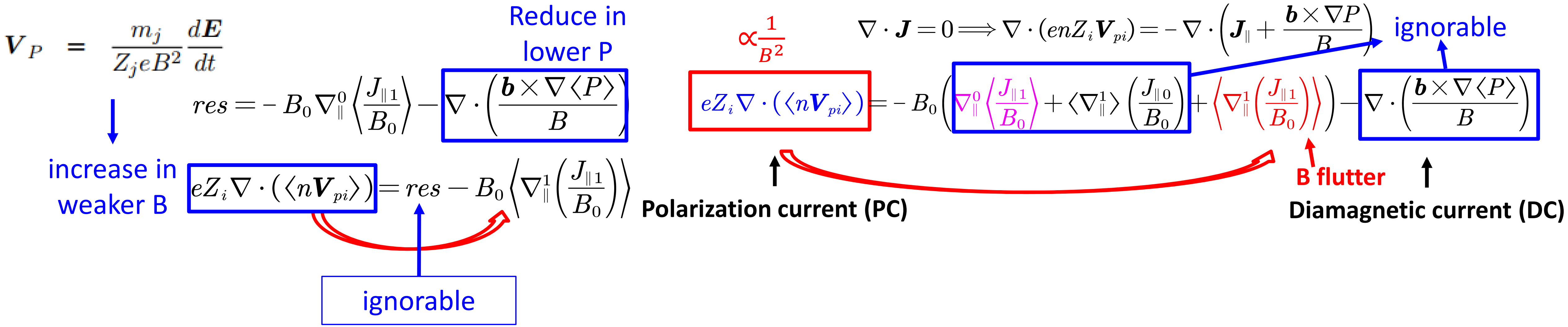
- **Influence on transport shows difference from the strong B case.**
  - **Both with and without $\langle J_{\parallel} \rangle$, magnetic flutter flux remains at a low level.**
  - **With $\langle J_{\parallel} \rangle$, B flutter flux becomes even larger at specific moment (strong perturbation in $\boldsymbol{E} \times \boldsymbol{B}$ flux).**

Particle flux on outer mid plane @$(\nabla P_0)_{max}$



Test for EAST

- Effect of polarization current should be considered.
- With strong B_0 & high P
  - PC can be neglected
  - Strong DC effect
- With relatively small B_0 & P
  - PC effect increases
  - Weaker DC effect
  - BC may mainly depend on PC



$$\boldsymbol{V}_P = \frac{m_j}{Z_j e B^2} \frac{d\boldsymbol{E}}{dt}$$

$$res = -B_0 \nabla_\parallel^0 \left\langle \frac{J_{\parallel 1}}{B_0} \right\rangle - \boxed{\nabla \cdot \left(\frac{\boldsymbol{b} \times \nabla \langle P \rangle}{B} \right)}$$

$$\boxed{eZ_i \nabla \cdot (\langle n\boldsymbol{V}_{pi} \rangle)} = res - B_0 \left\langle \nabla_\parallel^1 \left(\frac{J_{\parallel 1}}{B_0} \right) \right\rangle$$

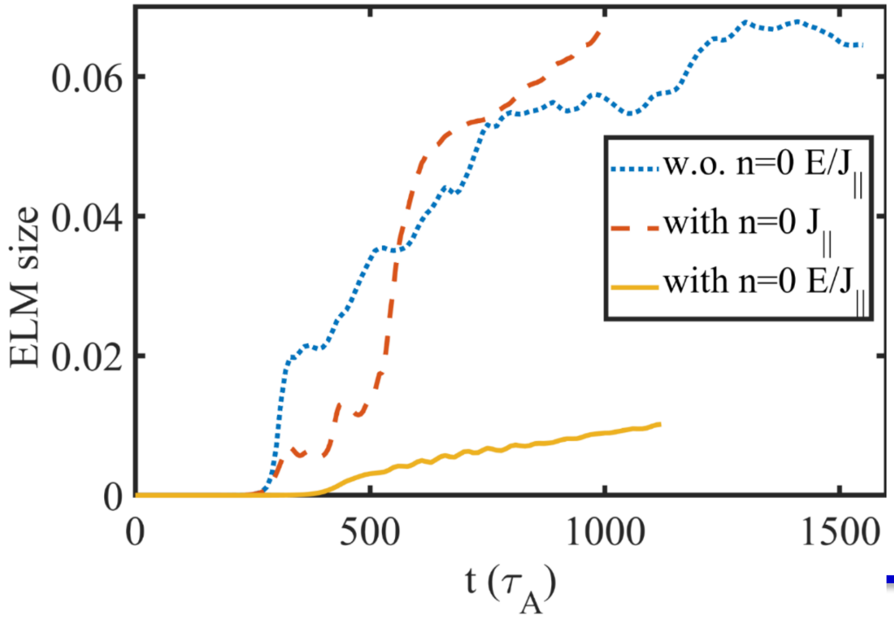
$$\nabla \cdot \boldsymbol{J} = 0 \Longrightarrow \nabla \cdot (enZ_i \boldsymbol{V}_{pi}) = -\nabla \cdot \left(\boldsymbol{J}_\parallel + \frac{\boldsymbol{b} \times \nabla P}{B} \right)$$

$$\underset{\propto \frac{1}{B^2}}{\boxed{eZ_i \nabla \cdot (\langle n\boldsymbol{V}_{pi} \rangle)}} = -B_0 \left(\boxed{\nabla_\parallel^0 \left\langle \frac{J_{\parallel 1}}{B_0} \right\rangle + \langle \nabla_\parallel^1 \rangle \left(\frac{J_{\parallel 0}}{B_0} \right)} + \left\langle \nabla_\parallel^1 \left(\frac{J_{\parallel 1}}{B_0} \right) \right\rangle \right) - \boxed{\nabla \cdot \left(\frac{\boldsymbol{b} \times \nabla \langle P \rangle}{B} \right)}$$

Test for EAST

ELM size is shown in figure.

- With $\langle J_{\parallel} \rangle$, the "first burst" rapidly ends with lower energy loss compared to without $\langle J_{\parallel} \rangle$ case.
- Both case reach about 7% in energy loss, inconsistent with type-III small ELM.
- Additional simulation with both $\langle E_r \rangle$ & $\langle J_{\parallel} \rangle$ included is carried out. Evolution of ELM size is completely different. Needs further research.



Summary

- **Based on BOUT++ 6fied-2fluid code and LaplaceXY method, evolution of $\langle J_{\parallel} \rangle$ is contained in ELM simulation.**
- **$\langle J_{\parallel} \rangle$ has little variation and cause little influence in linear phase. It varies significantly and has remarkable impact on turbulence evolution in nonlinear phase.**
- **According to toroidally averaged current continuity equation**
  - **With strong B & high P : $\langle J_{\parallel} \rangle$ will directly affect magnetic flutter flux.**
  - **With weak B & low P : $\langle J_{\parallel} \rangle$ affects little, while PC effect increases.**
- **Variation of $\langle J_{\parallel} \rangle$ has influence on evolution of energy loss. Instant collapse is reduced. But saturation phase still has large energy loss.**
- **With both $\langle J_{\parallel} \rangle$ and $\langle E \rangle$ included, ELM evolves completely different from previous cases, needs further investigation.**

Thanks for your attention!!

Backups

Boundary conditions

- **For n, T, ϕ, $J_{\|}$, $v_{\|}$profile**
  - **Neumann in core / Dirichlet in vaccum.**
- **For ψ**
  - **Zero Laplace in radial boundaries.**
- **For ϖ**
  - **Dirichlet (with exponential sink) in radial boundaries.**

Magnetic flutter flux in continuity equation

Continuity equation $$\frac{\partial n_j}{\partial t} = -\nabla \cdot [n_j \boldsymbol{V}_j]$$

Velocity components $$\boldsymbol{V}_j = \boldsymbol{V}_E + \boldsymbol{V}_{dj} + \boldsymbol{V}_{\parallel j}$$

***E* × *B* drift** $$\boldsymbol{V}_E = \frac{\boldsymbol{b} \times \nabla\phi}{B}$$

Diamagnetic drift $$\boldsymbol{V}_{dj} = \frac{\boldsymbol{b} \times \nabla P_j}{q_j n B}$$

Parallel velocity $$\boldsymbol{V}_{\parallel j} = V_\parallel \boldsymbol{b}$$

Particle flux
$$\boldsymbol{\Gamma}_j = n_j \boldsymbol{V}_j = n_j\left(\boldsymbol{V}_E + \boldsymbol{V}_{dj} + \boldsymbol{V}_{\parallel j}\right)$$
$$= n_j \frac{\boldsymbol{b} \times \nabla\phi}{B} + \boxed{\frac{\boldsymbol{b} \times \nabla P_j}{q_j B} + \boxed{V_{\parallel j}\boldsymbol{b}}} \leftarrow$$

Magnetic flutter flux

Particle flux is related to type of particle

If not ignorable
Directly adding magnetic flutter flux is inconsistent with quasi-neutral condition

Magnetic flutter flux in continuity equation

Continuity equation

$$\frac{\partial n_i}{\partial t} = -\nabla\cdot\left[n_i\left(\boldsymbol{V}_E + \boldsymbol{V}_{de} + \boldsymbol{V}_{\parallel i}\right) - \frac{\boldsymbol{J}_\parallel}{Z_i e}\right]$$

Particle flux

$$\boldsymbol{\Gamma_j} = n_j\boldsymbol{V}_j = n_j\left(\boldsymbol{V}_E + \boldsymbol{V}_{dj} + \boldsymbol{V}_{\parallel j} + \boldsymbol{V}_{pj}\right)$$
$$= n_j\frac{\boldsymbol{b}\times\nabla\phi}{B} + \frac{\boldsymbol{b}\times\nabla P_e}{q_e B} + V_{\parallel e}\boldsymbol{b}$$

Magnetic flutter flux

In this model

- **Ion polarization flux is retained but magnetic flutter flux is ignored.**
- **Electron polarization flux is ignored but magnetic flutter flux is retained.**

- **Electron has significant magnetic flutter transport due to its parallel velocity. And it is mainly compensated by ion polarization flux to avoid charge separation.**

For further simplification
Ion magnetic flutter flux is ignored

$$V_{\parallel e} \gg V_{\parallel i}$$

$$\boldsymbol{\Gamma_j} = n_j\boldsymbol{V}_j = n_j\left(\boldsymbol{V}_E + \boldsymbol{V}_{dj} + \boldsymbol{V}_{\parallel j} + \boldsymbol{V}_{pj}\right)$$
$$\cong n_j\frac{\boldsymbol{b}\times\nabla\phi}{B} + \frac{\boldsymbol{b}\times\nabla P_e}{q_e B} + \boxed{\frac{J_\parallel}{e}\boldsymbol{b}}$$

Magnetic flutter flux

Magnetic flutter flux in continuity equation

Polarization velocity $\boldsymbol{V}_{pj} = \frac{b}{\Omega_{\mathrm{j}}} \times \left(\frac{d\boldsymbol{u}_j}{dt} + \frac{\nabla \cdot \pi_j - \boldsymbol{R}_j}{m_j n_j} \right) \sim \frac{\omega}{\Omega_j} V_j$

Instability time scale

Gyrofrequency

$\Omega_j \sim \frac{1}{m_j}$□ $\boldsymbol{V}_{pi} \gg \boldsymbol{V}_{pe}$ → **Reasonable to ignore in electron velocity but retain in ion**

With ion polarization velocity

$$\frac{\partial n_i}{\partial t} = -\nabla \cdot \left[n_i \left(\boldsymbol{V}_E + \boldsymbol{V}_{dj} + \boldsymbol{V}_{\parallel j} + \boldsymbol{V}_{pe} \right) \right] \cong \frac{\partial n_e}{\partial t} = -\nabla \cdot \left[n_e (\boldsymbol{V}_E + \boldsymbol{V}_{de} + \boldsymbol{V}_{\parallel e}) \right]$$

After simplification

$$\nabla \cdot \left(Z_i e n \boldsymbol{V}_{pi} \right) = -\nabla \cdot \left(\boldsymbol{J}_{\parallel} + \frac{\boldsymbol{b}}{B} \times \nabla p \right)$$

Continuity equation

$$\frac{\partial n_i}{\partial t} = -\nabla \cdot \left[n_i (\boldsymbol{V}_E + \boldsymbol{V}_{de} + \boldsymbol{V}_{\parallel i}) - \frac{\boldsymbol{J}_{\parallel}}{Z_i e} \right]$$

Boundary condition for $J_{||}$

- **To avoid numerical instability.**
  - **$J_{||}$ is masked in both sides of radial boundary with tanh function.**
  - **For consistency with ψ, relaxing method is used on ohm equation(like in 3field code).**

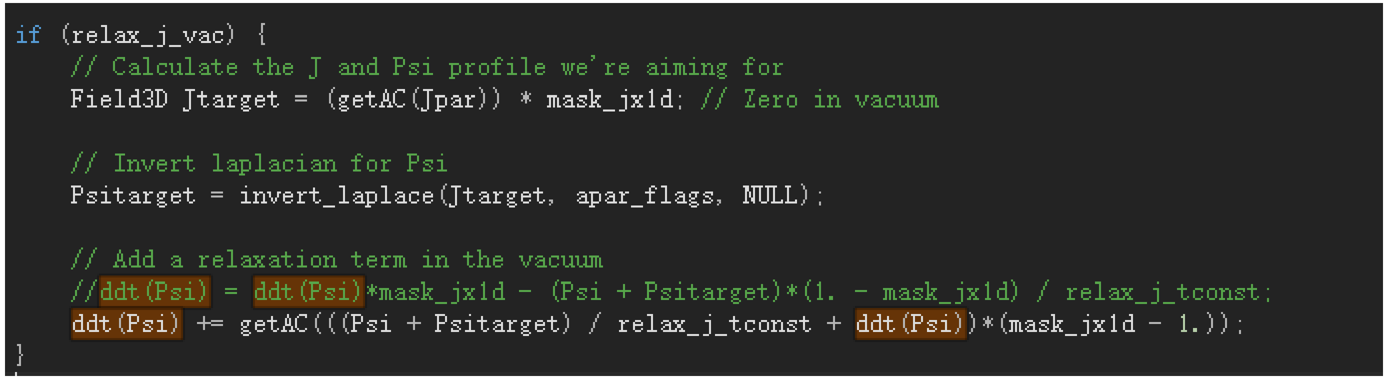
- **Masking is only made for n≠0 component for difficulty in inverting laplace for n=0 ψ.**

$$\Theta = \frac{1}{2}\left[1 - \tanh\left(\frac{P - P_{vac}}{\Delta P_{vac}}\right)\right]$$

$$J_{||}^{sol} = -\frac{1}{\mu_0} B_0 \nabla_\perp^2 \psi$$

$$\psi^{target} = \nabla_\perp^{-2}\left(\mu_0 J_{||}^{target} / B_0\right)$$

$$\frac{\partial \psi}{\partial t} = -\left(1 - \Theta\right)\frac{1}{B_0}\nabla_{||}\phi + \Theta\left(\psi^{target} - \psi\right)/\tau_{jvac}$$



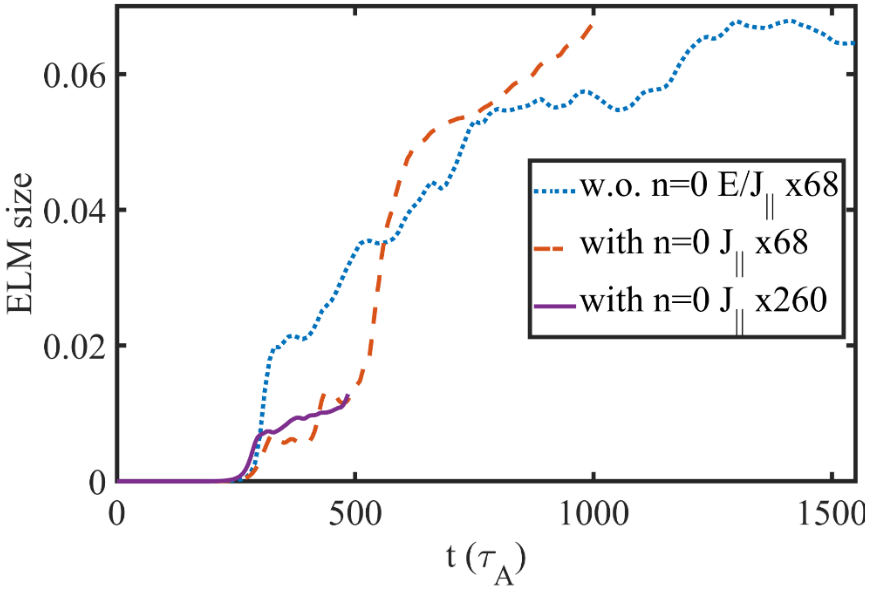
```
if (relax_j_vac) {
    // Calculate the J and Psi profile we're aiming for
    Field3D Jtarget = (getAC(Jpar)) * mask_jx1d; // Zero in vacuum

    // Invert laplacian for Psi
    Psitarget = invert_laplace(Jtarget, apar_flags, NULL);

    // Add a relaxation term in the vacuum
    //ddt(Psi) = ddt(Psi)*mask_jx1d - (Psi + Psitarget)*(1. - mask_jx1d) / relax_j_tconst;
    ddt(Psi) += getAC(((Psi + Psitarget) / relax_j_tconst + ddt(Psi))*(mask_jx1d - 1.));
}
```

Preliminary simulation on fine grid

- Currently numerical method is only stable for coarse grid.
- For finer grid, still on optimization.
- Evolution of ELM size is similar in larger time scale, but differ in rapid collapse.



$\langle J_\parallel \rangle$, $\langle E_r \rangle$ separation

- $\langle J_\parallel \rangle$ is evolved from $\langle\ \rangle$ component of ohm eq.. → needs $\nabla_\parallel \langle \phi \rangle$
- By ordering : $\langle E_\theta \rangle$(associated with $\nabla_\parallel \langle \phi \rangle$) can be neglected in drift velocity calculating.
- Thus $\langle E_r \rangle$(or $\nabla_\perp \langle \phi \rangle$) and $\nabla_\parallel \langle \phi \rangle$
  - Evolve according to different physical mechanisms.
  - Has different ordering.
- It`s reasonable to decouple evolution of $\langle E_r \rangle$ and $\langle J_\parallel \rangle$. Effect of $\langle J_\parallel \rangle$ is separately considered during ELM.

$$J_\parallel \sim 10^6 \ \text{A/m}^2$$

$$\eta_\parallel \sim 10^{-7} \ \Omega \cdot \text{m}$$

$$E_\parallel = \eta_\parallel J_\parallel \sim 10^{-1} \ \text{V/m}$$

$$E_{\theta,J} \sim E_\parallel \frac{B_0}{B_\theta} \sim 10 \ \text{V/m}$$

$$\nabla_\perp \phi \gg \nabla_\parallel \phi$$

Ohm law: $$\frac{\partial}{\partial t}\psi = -\frac{1}{B}\nabla_\parallel \Phi + \frac{\eta_\parallel}{\mu_0}\nabla_\perp^2 \psi + \frac{1}{en_eB}\nabla_\parallel P_e + \frac{0.71k_B}{eB}\nabla_\parallel T_e.$$

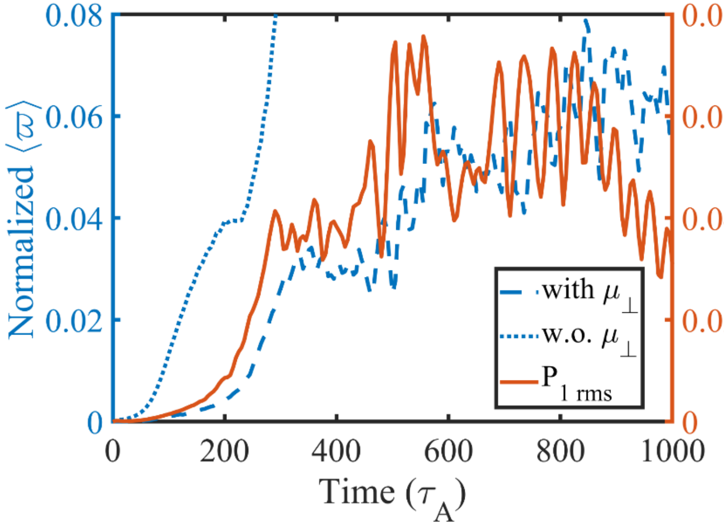
$$E_r \sim \frac{1}{n_i e}\frac{\partial P_i}{\partial r} \sim 10^4 \ \text{kV/m}$$

$$E_r \gg E_{\theta,J}$$

$\langle J_\parallel \rangle$, $\langle E_r \rangle$ separation

- **Evolution of $\langle E_r \rangle$ is mainly based on background turbulence**
  - **Any variation of ⟨ ⟩ is from nonlinear effect of perturbation components.**
  - **Without $\mu_\perp$, $\langle \varpi \rangle$ varies earlier than local perturbation strength.**
  - **With $\mu_\perp$, the evolution become synchronous.**
- **$\langle J_\parallel \rangle$ is almost synchronous with $\nabla\langle P \rangle$, balancing diamagnetic current.**

Evolution of $\langle \varpi \rangle$ & P_{rms} on outer mid plane $(\nabla P_0)_{max}$ (with/w.o. $\mu_\perp$)



Evolution of $\langle J_\parallel \rangle$&$\nabla\langle P \rangle$ on outer mid plane $(\nabla P_0)_{max}$