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Plasma elongation effect on the parity change in electromagnetic ITG modes and the generation of intrinsic rotation in the tokamak plasmas

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Motivation

Turbulence driven intrinsic rotation:

- Ubiquitous in fusion plasmas (Ohmic, H-mode, ITB, etc...)
- Stabilize macroscopic plasma modes → play an important role in ITER and beyond.

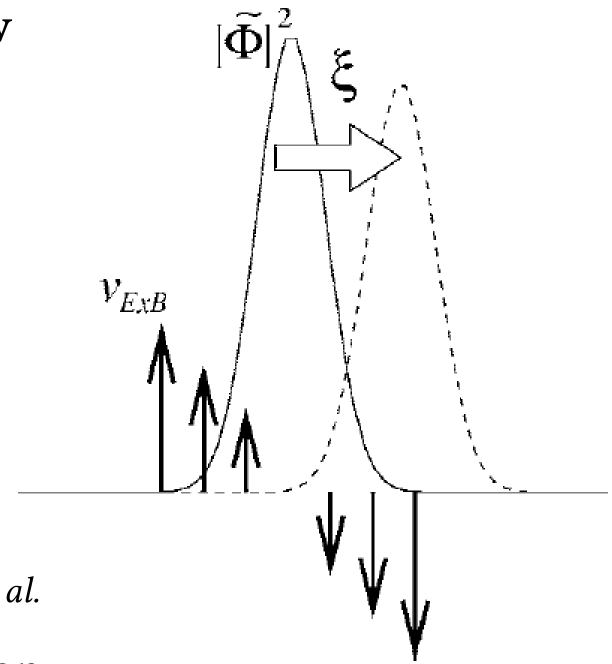
Symmetry breaking : radial in-homogeneity ⇒ parallel flow

- In the context of quasi-linear (QL) theory,
  the intrinsic rotation can be generated by Reynolds stress.

$$\frac{\partial}{\partial \hat{t}}\langle u_{\parallel}\rangle = -\nabla\cdot\Pi_{Rey} = -\nabla\cdot\left[\chi_{\phi}\frac{\partial\langle V_{\parallel}\rangle}{\partial r} + V_{Pinch}\langle V_{\parallel}\rangle + \Pi_{res}\right]$$

$$\Pi_{Rey} \propto \sum k_{\parallel}|\phi|^2 \rightarrow \text{ need asymmetry in } |\phi|^2 \text{ to be nonzero.}$$

- symmetry breakers : EXB shear *[Dominguez&Staebler(PoF,1993); Gurcan et al.(PoP,2007)]*, Turbulence intensity gradient *[Gurcan et al.(PoP,2010)]*, polarization drift *[McDevitt et al.(PRL,2009)]*, density gradient *[Singh et al.(PoP,2012)]*, reversed q-shear *[Singh et al.(PoP,2013)]*, poloidal tilting of a mode *[Camenen et al.(NF,2011)]*



Symmetry breaking by V'_{ExB}
[Gurcan et al.(PoP,2007)]

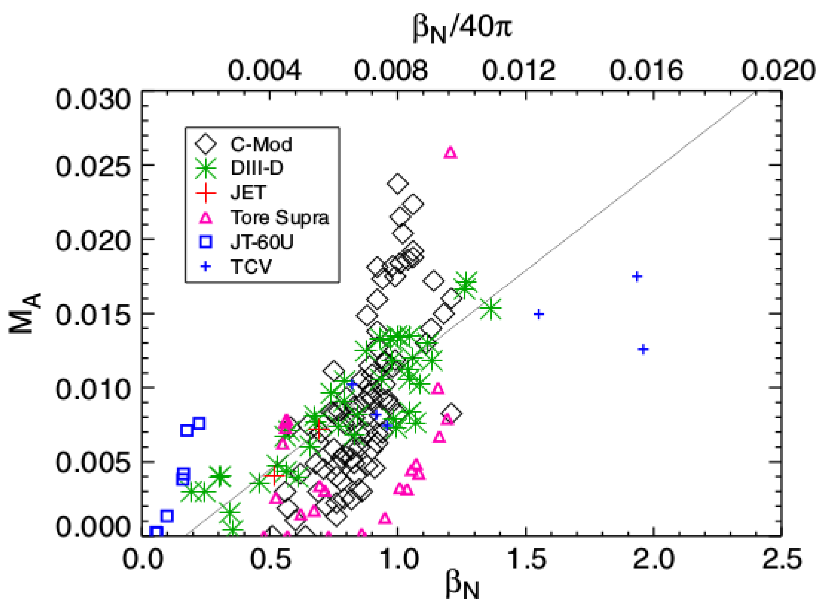
Motivation for EM effects

Recent experiments indicate the importance of EM effects.

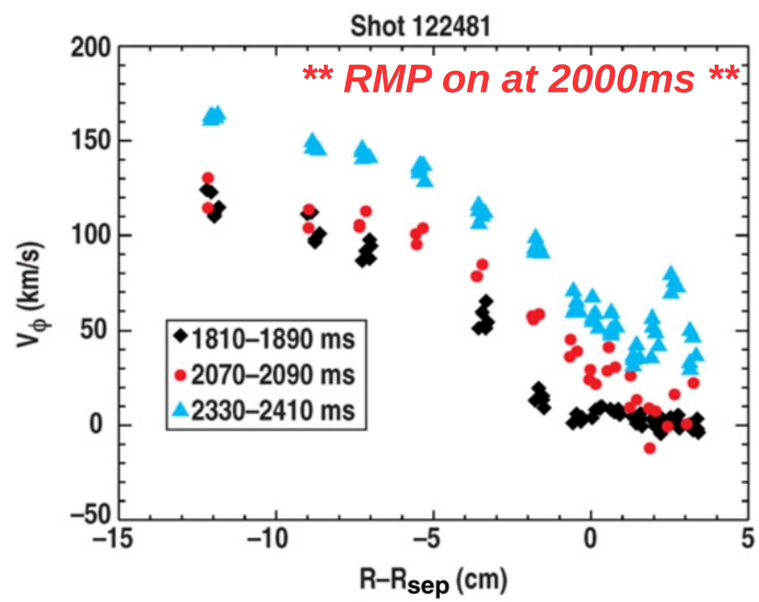
: Intrinsic rotation is strongly generated when

1. β is high due to enhanced confinement (H-mode, ITB) [Rice *et al.*(NF,2007); Solomon *et al.*(PoP,2010)]

2. external magnetic perturbations is applied [Burrell et al. (PPCF,2005); Zhao et al.(NF,2015)]



[Rice *et al.*(NF,2007)]



[Burrell *et al.*(PPCF,2005)]

Two fluid model for ions and electrons for ITG mode

Vorticity eq. :
$$\frac{\partial \tilde{U}}{\partial \hat{t}} = \frac{1}{\hat{n}_{eq}} \hat{\nabla}_{\parallel} \tilde{J}_{\parallel} - \frac{1}{\hat{n}_{eq}} \hat{\omega}_{\nabla B} \left(\tilde{P}_i + \tilde{P}_e \right) - D_U \hat{\nabla}_{\perp}^4 \tilde{U}$$

continuity eq. :
$$\frac{\partial \tilde{n}}{\partial \hat{t}} = -\hat{n}_{eq} \hat{\nabla}_{\parallel} \tilde{u}_{\parallel} + \hat{\nabla}_{\parallel} \tilde{J}_{\parallel} + \hat{\omega}_{\nabla B} \left(\hat{n}_{eq} - \tau \tilde{P}_e \right) - D_n \hat{\nabla}_{\perp}^4 \tilde{n}$$

Ion pressure eq. :
$$\frac{\partial \tilde{P}_i}{\partial \hat{t}} = -\left[\tilde{\phi}, \hat{P}_{i,eq} \right] - \frac{5}{3} \hat{P}_{i,eq} \left\{ \hat{\nabla}_{\parallel} \tilde{u}_{\parallel} - \frac{1}{\hat{n}_{eq}} \hat{\nabla}_{\parallel} \tilde{J}_{\parallel} - \hat{\omega}_{\nabla B} \left(\tilde{\phi} - \frac{\tau}{\hat{n}_{eq}} \tilde{P}_e \right) \right\} - \frac{2}{3} \sqrt{\frac{8 \hat{T}_{i,eq}}{\pi}} |\hat{\nabla}_{\parallel}| \tilde{T}_i - D_P \hat{\nabla}_{\perp}^4 \tilde{P}_i$$

Ion parallel momentum:
$$\frac{\partial \tilde{u}_{\parallel}}{\partial \hat{t}} = -\frac{1}{\hat{n}_{eq}} \hat{\nabla}_{\parallel} \left(\tilde{P}_i + \tau \tilde{P}_e \right) - D_v \hat{\nabla}_{\perp}^4 \tilde{u}_{\parallel}$$

Ohm's law :
$$\frac{\beta_e}{2} \frac{\partial \tilde{A}_{\parallel}}{\partial \hat{t}} = -\hat{\nabla}_{\parallel} \left(\tilde{\phi} - \frac{\tau}{\hat{n}_{eq}} \tilde{P}_e \right) - \eta \tilde{J}_{\parallel}$$

where, $\tilde{U} = \frac{1}{\hat{B}_0^2} \left(\hat{\nabla}_{\perp}^2 \tilde{\phi} + \frac{1}{\hat{n}_{eq}} \hat{\nabla}_{\perp}^2 \tilde{P}_i \right)$, $\tilde{J}_{\parallel} = -\hat{\nabla}_{\perp}^2 \tilde{A}_{\parallel}$, $\beta_e = \frac{8\pi n_0 T_{e,0}}{B_0^2}$, $\tau = T_{e,0}/T_{i,0}$,

$[f,g] = \frac{1}{r} \left(\frac{\partial f}{\partial r} \frac{\partial g}{\partial \theta} - \frac{\partial f}{\partial \theta} \frac{\partial g}{\partial r} \right)$, $\hat{\omega}_{\nabla B} \tilde{f} = 2 \frac{a}{R} \left[\hat{r} cos\theta, \tilde{f} \right]$, $\hat{\nabla}_{\parallel} \tilde{f} = \hat{\nabla}_{\parallel,0} \tilde{f} - \frac{\beta_e}{2} \left[\tilde{A}_{\parallel}, \tilde{f} \right]$.

Contents

I. Circular tokamak geometry

: EM effects on ITG mode and intrinsic rotation generation

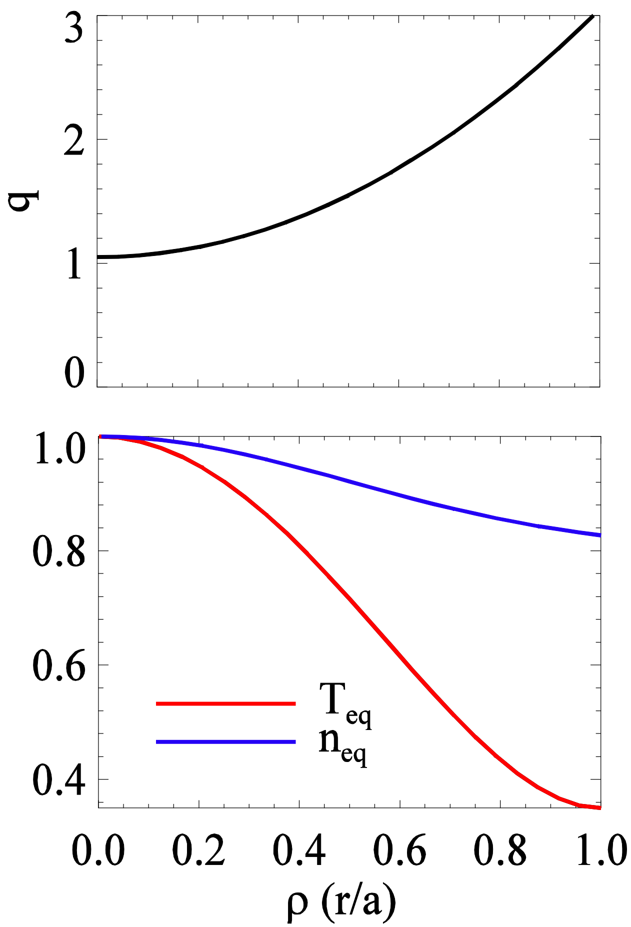
[Kaang *et al.*, Phys. Plasmas **25**, 012505 (2018)]

II. Elongated tokamak geometry

: Elongation effects on the EM ITG and intrinsic rotation generation

[Kaang *et al.*, Phys. Plasmas **accepted**, (2023)]

We solve a set of two fluid equations in BOUT++ frame work.



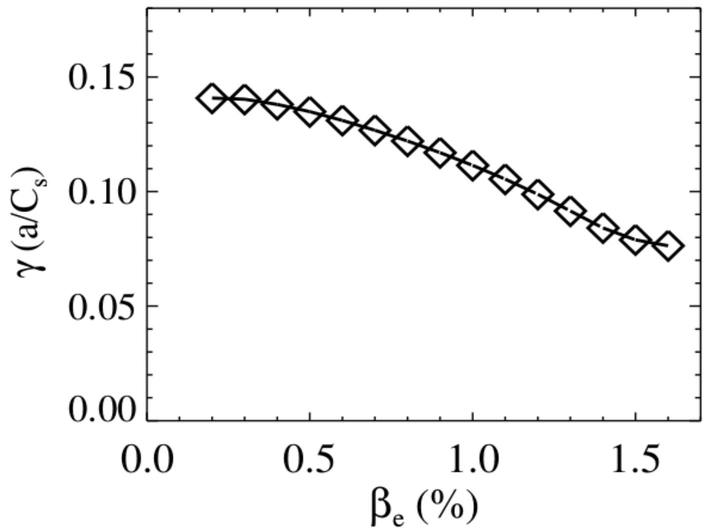
- **Circular tokamak geometry (r, θ, ζ).**

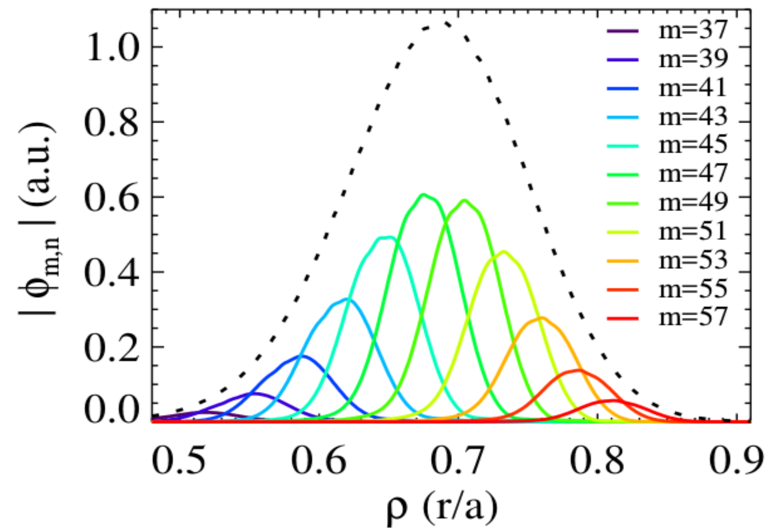
$$q = 1.05 + 2.0\rho^2$$
$$\hat{T}_{eq} = 0.35 + 0.65\left(1 - \rho^2\right)^2$$
$$\hat{n}_{eq} = 0.8 + 0.2e^{-2\rho^2}$$

[Miyato *et al.*(PoP, 2004), Kaang *et al.*(PoP, 2018)]

- **Simulation parameters**
  - · Grid # : nx=644, ny=64, nz=17
  - · R=2.0m, a=0.5m ⟹ a/R= 0.25
  - · T_c=2keV, B_0=1T ⟹ ρ_s=0.0125
  - · $n_c = \beta_e \frac{B_0^2}{\mu_0} \frac{1}{T_c}$

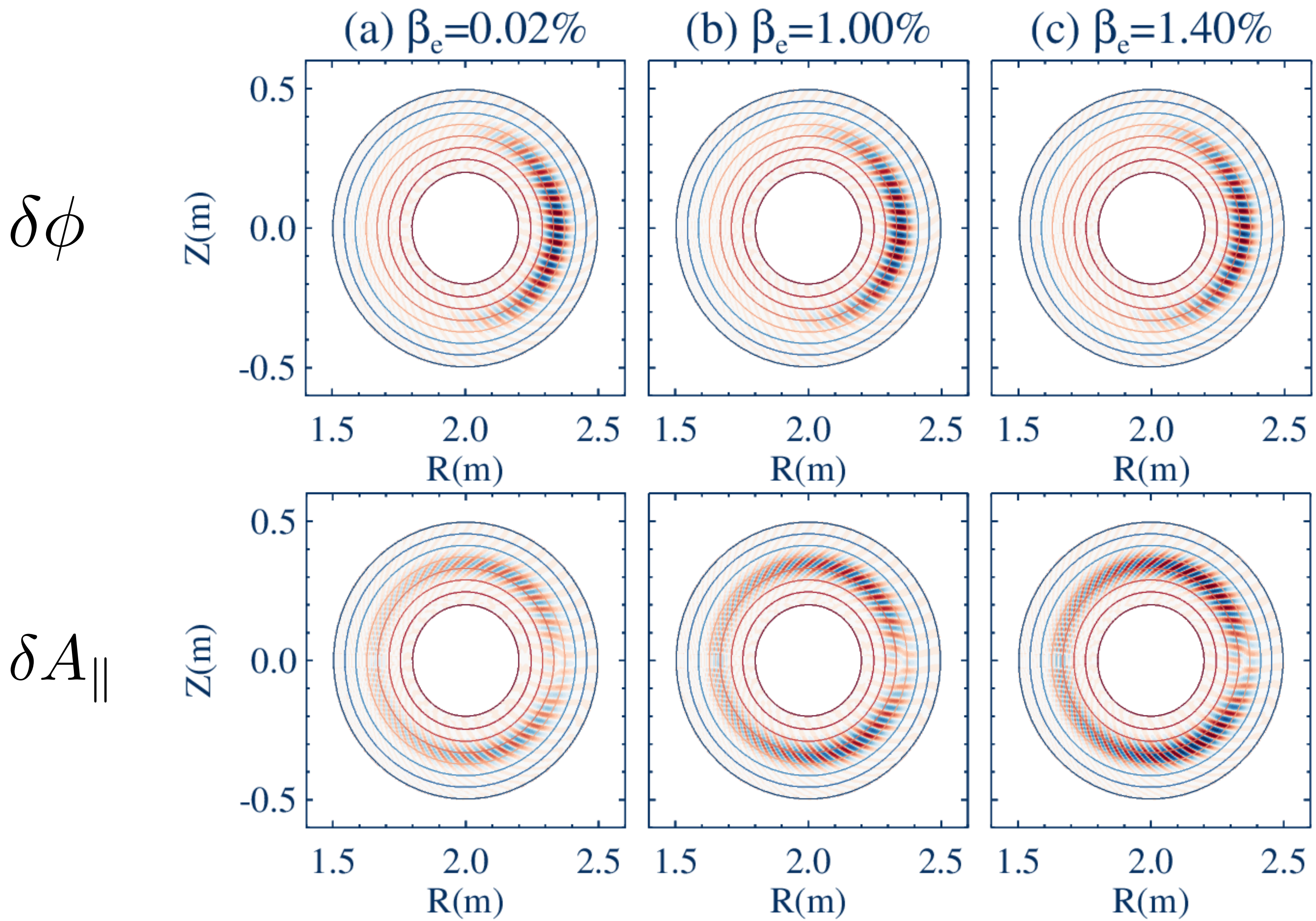
Stabilization with β_e and global structure of global EM-ITG





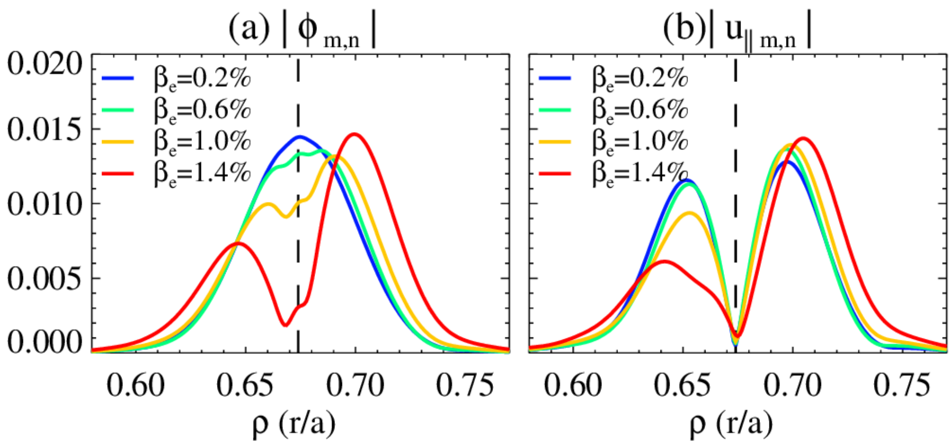
- Finite β_e stabilizes the global EM-ITG mode: consistent with previous results *[Dong et al. (PoF,1987); Miyato and Kishimoto (PoP,2004); Beli and Candy (PoP,2010)]*
- EM-ITG eigen-mode shows ballooning structure in the radial direction (dotted line in the right figure)
- Poloidal Fourier harmonics are located at corresponding rational surfaces and toroidally coupled.

Global eigenmodes on a poloidal cross section when $|\varphi|^2$=const



Parity mixing with β_e increases mode asymmetry

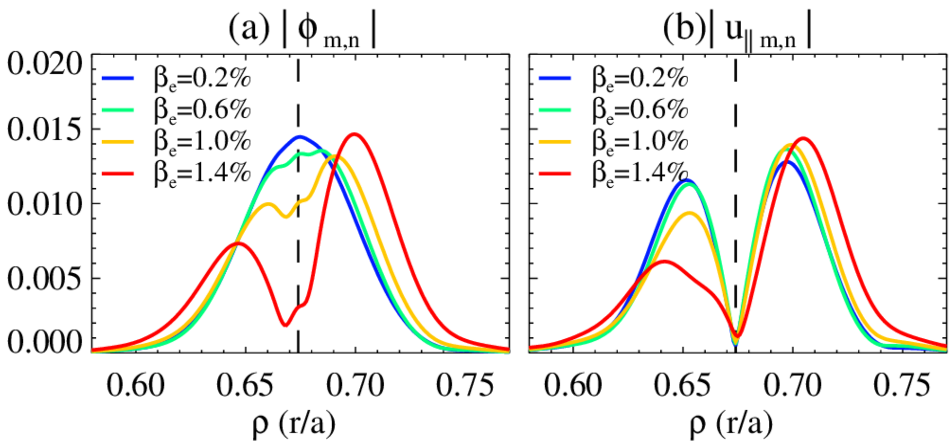
- Decompose global eigenfunctions into poloidal harmonics and choose one poloidal mode (m=47)

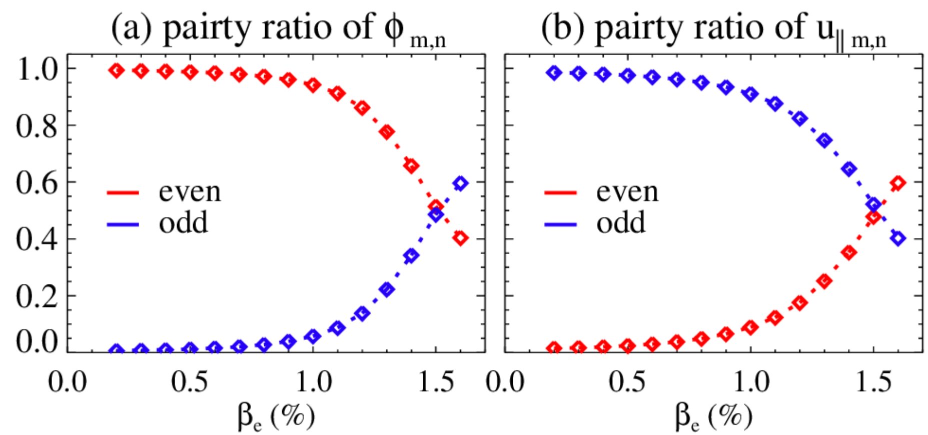


- In the low β_e, φ and $u_\parallel$ are nearly symmetric. However, both become asymmetric at high β_e.
  → **EM effects (β_e) plays a role of a symmetry breaker in global tokamak geometry.**

Parity mixing with β_e increases mode asymmetry

- Decompose global eigenfunctions into poloidal harmonics and choose one poloidal mode (m=47)





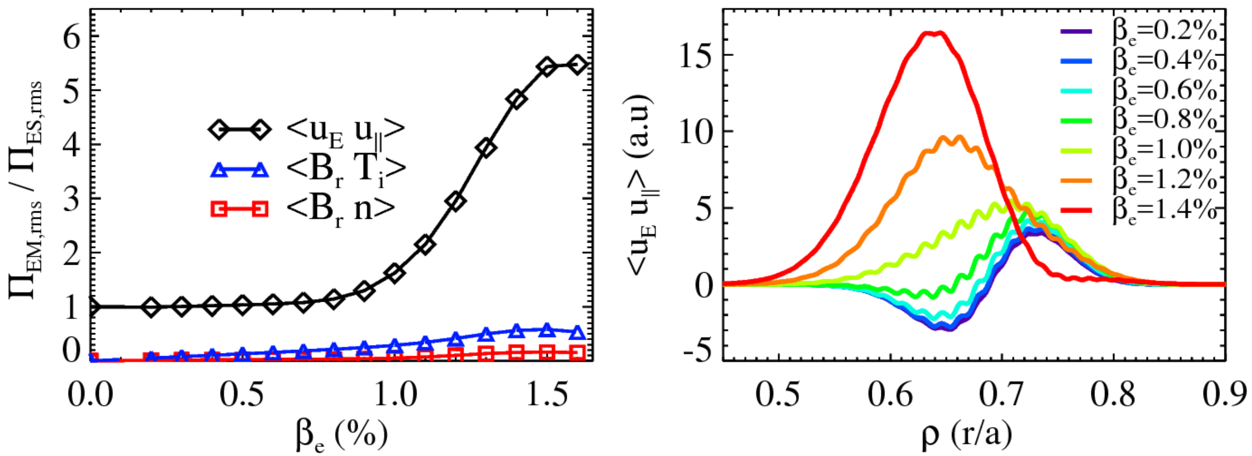
Where, $\varphi^{even}(\rho) = \varphi^{even}(-\rho)$ & $\varphi^{odd}(\rho) = -\varphi^{odd}(-\rho)$

- In the low β_e, φ and $u_{\parallel}$ are nearly symmetric. However, both become asymmetric at high β_e.
  → **EM effects (β_e) plays a role of a symmetry breaker in global tokamak geometry.**

- When we decompose the (m,n) mode into even and odd components with respect to the rational surface position, the parity gradually changes from even[odd] to odd[even] for φ [$u_{\parallel}$].
  → **The mode asymmetry is enhanced as both even and odd components coexist at high β_e**

Residual stress is governed by the mode asymmetry.

$$\Pi_{res} = \langle \tilde{u}_E \tilde{u}_\parallel \rangle + \langle \tilde{B}_r \tilde{T}_i \rangle + (\tau + 1)\frac{\hat{T}_{eq}}{\hat{n}_{eq}} \langle \tilde{B}_r \tilde{n} \rangle$$

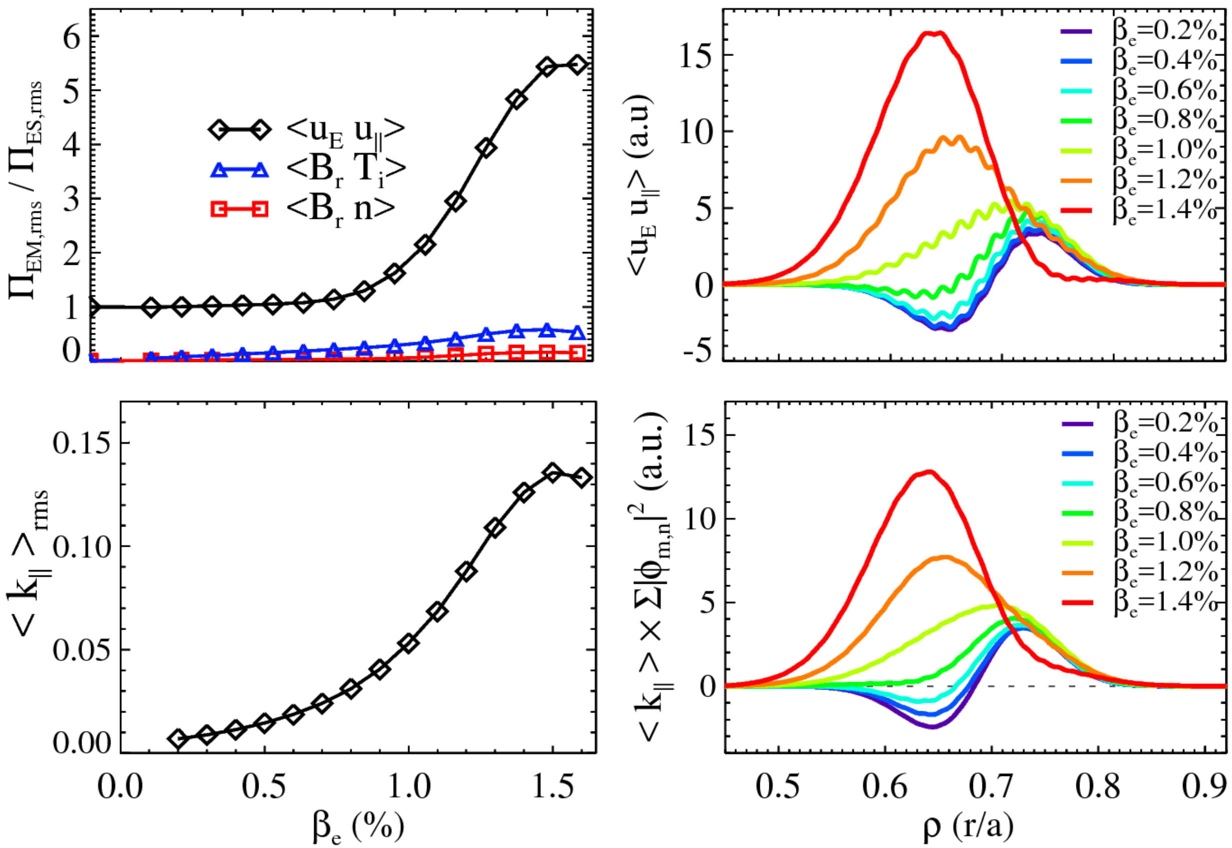


- Conventional residual stress part is dominant.
- As β_e increases, 1. residual stress is strongly enhanced

  2. radial profile changes from the dipolar to the unipolar one.
- Residual stress significantly increases in the high β_e regime where the radial profile changes into unipolar ($\beta_e \geq 1.0\%$)

Residual stress is governed by the mode asymmetry.

$$\Pi_{res} = \langle \tilde{u}_E \tilde{u}_\parallel \rangle + \langle \tilde{B}_r \tilde{T}_i \rangle + (\tau + 1)\frac{\hat{T}_{eq}}{\hat{n}_{eq}} \langle \tilde{B}_r \tilde{n} \rangle$$



$$\langle k_\parallel \rangle \equiv \frac{1}{qR_0} \frac{\sum_{m,n} (n/|n|)(nq - m)|\tilde{\phi}_{m,n}|^2}{\sum_{m,n} |\tilde{\phi}_{m,n}|^2}$$

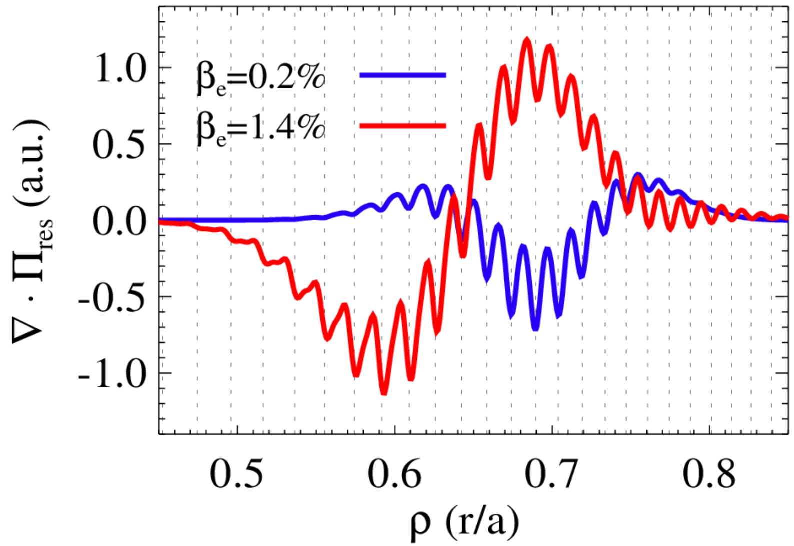
[Wang *et al.*(PRL, 2009)]

Residual stress and global mode asymmetric factor $<k_\parallel>$ are in good agreements

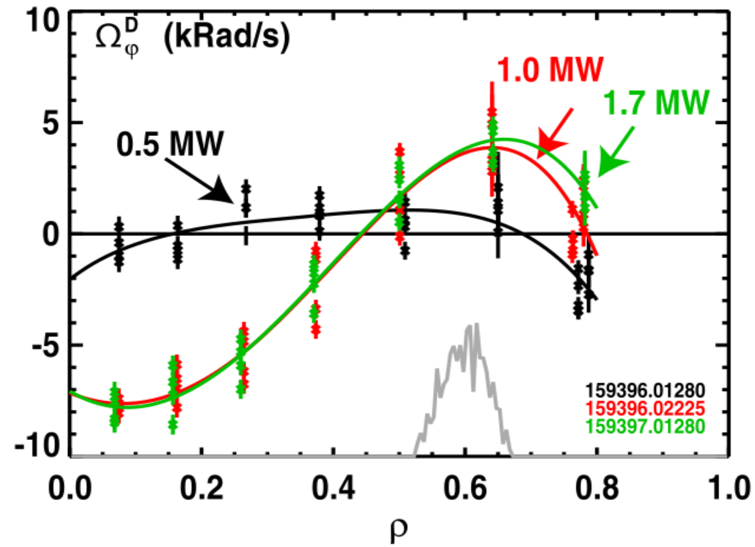
→ **residual stress is generated via the symmetry breaking also in tokamak geometry.**

Generated intrinsic rotation is similar to the observation in tokamak

< In the simulation >



< In the DIII-D tokamak >



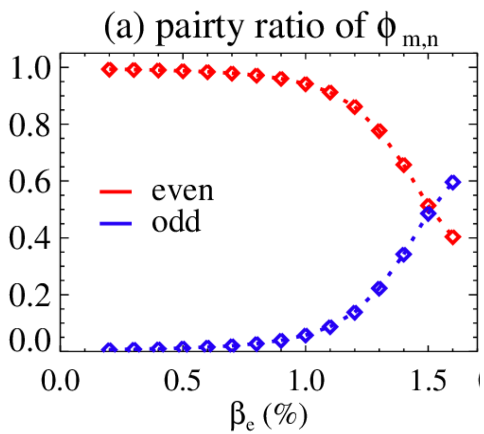
[Grierson *et al.*(PRL, 2017)]

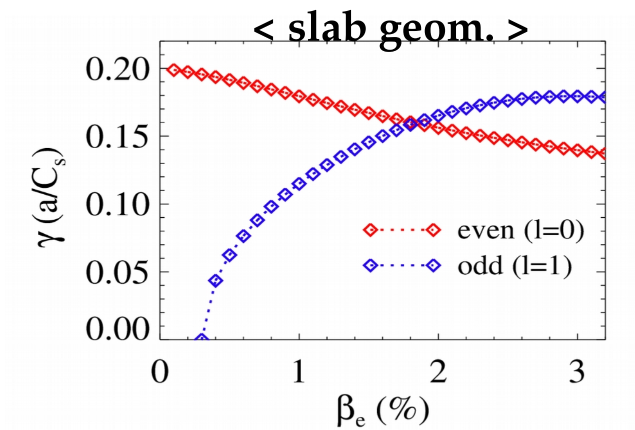
Generated intrinsic rotation shows a similarity with the observed one in DIII-D tokamak with ECH heating

- Amplitude is enhanced as β_e increases.
- Radial structure change unipolar ⟶ dipolar as β_e increases.

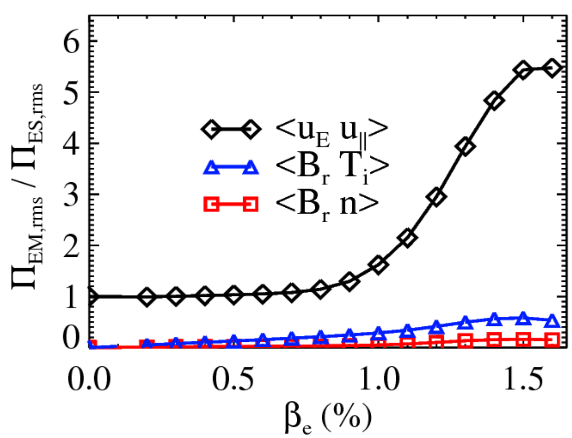
EM toroidal effects enhance the residual stress via mode parity mixing.

- EM effects induce mode parity mixing (= mode asymmetry) in toroidal geometry, while it changes the mode from the pure even to the pure odd parity concurrently in slab geometry.

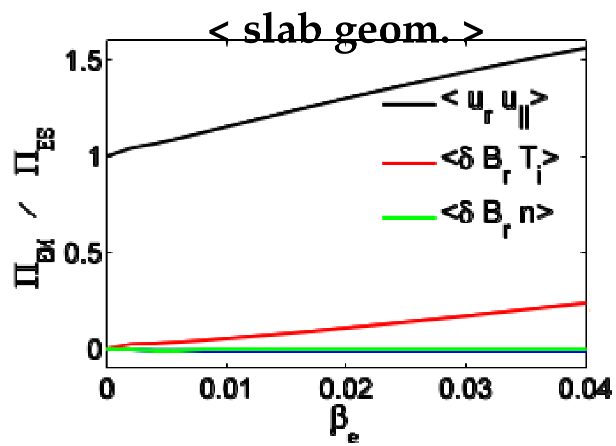




- The residual stress is strongly enhanced through the mode parity mixing in tokamak geometry.

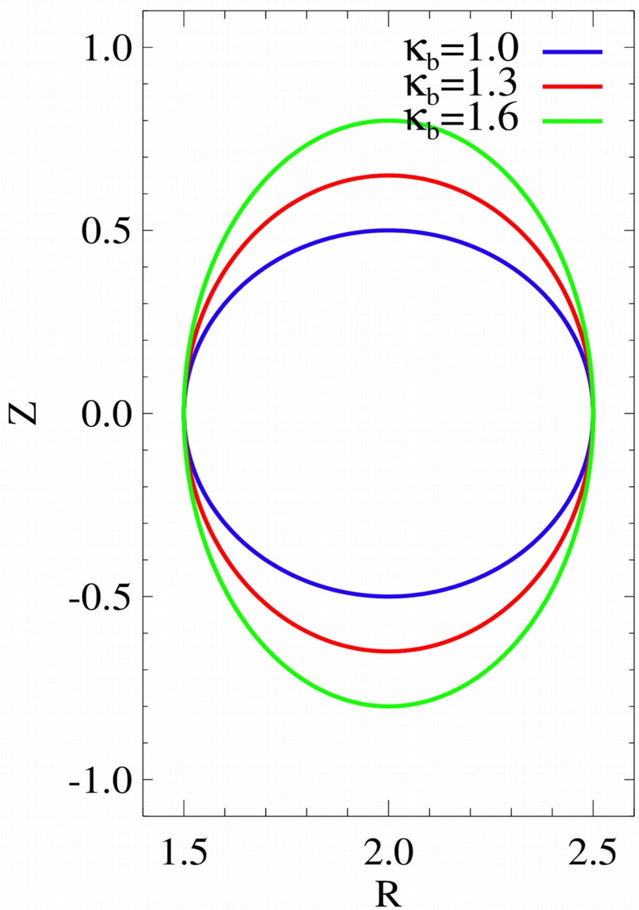


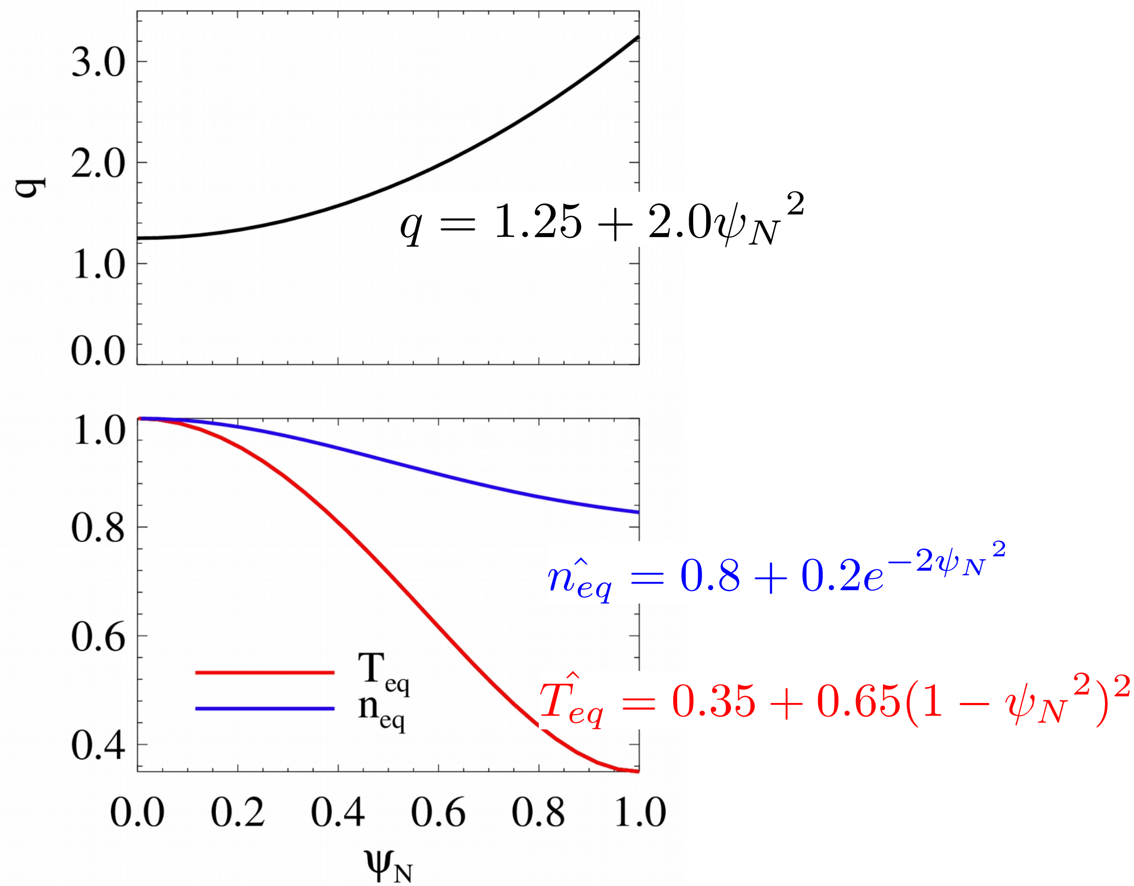
~500%↑ at β_e=1.5%



~35%↑ at β_e=1.5%

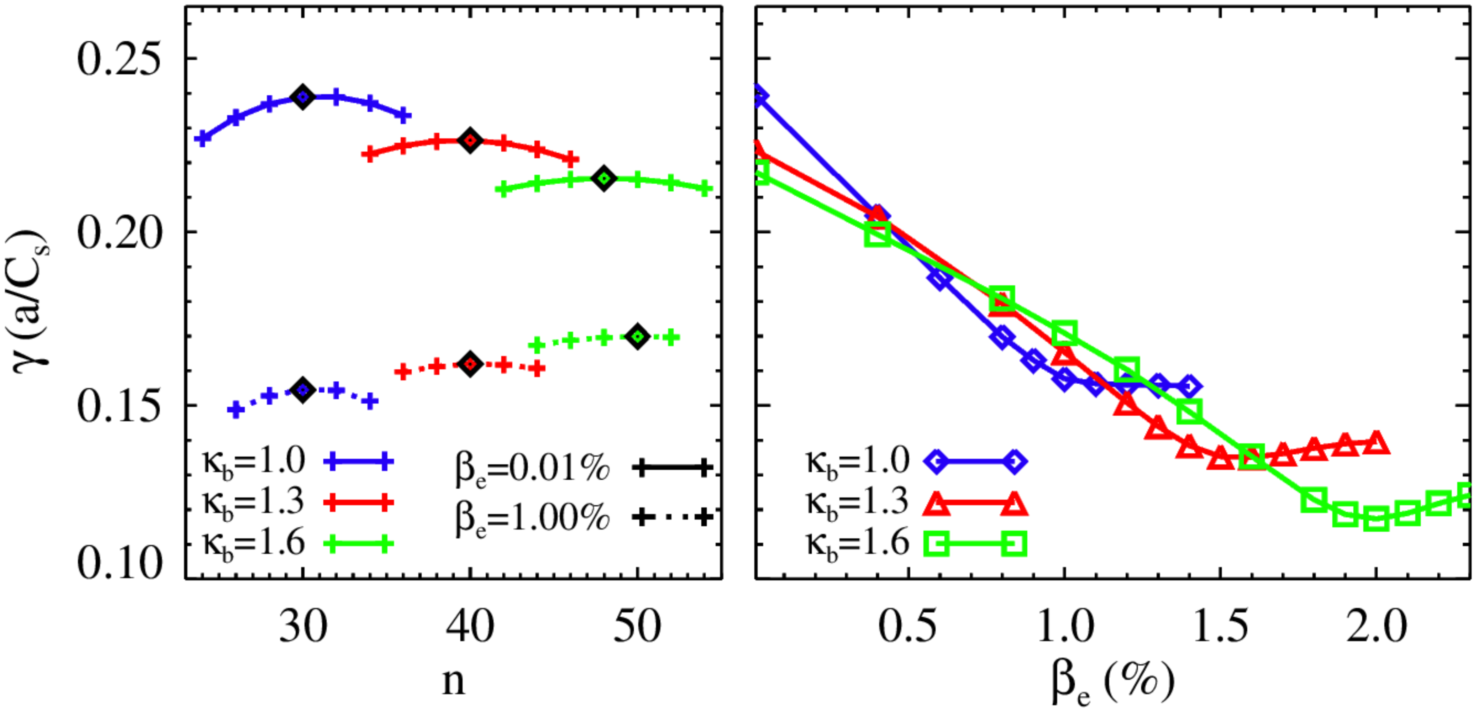
- We use elongated toroidal axisymmetric geometry (via. CHEASE)





· Grid # : nx=644, ny=64, nz=17 · a=0.5m · T_c=2keV, B_0=1T ⟹ ρ_s=0.0125 · $n_c = \beta_e \frac{B_0{}^2}{\mu_0}\frac{1}{T_c}$

Elongation effects on the linear growth-rate



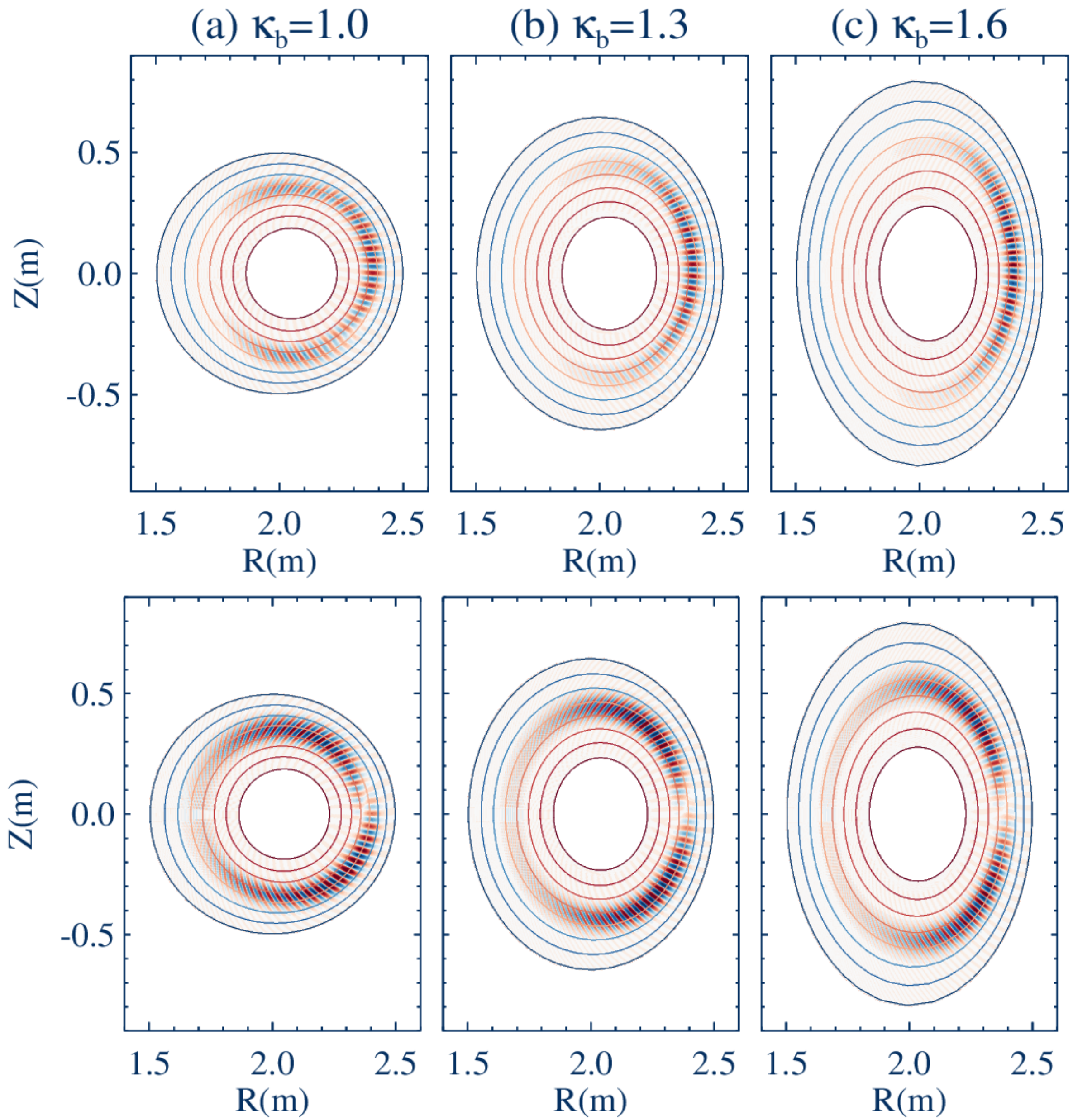
- The EM effects stabilize ITG mode for all κ_b's. But, the growth-rate decreases with β_e more slowly for higher-κ plasmas.[Citrin *et al.* (NF, 2022)]

  → **The plasma elongation reduces the EM stabilizing effects.**

Global eigenmodes on a poloidal cross section when β_e=1%

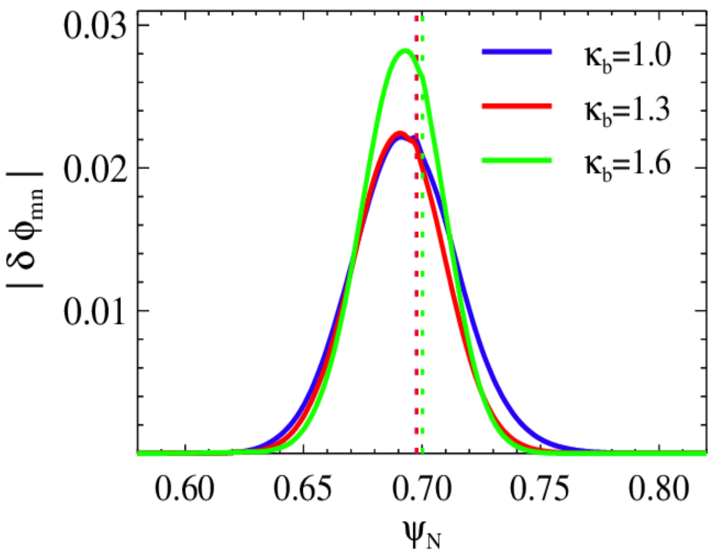
$\delta\phi$

$\delta A_{\parallel}$

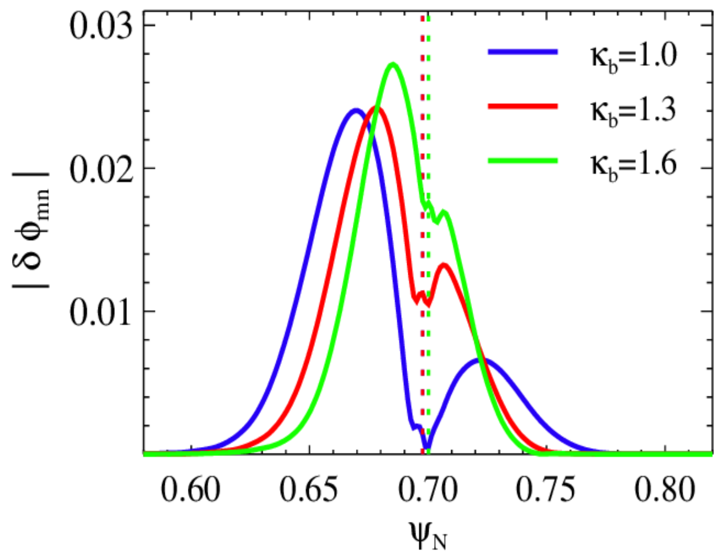


Elongation effects on mode asymmetry via parity mixing with β_e

β_e=0.01%



β_e=1.0%



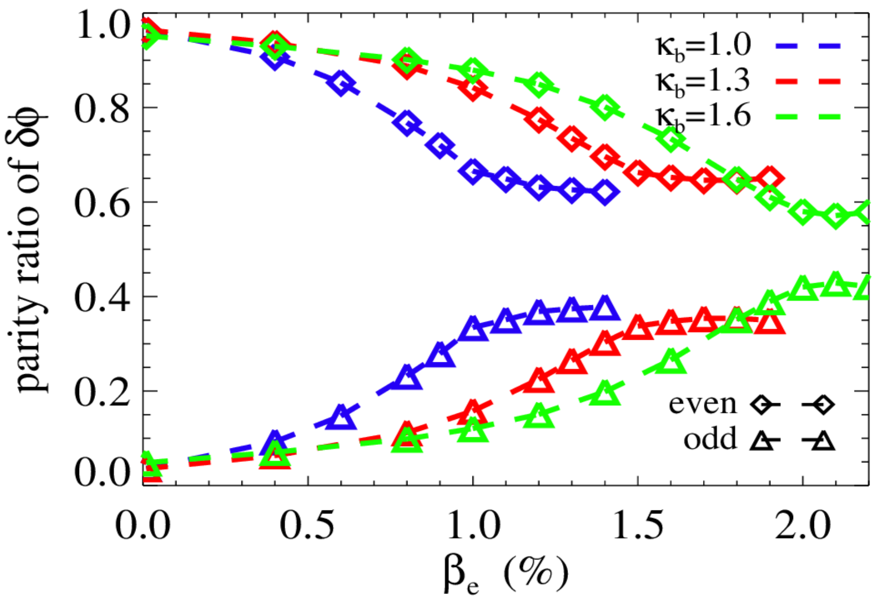
- Decompose global eigenfunctions into poloidal harmonics and choose poloidal modes located at ψ_N=0.7, where R_0/L_{Ti} is maximum.

  - At 0.01%, $\delta\varphi_{mn}$ are symmetry for all κ_b.

  - At 1.0%, $\delta\varphi_{mn}$ become asymmetry for all κ_b. The mode is more asymmetric for low κ_b.

  → **EM effects (β_e) as a symmetry breaker becomes week for high κ_b.**

Elongation effects on mode asymmetry via parity mixing with β_e



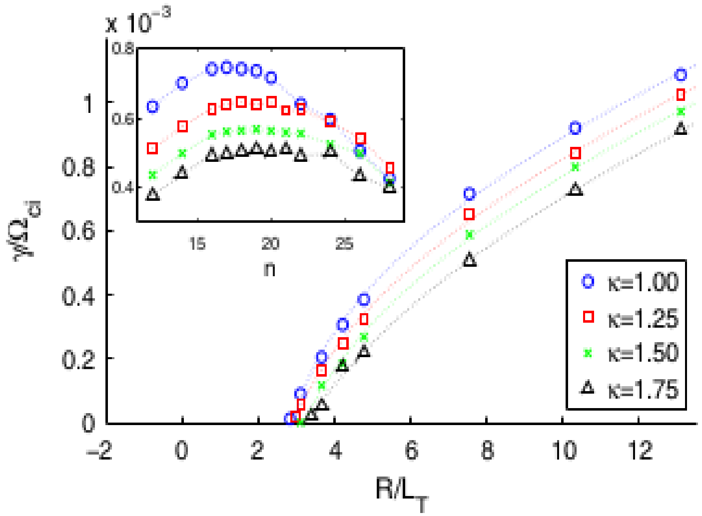
Where, $\varphi^{even}(\rho) = \varphi^{even}(-\rho)$ & $\varphi^{odd}(\rho) = -\varphi^{odd}(-\rho)$

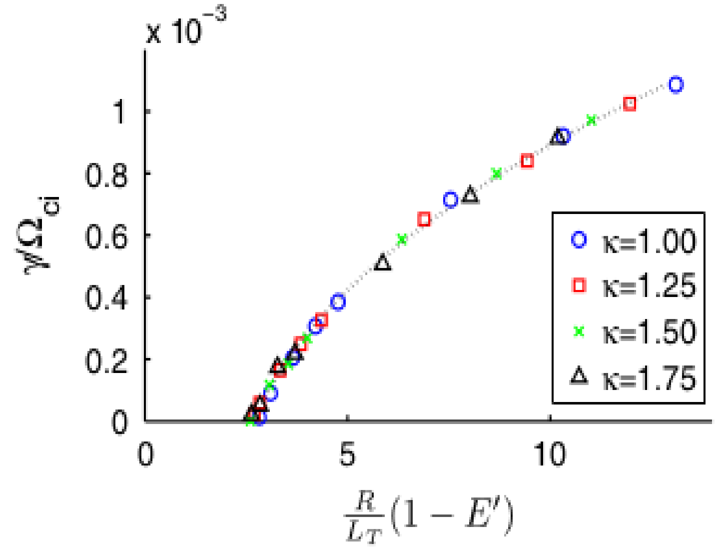
- Divide $\delta\varphi_{mn}$ into even and odd components with respect to the rational surface for each m and globally average.
  - the parity gradually changes from even to odd for φ.
  - The odd parity ratio reaches about 0.4 and saturates at $\beta_e \approx 1.0$, 1.5, and 2.0% when $\kappa_b = 1.0$, 1.3, and 1.6, respectively.

  → **The parity mixing occurs slowly as κ_b increases.**

Elongation stabilization effects via reduction of R_0/L_{Ti} : ES

[Angelino *et al.*(PRL,2009)]

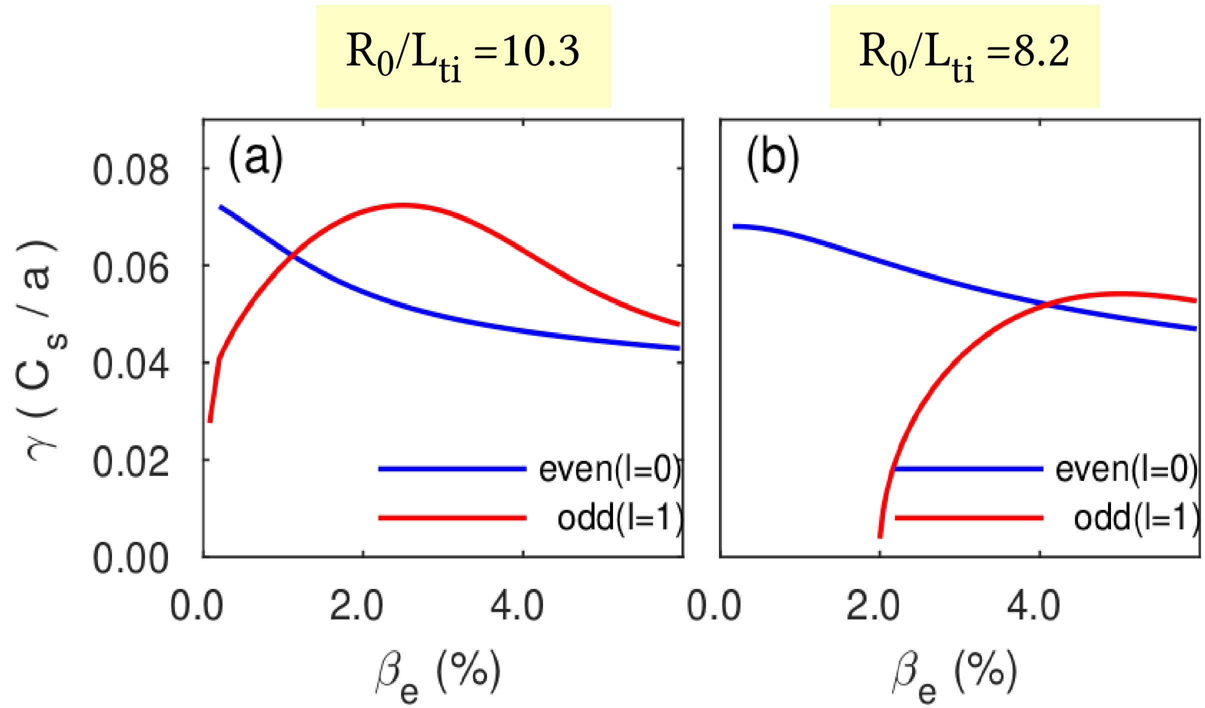




$$\frac{R_0}{L_{T_i}} \rightarrow \frac{R_0}{L_{T_i}}(1-E'), \quad where \quad E = r\frac{\kappa-1}{\kappa+1}$$

The elongation can stabilize the ITG instability by reducing the ion temperature gradient.

Elongation stabilization effects via reduction of R_0/L_{Ti} : in slab

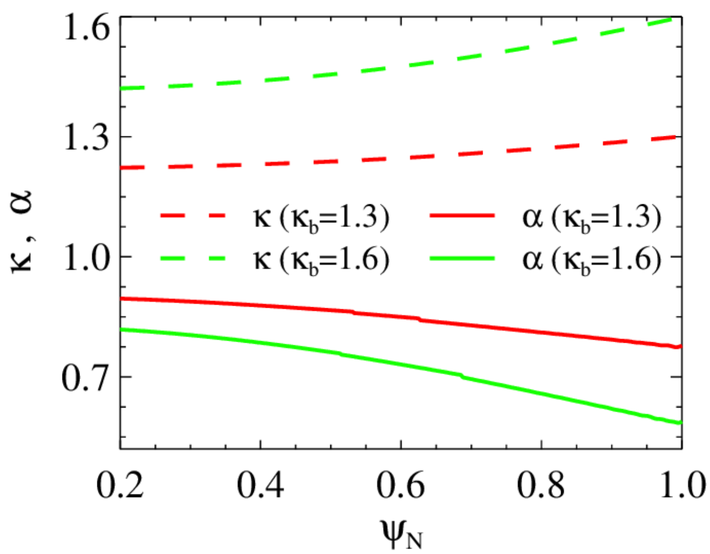


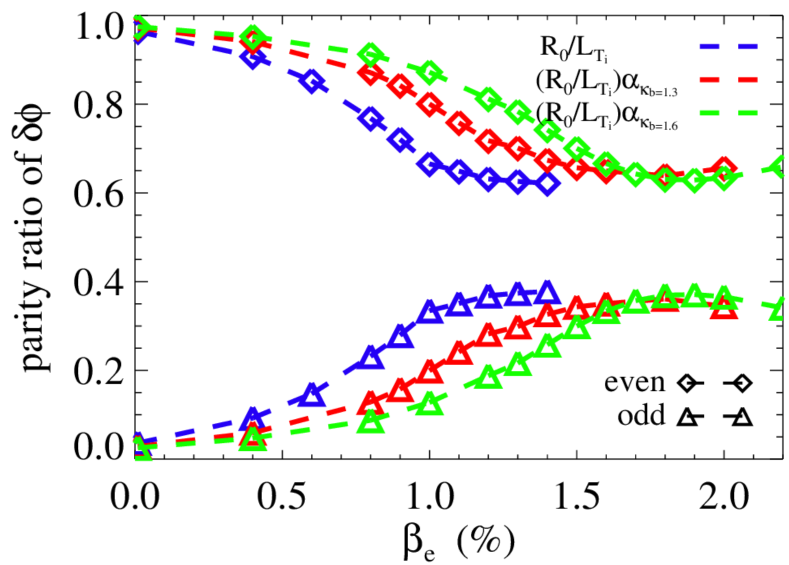
- The slab ITG with different R_0/L_{Ti}.

  - The parity switch is delayed by the reduction of R_0/L_{Ti} : requires a higher β_e at lower R_0/L_{Ti} .

  → The parity change decline by elongation might be caused by the reduction of the effective R_0/L_{Ti}.

Elongation stabilization effects via reduction of R_0/L_{Ti} : in global

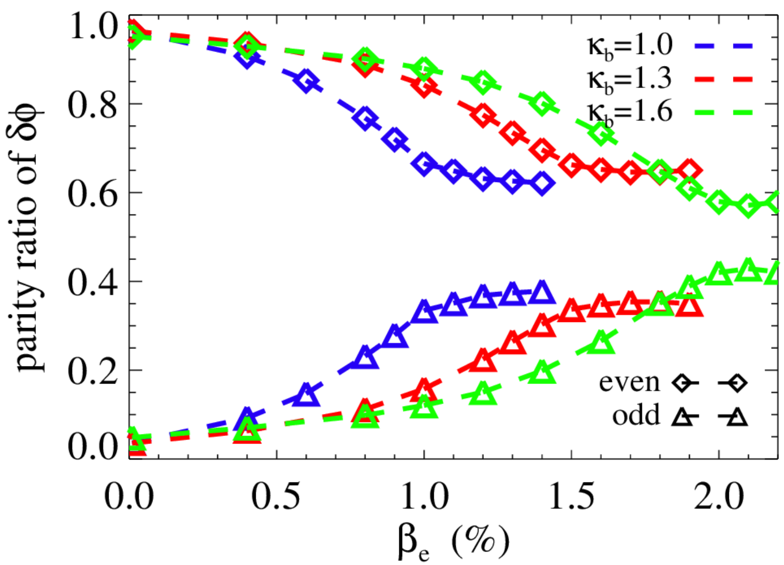


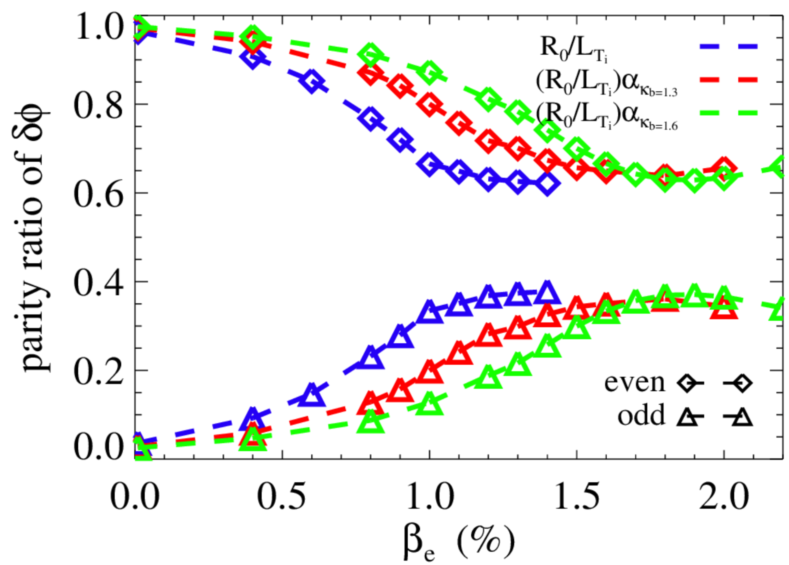


- Global simulation with $(R_0/L_{Ti})\alpha_{\kappa_b=1.3}$, $(R_0/L_{Ti})\alpha_{\kappa_b=1.6}$ in circular plasmas.

$$R_0/L_{T_i} \to (R_0/L_{T_i})\,\alpha, \quad where \quad \alpha = 1 - \left(r\frac{\kappa-1}{\kappa+1}\right)'$$

Elongation stabilization effects via reduction of R_0/L_{Ti} : in global

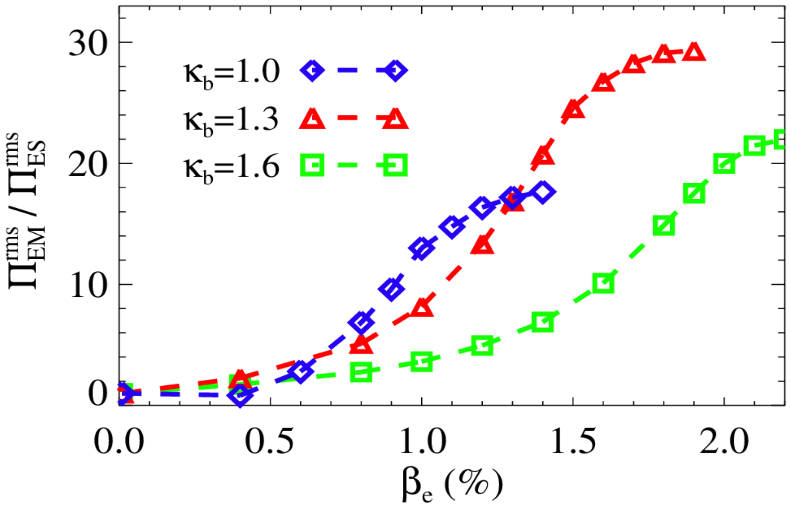


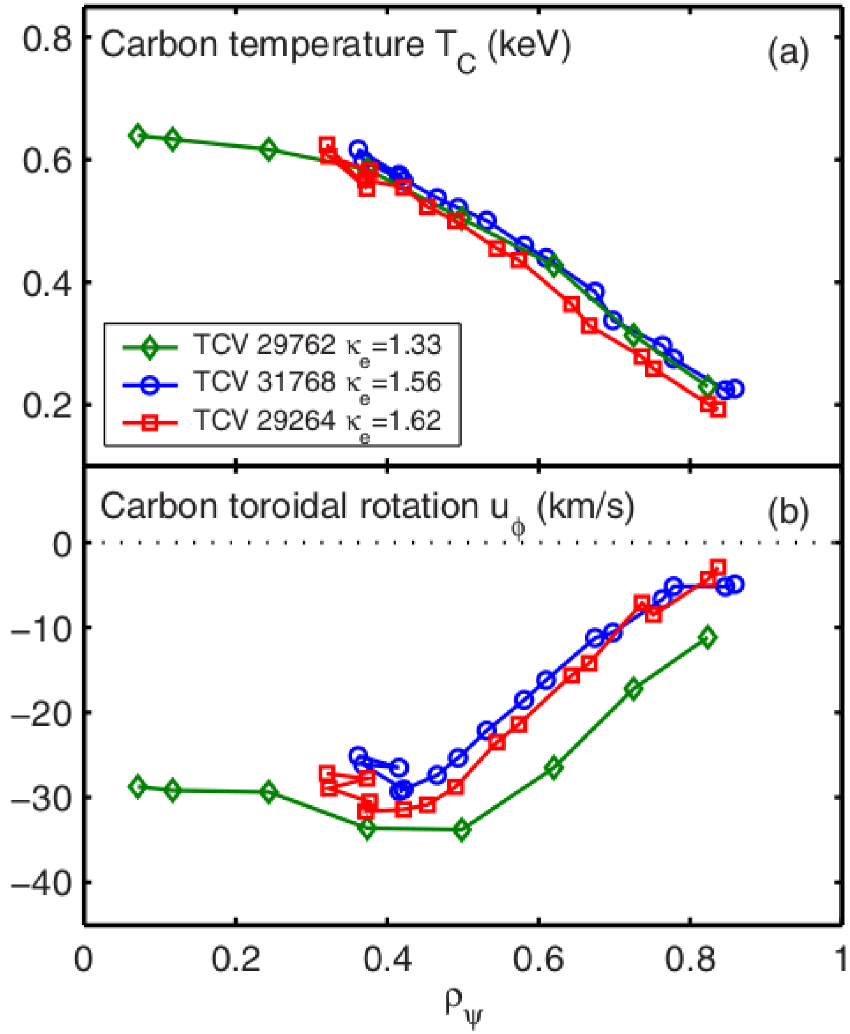


- Global simulation with $(R_0/L_{Ti})\alpha_{\kappa_b=1.3}$, $(R_0/L_{Ti})\alpha_{\kappa_b=1.6}$ in circular plasmas.
- The parity ratios of δφ for the different R/L_{Ti} in circular plasmas are similar to the parity ratios of δφ for same R_0/L_{Ti} with different κ.

  → **The plasma elongation affects the mode structure via the reduction of the effective R_0/L_{T_i} in EM regime, also.**

Elongation effects on the generation of intrinsic rotation





[Duval *et al.*(PoP,2008)]

- The enhancement of Π_{EM} appears at high β_e as κ_b increases
  - The reduction of the parity change → the reduction of mode asymmetry.
- Possible mechanism for the reduction of the toroidal rotation with increasing elongation in TCV experiments.

Global EM effects on mode parity change and intrinsic rotation generation

- EM effect gives asymmetry to the eigenmode as even and odd components co-exist.
- When even and odd parity components are comparable to each other,

1. the mode asymmetry and the resulting residual stress are maximized.
2. the global profile of the residual stress changes from the dipolar to the unipolar shape.

Elongation effects on mode parity change and intrinsic rotation generation

- The parity mixing of eigenmode occurs slowly as the plasma elongation increases.
- The plasma elongation effects on the parity change via the reduction of effective ion temperature gradient.
- As the elongation stabilizes the mode asymmetry induced by parity change, the generation of intrinsic rotation is reduced.