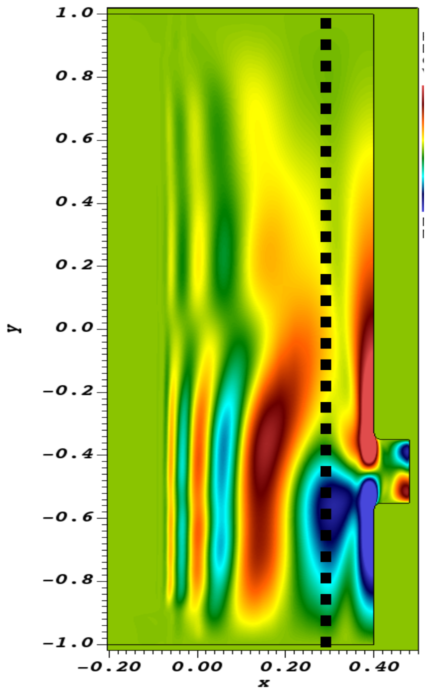
Modeling RF-induced ponderomotive effects on edge/SOL transport



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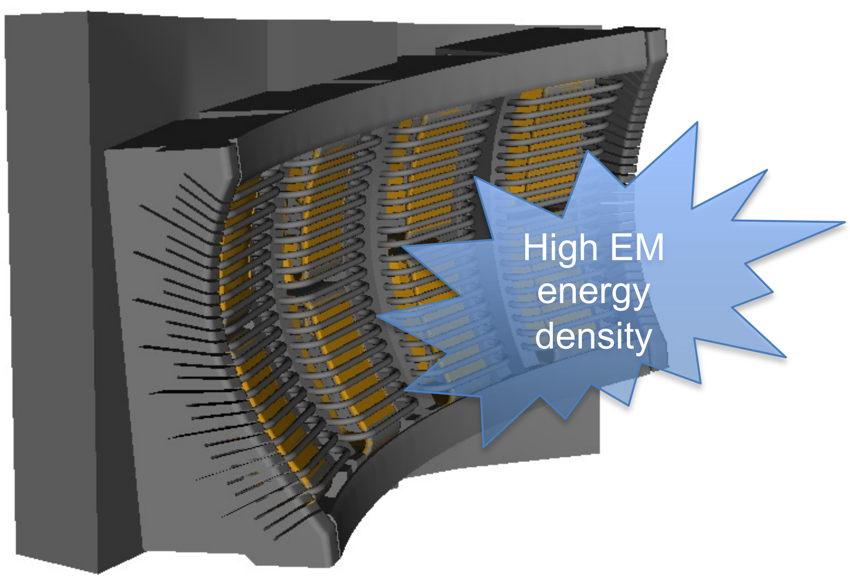
What is ponderomotive force?

- Electric field energy density $\frac{\epsilon_0 E^2}{2} \sim \frac{[J]}{[m^3]} \sim \frac{[N]}{[m^2]} \sim$ pressure
- Gradient of pressure or energy density = force density
- Pressure gradients drive momentum transport, e.g. in species fluid equation

$$m_\alpha n_\alpha \left(\frac{\partial \vec{V}_\alpha}{\partial t} + (\vec{V}_\alpha \cdot \nabla)\vec{V}_\alpha \right) + \nabla \cdot \overleftrightarrow{P}_\alpha = q_\alpha n_\alpha (\vec{E} + \vec{V}_\alpha \times \vec{B}) + \vec{R}_\alpha$$



- In plasma edge/SOL, energy density gradients that arise from RF antenna operation can also drive transport

Can pondermotive forces significantly affect fusion plasma boundaries?

Questions of interest:

(1) What physics do ponderomotive forces add to edge/SOL dynamics?

(2) Can ponderomotive effects become significant enough to affect edge/SOL transport near a high-power RF antenna (~1 MW/m^2)?

RF interaction with PFCs (sheaths, impurity production, etc.) may also affect edge/SOL physics, but plasma-material interactions are not the focus here.

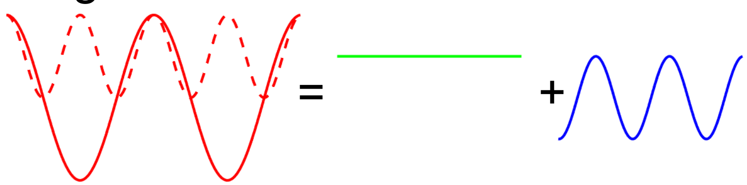
How do RF effects influence physics on edge plasma transport timescales?

- When injecting RF we have both fluid (slow, 0-subscripted) and RF wave (fast, 1-subscripted) timescales

$$\vec{E} = \vec{E}_0 + \vec{E}_1 \rightarrow \qquad f_\alpha = f_{0\alpha} + f_{1\alpha} \rightarrow \qquad \rho_\alpha = \rho_{0\alpha} + \rho_{1\alpha}$$
$$\vec{B} = \vec{B}_0 + \vec{B}_1 \qquad\qquad\qquad\qquad \vec{J}_\alpha = \vec{J}_{0\alpha} + \vec{J}_{1\alpha}$$
$$\ldots$$

- Terms quadratic in fast-time quantities contribute physics on slow timescales, as when DC-like terms arise in the trigonometric relation

$$[\cos(\omega t)]^2 = \frac{1}{2} + \frac{1}{2}\cos(2\omega t)$$



- Fluid velocity, in RF variables, has slow, fast, quadratic (mixed) components

flow velocity $\rho_{0\alpha}\vec{V}_{0\alpha} = \vec{J}_{0\alpha}$ “jitter velocity” $\rho_{1\alpha}\vec{V}_{0\alpha} + \rho_{0\alpha}\vec{V}_{1\alpha} = \vec{J}_{1\alpha}$

total velocity

$$\vec{V}_\alpha = \frac{\vec{J}_\alpha}{\rho_\alpha} = \frac{\vec{J}_{0\alpha} + \vec{J}_{1\alpha}}{\rho_{0\alpha} + \rho_{s\alpha}} = \vec{V}_{0\alpha} + \vec{V}_{1\alpha} - \frac{\rho_{1\alpha}\vec{V}_{1\alpha}}{\rho_{0\alpha} + \rho_{1\alpha}}$$

RF-induced ponderomotive effects contribute to slow-timescale momentum transport

- Momentum equation, on slow (fluid) timescale, thus has an added source term

$$m_\alpha n_{0\alpha}\left[\frac{\partial \vec{V}_{0\alpha}}{\partial t} + (\vec{V}_{0\alpha}\cdot\nabla)\vec{V}_{0\alpha}\right] + \nabla\cdot\overleftrightarrow{\mathbb{P}}_{0\alpha} = q_\alpha n_{0\alpha}(\vec{E}_0 + \vec{V}_{0\alpha}\times\vec{B}_0) + \langle\vec{F}_\alpha\rangle_0 + momentum\ sources/sinks$$

in the form of a new ponderomotive force density:

$$\vec{F}_\alpha = \rho_{1\alpha}\vec{E}_1 + \vec{J}_{1\alpha}\times\vec{B}_1 - \nabla\cdot\left[\frac{m_\alpha}{q_\alpha}\frac{\rho_{0\alpha}^2\vec{V}_{1\alpha}\vec{V}_{1\alpha}}{(\rho_{0\alpha}+\rho_{1\alpha})}\right]$$

arising from RF fields, charge densities, current densities.

- Volumetric ponderomotive source terms also arise in species energy equations, as RF waves damp and transfer power to the plasma

The physics of RF-induced ponderomotive forces is rich and complex

$$\mathbf{F}_\alpha = -\frac{m_\alpha n_{0\alpha}}{4}\nabla(|\boldsymbol{V}_{1\alpha}|^2) - \frac{q_\alpha n_{0\alpha}}{2\omega}\nabla\cdot[Im(\boldsymbol{V}_{1\alpha}\boldsymbol{V}_{1\alpha}{}^*)\times\boldsymbol{B}_0]$$

$$+\frac{q_\alpha n_{0\alpha}}{4\omega}\nabla[Im(\boldsymbol{V}_{1\alpha}\times\boldsymbol{V}_{1\alpha}{}^*)\cdot\boldsymbol{B}_0] - \frac{q_\alpha}{2\omega}\boldsymbol{B}_0\times[Im(\boldsymbol{V}_{1\alpha}\boldsymbol{V}_{1\alpha}{}^*)]\cdot\nabla n_{0\alpha}$$

$$+\frac{m_\alpha n_{0\alpha}\nu_\alpha}{2\omega}\nabla\cdot[Im(\boldsymbol{V}_{1\alpha}\boldsymbol{V}_{1\alpha}{}^*)] + \frac{m_\alpha n_{0\alpha}\nu_\alpha}{2\omega}Im([\nabla\boldsymbol{V}_{1\alpha}]\cdot\boldsymbol{V}_{1\alpha}{}^*)$$

$$-\frac{m_\alpha}{2\omega}Im(\boldsymbol{V}_{1\alpha}\boldsymbol{V}_{1\alpha}{}^*)\cdot\nabla(n_{0\alpha}\nu_\alpha)$$

(after a Fourier transform in time, and considerable mathematical manipulation)

- Grad-V^2 term: like single-particle picture of ponderomotive force, with V_1 the "jitter velocity".
- Density gradient term: only non-zero for circular polarization, carries sign of charge. (Maybe important for RF waves launched into H-mode plasmas?)
- Green terms: also single-particle terms; only non-zero for circular polarization, and also carry sign of the charge.
- Black terms are associated with neutral collisions.

RF-induced parallel momentum sources are the primary focus of this talk

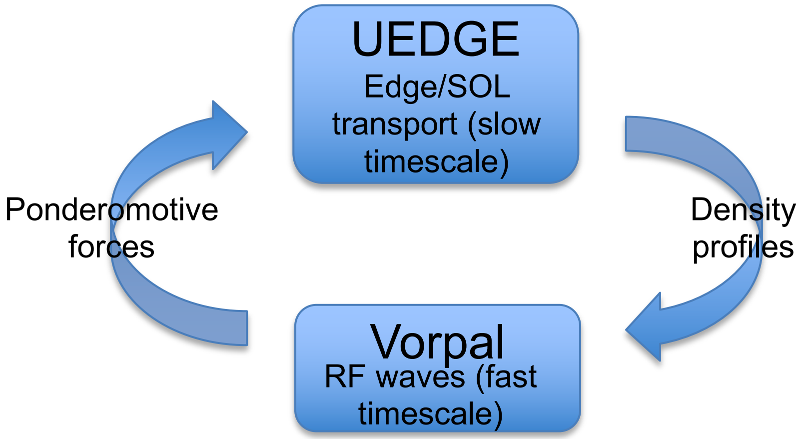
- Heuristic estimate: ponderomotive forces in plasma edge/SOL will most significantly influence parallel momentum transport
    - Dominant effect – changes to plasma density in front of the antenna.
- In this talk: neglect ponderomotive contributions to energy equations, cross-field momentum transport, convective cell dynamics, etc.
- Focus: how ponderomotive parallel momentum sources modify edge/SOL transport

Our computations couple an edge plasma model (UEDGE) and an RF wave model (Vorpal)

- Vorpal = FDTD code, models RF antenna geometry and wave propagation
- UEDGE = implicit finite-volume code, models edge transport of plasma/neutrals

Coupling scheme:

- Run UEDGE (equilibrium edge/SOL)
  -Map profiles to Vorpal grid (uniform grid, large domain)
- Run Vorpal (RF/ponderomotive)
  -Map solution to UEDGE grid (variable grid, smaller domain)
- Update UEDGE, adding PF source terms
  -new equilibrium density profile / solution for edge/SOL transport
- Update Vorpal; pass RF through new density profile
  -new RF wave pattern; new ponderomotive forces
- Repeat cycle to convergence, if attainable



- Vorpal: ~48 hours on 32 cores, ~1M grid cells, dt = 1e-12
- UEDGE: ~10 hours in serial (for scan), ~2k grid cells

TECH-X

UEDGE solves a system of collisional (Braginskii) fluid equations in axisymmetric tokamak geometry

plasma density

$$\frac{\partial}{\partial t}(n_i) \quad + \quad \nabla \bullet (n_i \vec{u}_i) = -S_r + S_i$$

$$n_i u_\perp = -D_{\perp i} \nabla_\perp n_i$$

ion || momentum

$$\frac{\partial}{\partial t}(m n_i u_{\parallel i}) \quad + \quad \nabla \bullet (m n_i u_{\parallel i} \vec{u}_i - \eta_i \nabla u_{\parallel i}) = -\nabla_\parallel P_i + m n_N n_i K_{cx}(u_{\parallel N} - u_{\parallel i}) + m S_r u_{\parallel N} - m S_i u_{\parallel N}$$

+ ponderomotive sources

electron thermal energy

$$\frac{\partial}{\partial t}(3/2 n_e T_e) \quad + \quad \nabla \bullet (\frac{5}{2} n_e T_e \vec{u}_e + \vec{q}_e) = \vec{u}_e \bullet \nabla (3/2 n_e T_e) - \Pi_e \bullet \nabla \vec{u}_e + Q_e$$

ion thermal energy

$$\frac{\partial}{\partial t}(3/2 n_i T_i) \quad + \quad \nabla \bullet (\frac{5}{2} n_i T_i \vec{u}_i + \vec{q}_i) = \vec{u}_i \bullet \nabla (3/2 n_i T_i) - \Pi_i \bullet \nabla \vec{u}_i + Q_i$$

$$q_\perp = -n \chi_\perp \nabla_\perp T$$

neutral density

$$\frac{\partial}{\partial t}(n_N) \quad + \quad \nabla \bullet (n_N \vec{u}_N) = S_r - S_i$$

ad hoc radial transport

$$n_N u_{\perp N} = -D_{\perp N} \nabla_\perp n_N$$

neutral || momentum

$$\frac{\partial}{\partial t}(m n_N u_{\parallel N}) \quad + \quad \nabla \bullet (m n_N u_{\parallel N} \vec{u}_N - \eta_N \nabla u_{\parallel N}) = -\nabla_\parallel P_N - m n_N n_i K_{cx}(u_{\parallel N} - u_{\parallel i}) - m S_r u_{\parallel N} + m S_i u_{\parallel N}$$

charge conservation

$$\nabla \bullet J(\phi) = 0$$

$$J_\parallel = \frac{en}{0.51 m \nu} \frac{B_x}{B} \left(\frac{1}{n} \frac{\partial P_e}{\partial x} - e \frac{\partial \phi}{\partial x} + 0.71 \frac{\partial T_e}{\partial x} \right)$$

+ ponderomotive sources

$$J_r = \sigma_\perp E_r$$

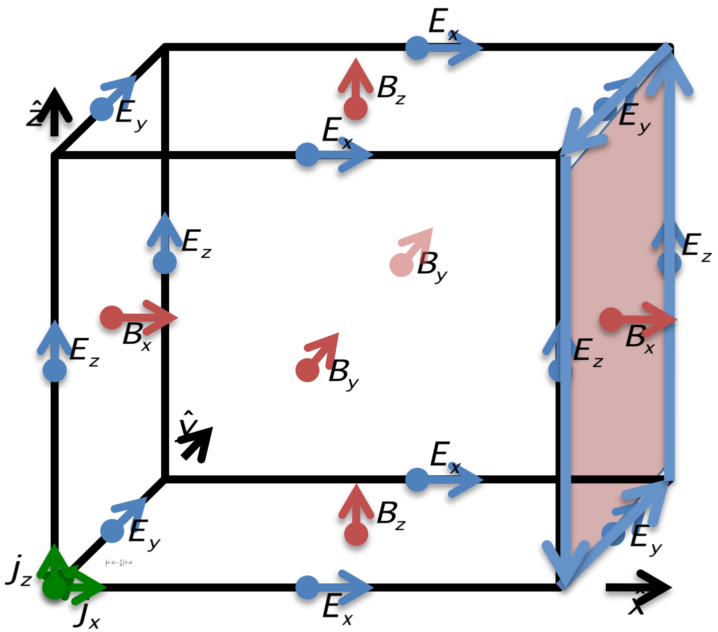
+ all cross-field drifts

sheath bound. cond.

$$\phi = \frac{-Te}{e} \ln \left[2\sqrt{\pi} \left(\frac{J_\parallel - e n u_{\parallel i}}{e n v_{te}} \right) \right]$$

*T. D. Rognlien et al., J. Nucl. Mater. 196–198, 347 (1992)

Vorpal models RF and plasma waves using finite-difference time-domain methods



K. S. Yee, *IEEE Trans. Antennas Propag.* **14**, 302 (1966).

- FDTD approach: preserve vector operations ($\nabla\times, \nabla\cdot, \nabla$); center fields in space & time, on discrete cells (***E*** @ cell edges, ***B*** @ cell faces)

$$\oint \vec{E} \cdot d\vec{l} = -\frac{d}{dt}\int \vec{B} \cdot d\vec{A}$$

- Time-centered - leap-frog time staggering puts ***E*** at full timesteps, ***B*** at half timesteps
- Adding current sources enables cold plasma waves to be modeled (J @ cell nodes). Semi-implicit method avoids $\omega_p \Delta t \leq 2$ constraint.

*Explicit Faraday step

$$\frac{\partial \vec{B}_1}{\partial t} = -\vec{\nabla}\times\vec{E}_1$$

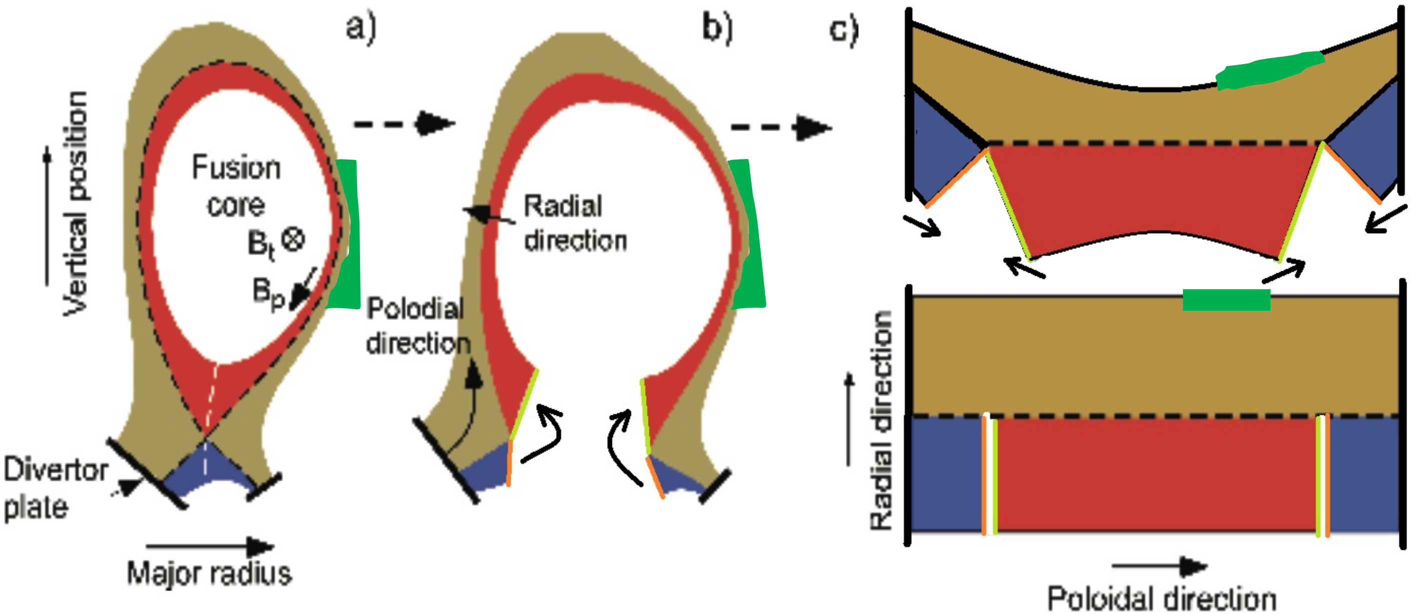
*Implicit step advances Ampere and cold current density equations in time [D. N. Smithe, Phys. Plasmas **14**, 056104 (2007)].

$$\frac{\partial \vec{E}_1}{\partial t} = c^2\vec{\nabla}\times\vec{B}_1 - \sum_{\alpha}\frac{\vec{J}_{1\alpha}}{\epsilon_0}$$

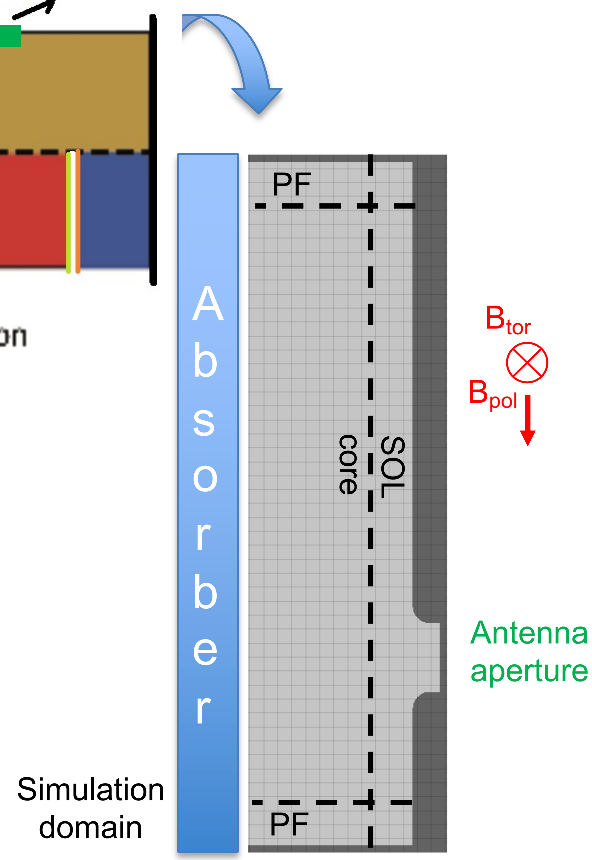
$$\frac{\partial \vec{J}_{1\alpha}}{\partial t} + \frac{q_\alpha \vec{B}_0}{m_\alpha}\times\vec{J}_{1\alpha} = \frac{q_\alpha \rho_{0\alpha}}{m_\alpha}\vec{E}_1$$

- Ponderomotive forces are computed directly from wave fields, then time-averaged in Vorpal post-processing

Vorpal/UEDGE cases are run in a 2D edge/SOL slab model, generalized to include an RF antenna (green)



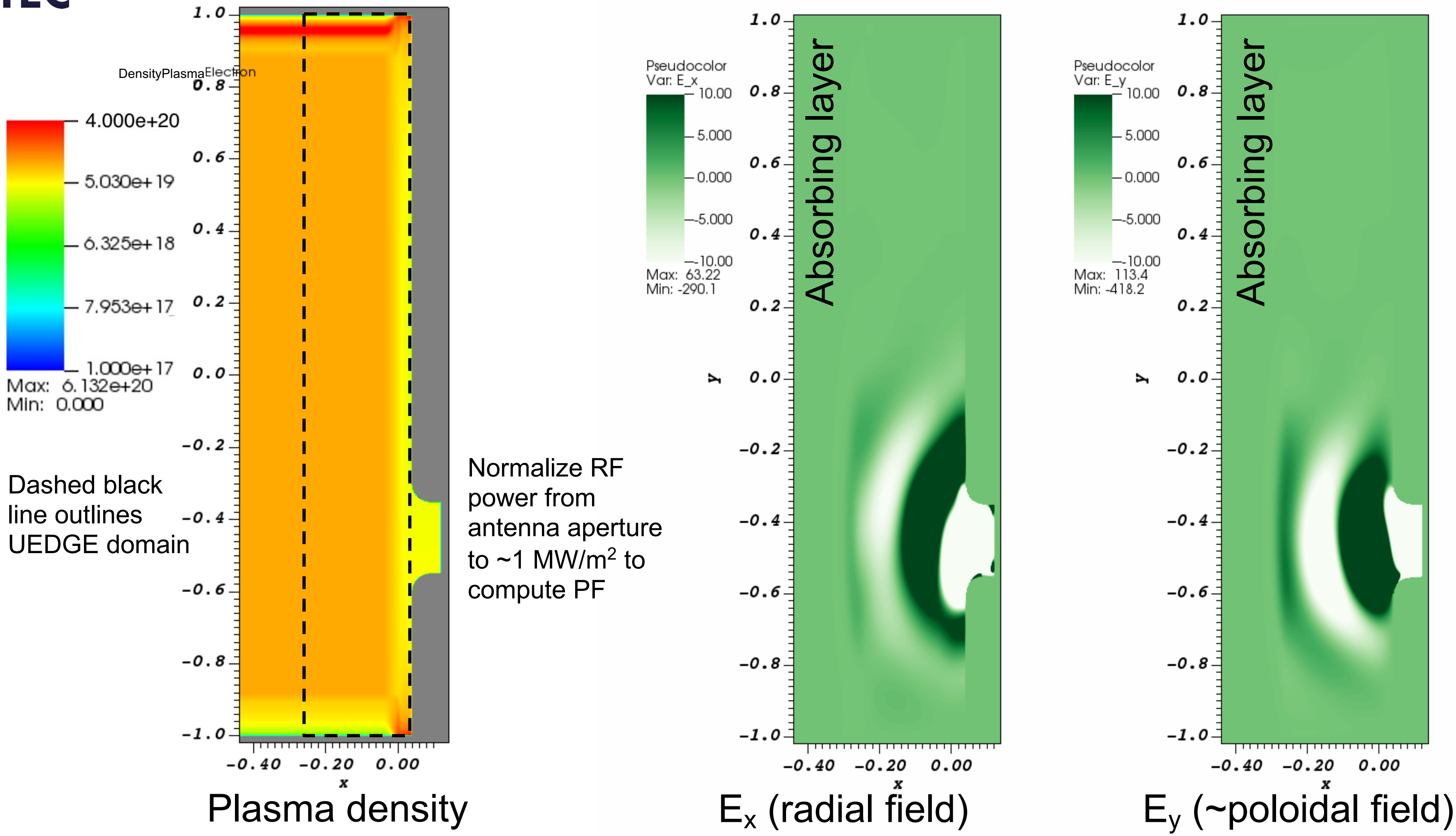
Modified from McCourt et al., Comp. Sci. Discov. **5**, 014012 (2012).



- Vorpal domain is broadened on the core side relative to UEDGE domain, with constant density and absorbing boundary conditions there

- Unlike a physical antenna, model is independent of toroidal angle

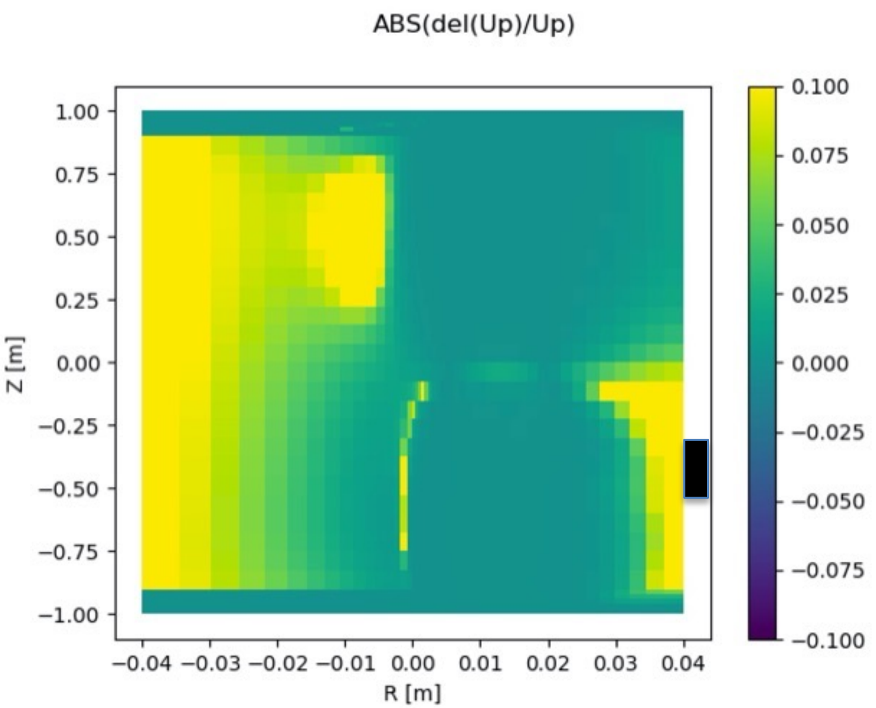
Fast waves propagate toward the core plasma after tunneling through the low-density evanescent region



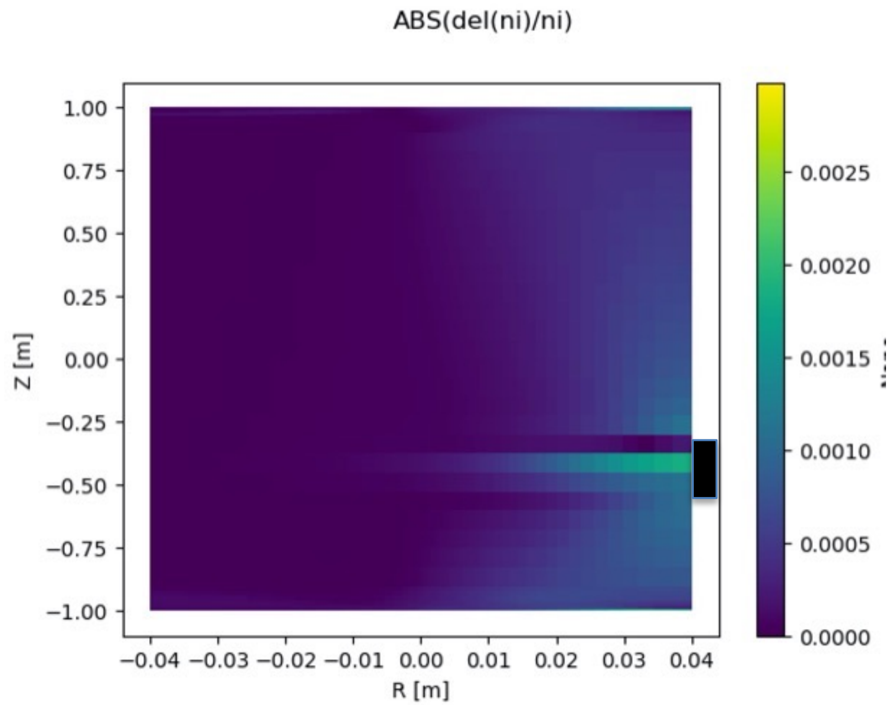
Parameters shown are for an Alcator C-Mod-like plasma:

B_{tor} = 5.0 T B_{pol} = 0.5 T $n_{core} \sim 1.0 \times 10^{20}$ m^{-3} f = 40 MHz

Forces, though detectable, have little effect on edge/SOL transport in this C-Mod-like scenario



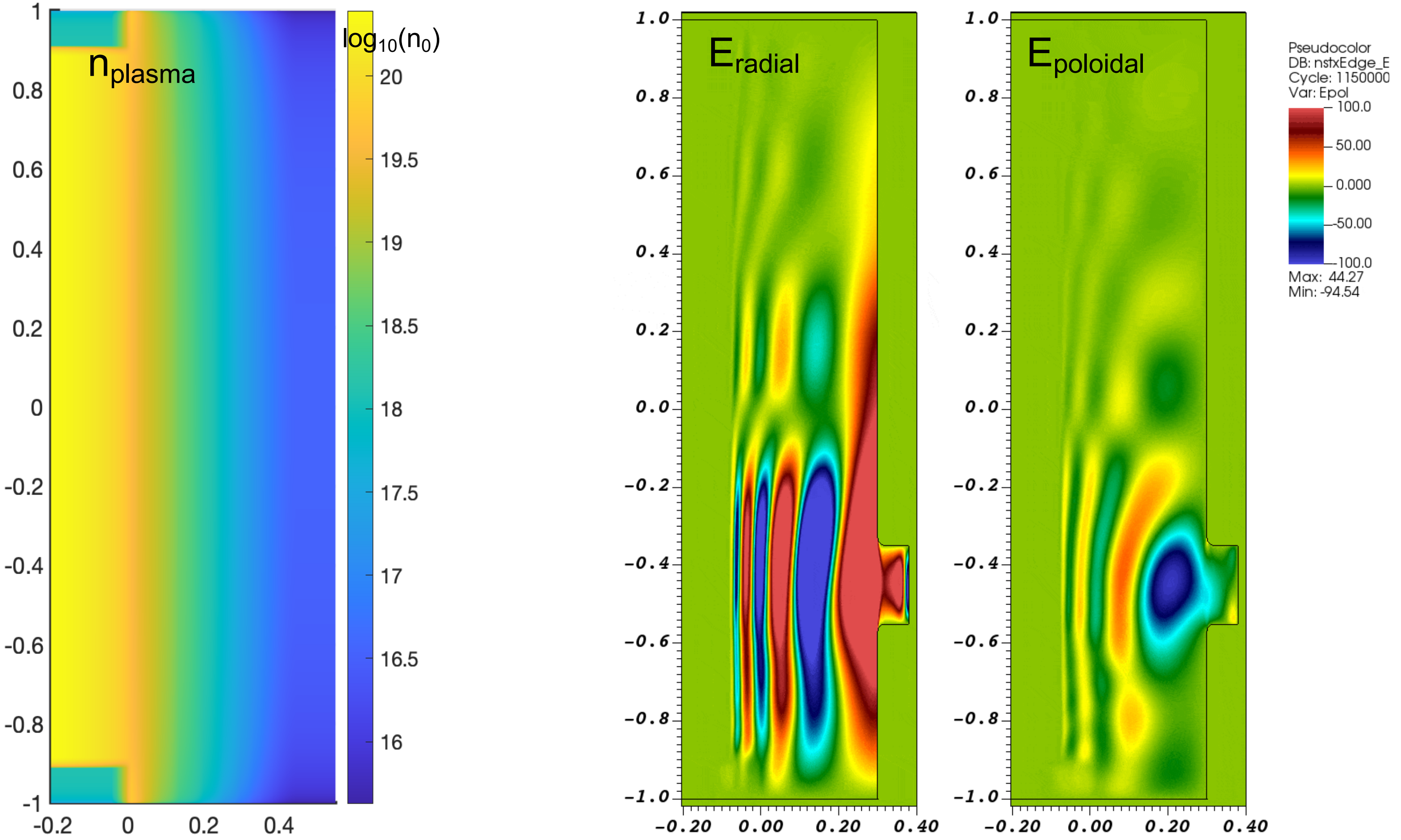
Ponderomotive effects induce > 10% changes in relative magnitude of parallel flow velocity



Overall effect on bulk plasma density is minimal (<1% change in relative magnitude)

- Generally, PF effects are smaller in cases with high plasma density and low density gradients near the antenna.

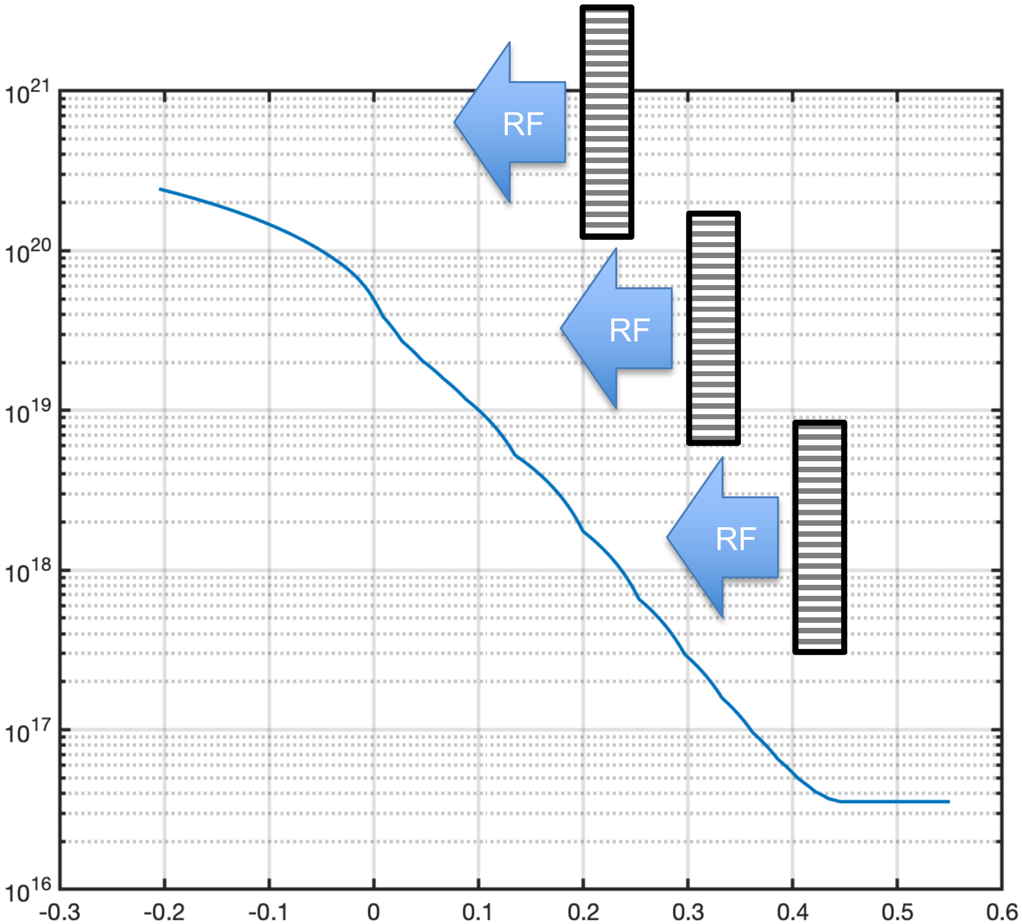
In NSTX-like scenarios, ponderomotive forces influence edge/SOL dynamics more strongly

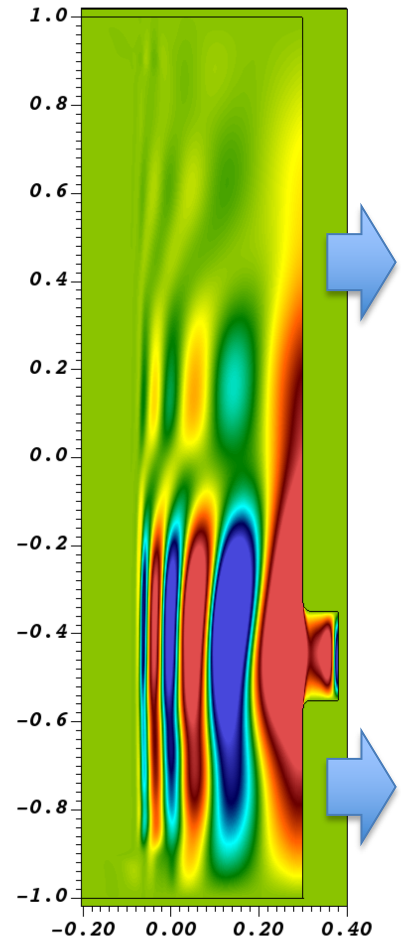


Plots shown hereafter are for NSTX-like plasma:

B_{tor} = 1.0 T B_{pol} = 0.5 T n_{core} ~ a few x 10^{20} m^{-3} f = 30 MHz

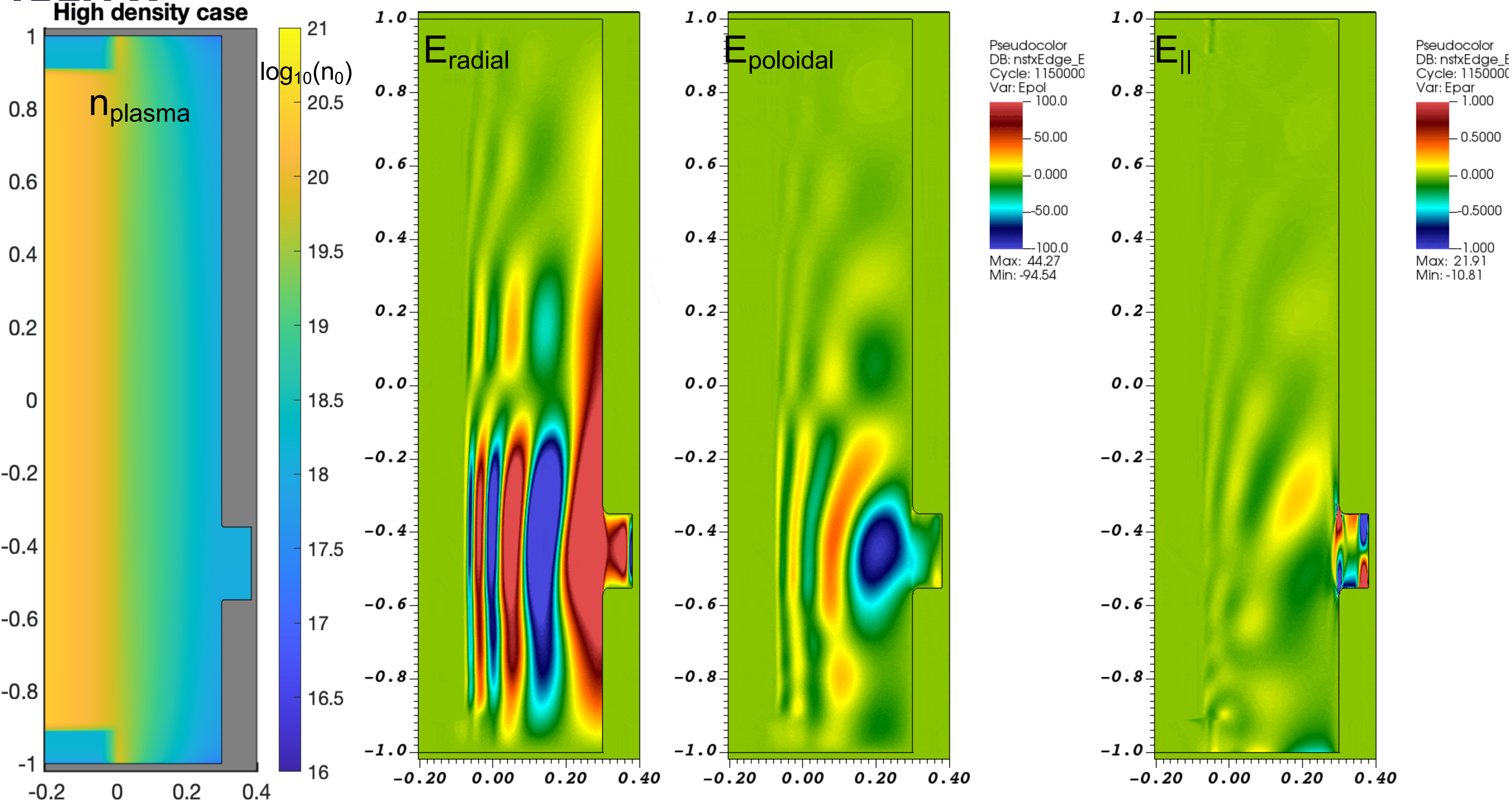
We situate the antenna at various points on the density profile and assess the ensuing PF effects





- By moving antenna and wall outward, can examine how ponderomotive effects influence edge/SOL transport as we move to lower-density regimes

Wave fields, high density (~10^{18} m^{-3} @ wall) NSTX-like case: propagating fast wave



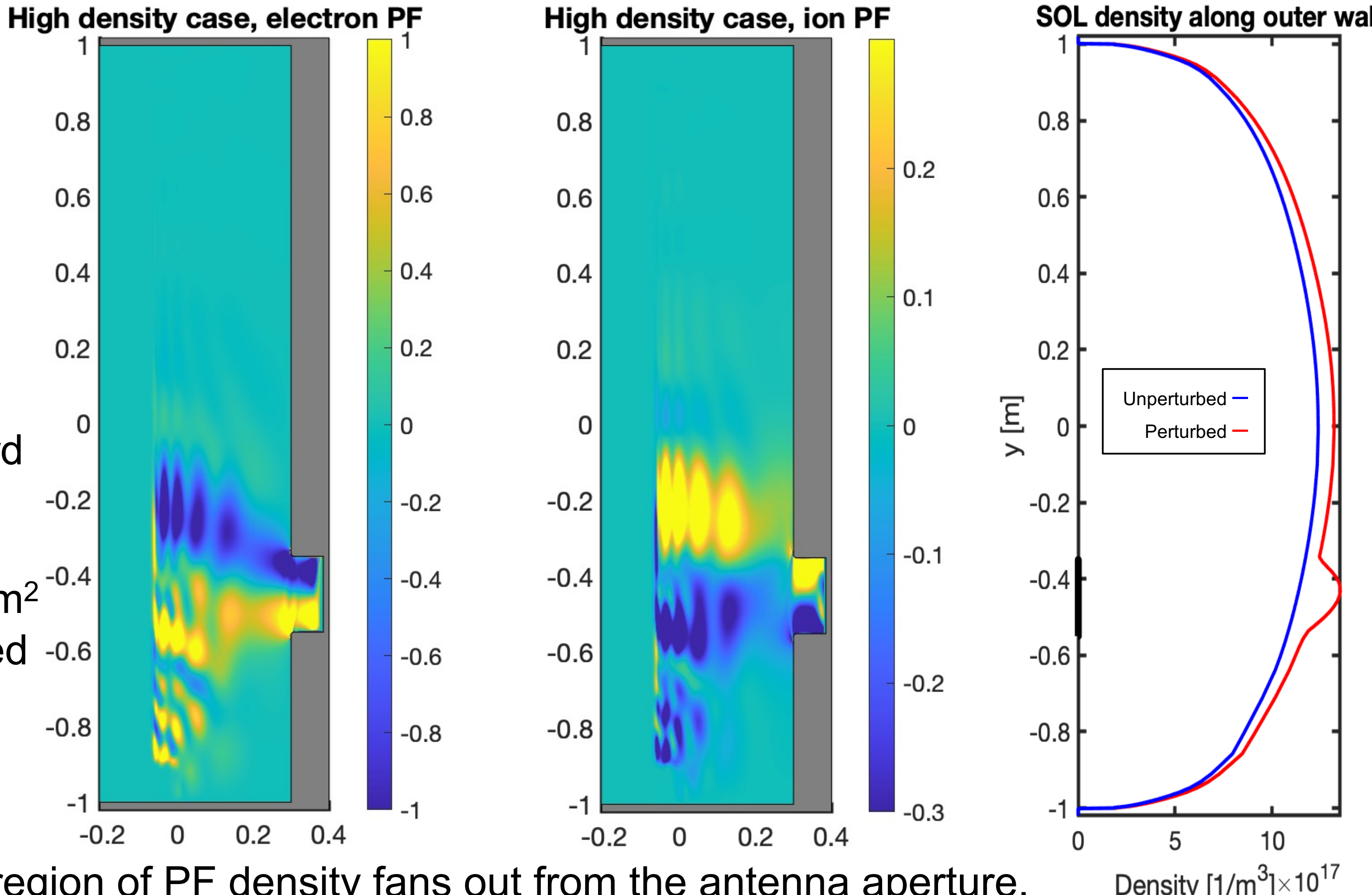
- Parallel E 100x smaller than radial, poloidal E (generally true for subsequent plots)
- Fast wave propagates at high densities with elliptical polarization

High-density (~10^{18} m^{-3}) scenario: ponderomotive forces increase density in front of the antenna

Yellow = upward force

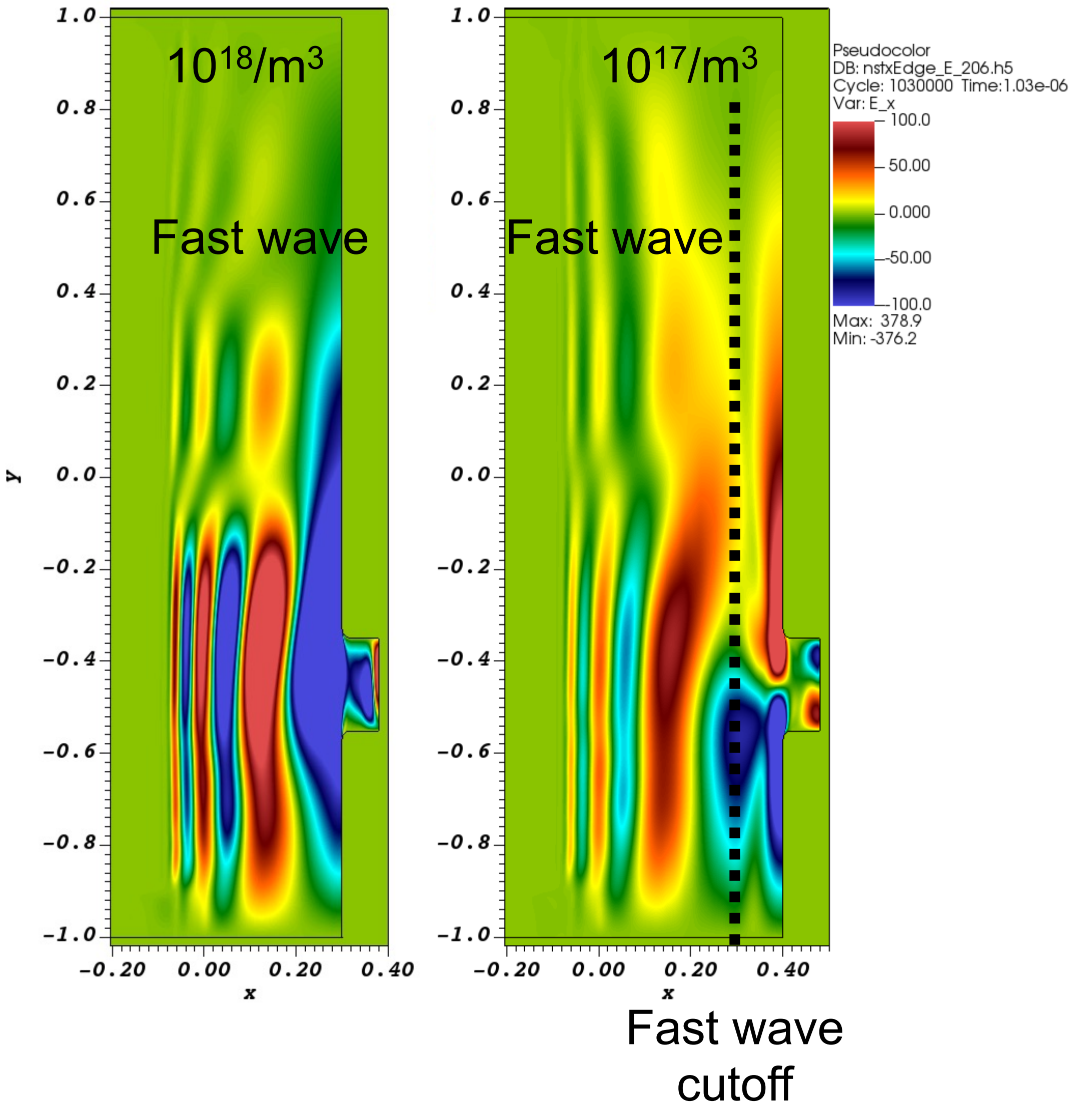
Blue = downward force

0.5 MW/m^2 converged solution

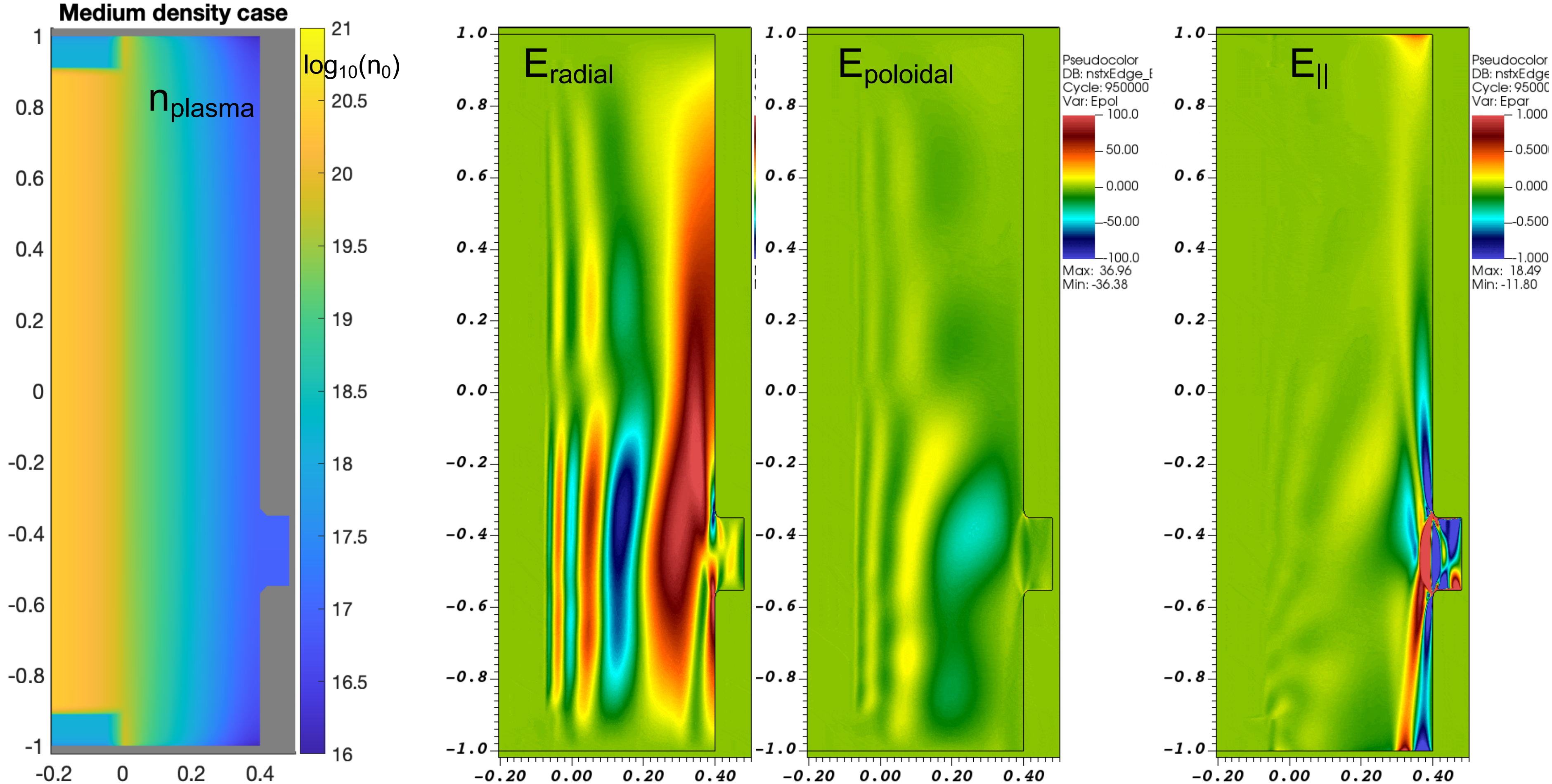


A broad region of PF density fans out from the antenna aperture, predominantly pulling electrons toward antenna and inducing locally increased density there. Broader effects on edge transport also push up the density profile generally, as both ion and electron forces impart momentum to the plasma.

Lowering the density cuts off the fast wave

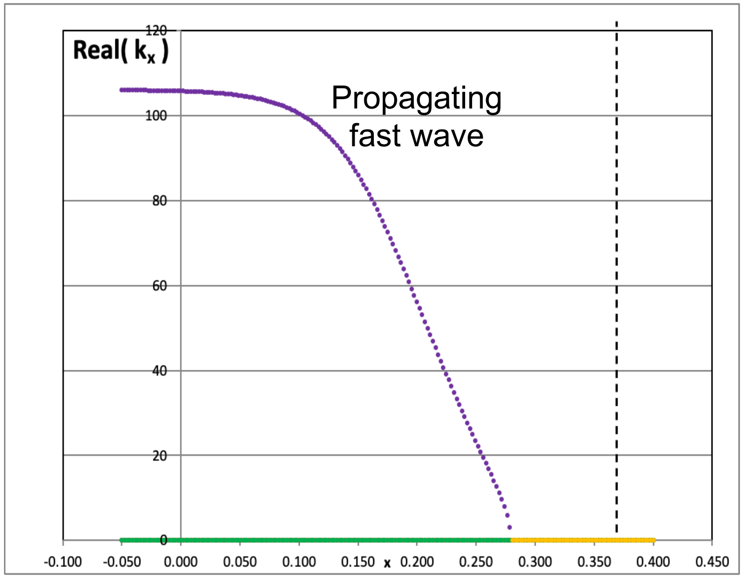


Wave fields, medium density (~10^{17} m^{-3} @ wall) NSTX-like case: long-wavelength fast wave is cut off near antenna

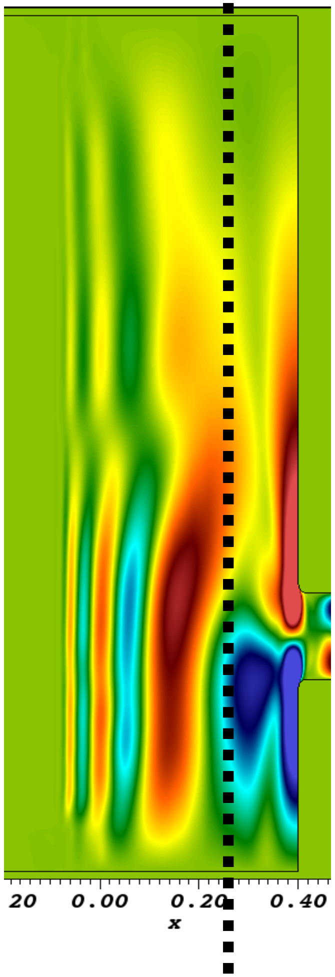


- Power coupling to plasma core (at left) is altered due to the evanescent fast wave in front of the antenna (bottom right of E_{radial} plot) – polarization is no longer elliptical in cutoff region

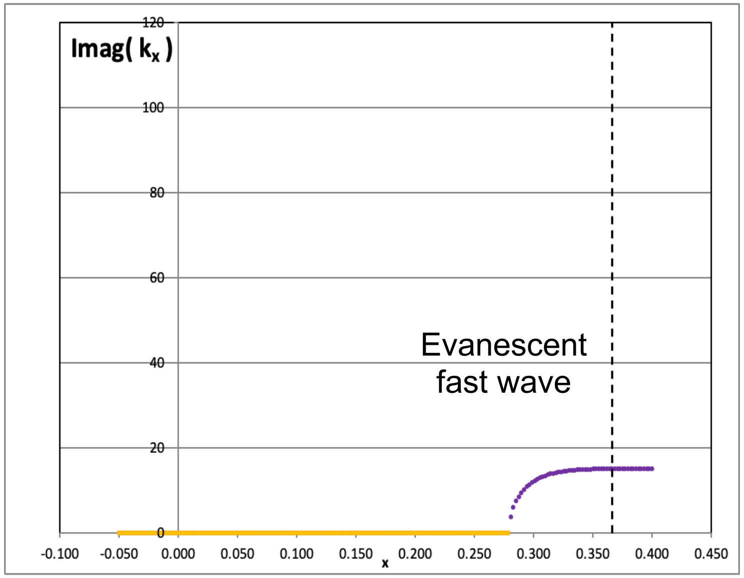
Numerically computed dispersion relations agree well with simulated wave phenomena



FW λ = 6 cm at left (w/ finite k_y=15)



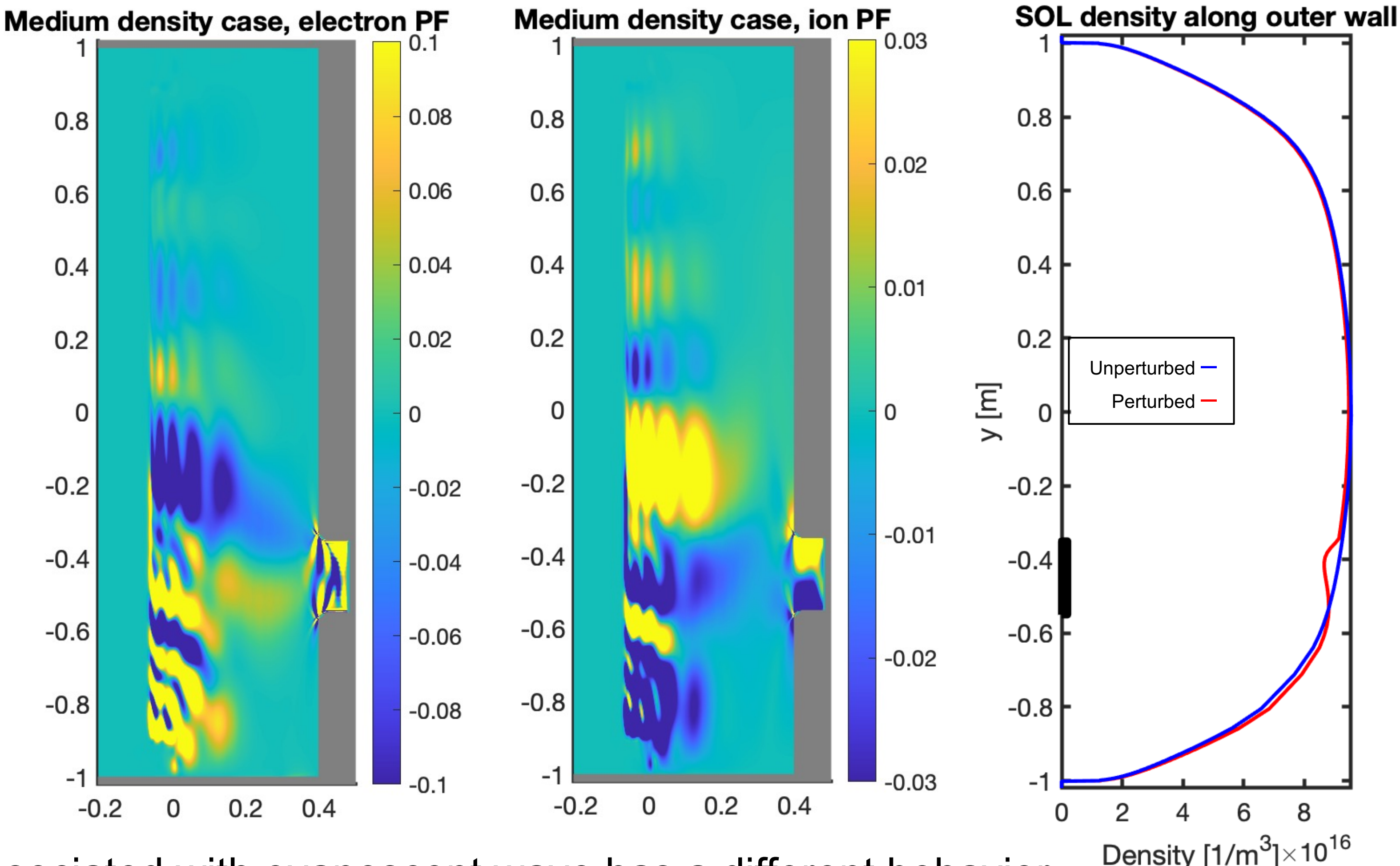
Fast wave cutoff



FW damping L ~ 7 cm at right

Medium-density (~10^{17} m^{-3}) scenario: ponderomotive forces begin to localize near the antenna and depress the density there

Input power flux 0.2 MW/m^2, converged solution

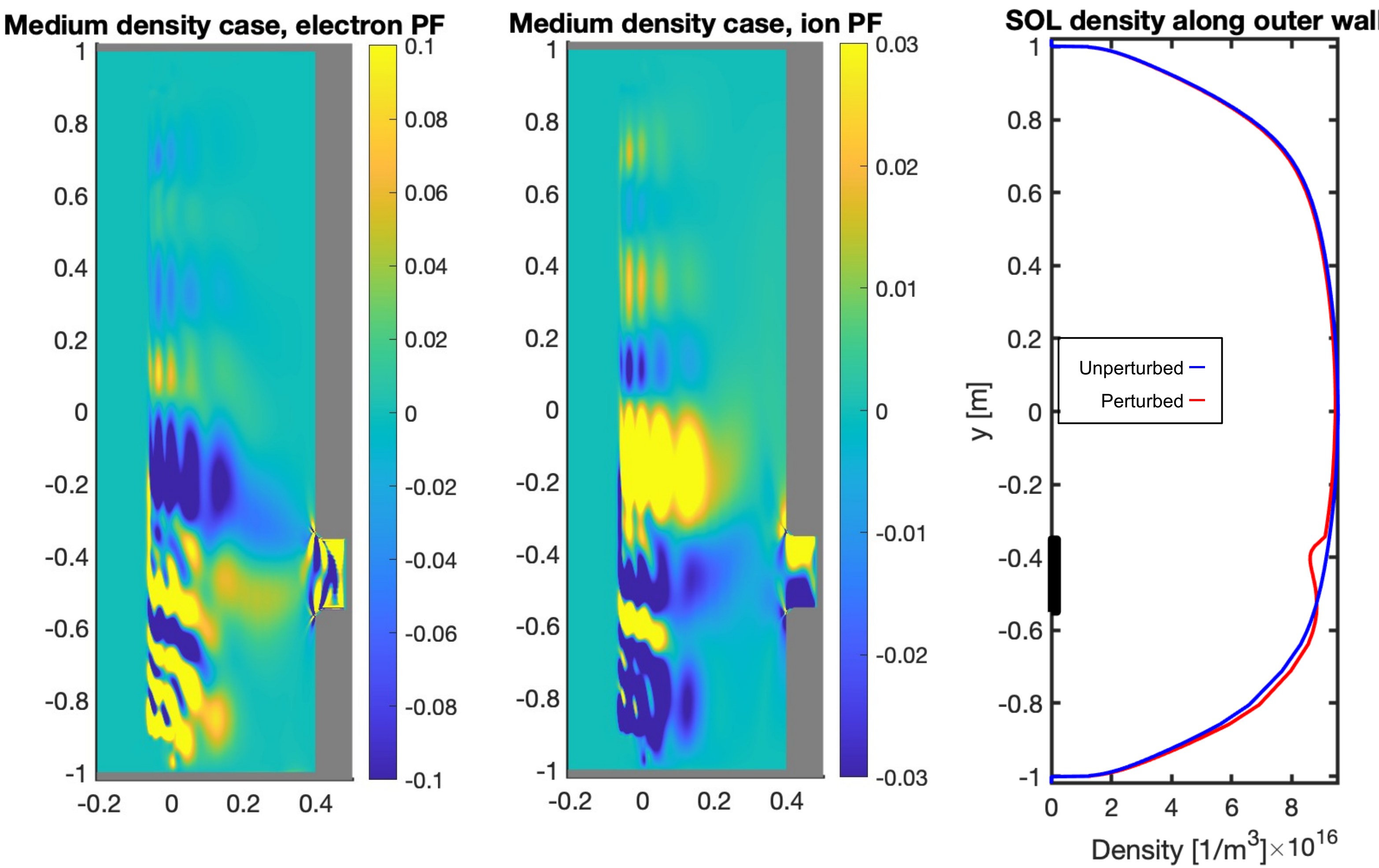


PF associated with evanescent wave has a different behavior.

Localized PF weakly expels density from the region immediately in front of the aperture (opposite to the effect observed at higher density).

What if we increase the RF power?

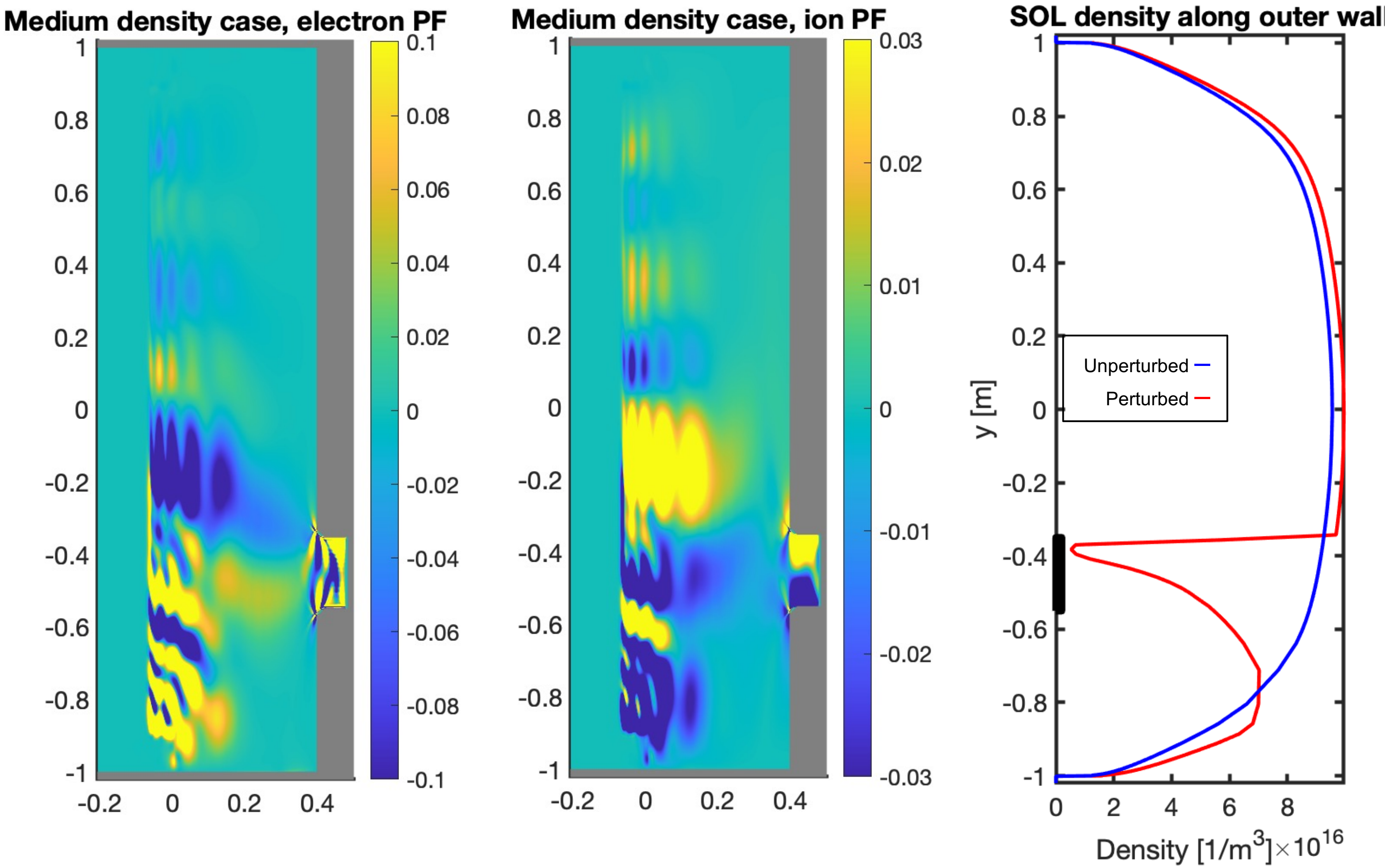
Input power flux 0.3 MW/m^2, converged solution



Increased RF power deepens the density depression in the region immediately in front of the aperture.

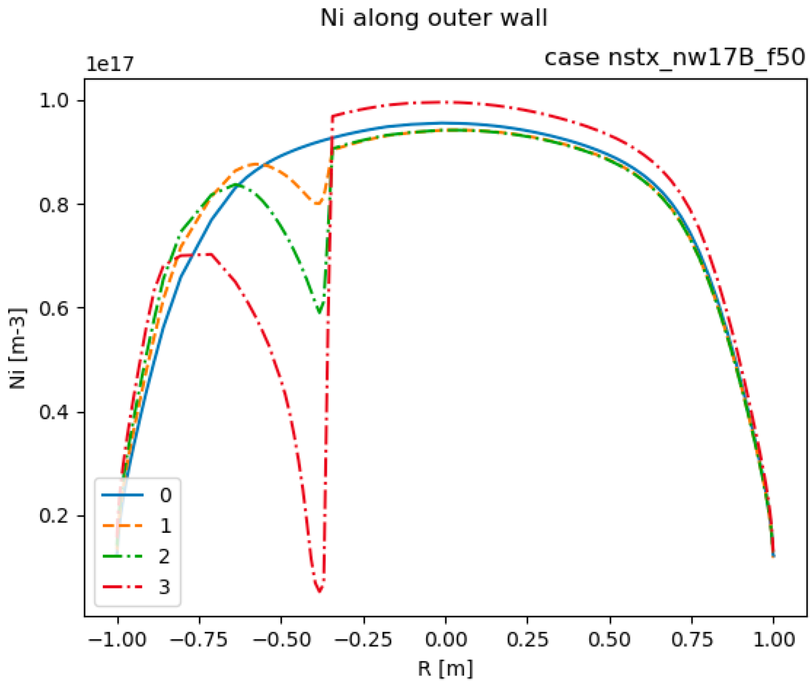
What if we increase the RF power even more?

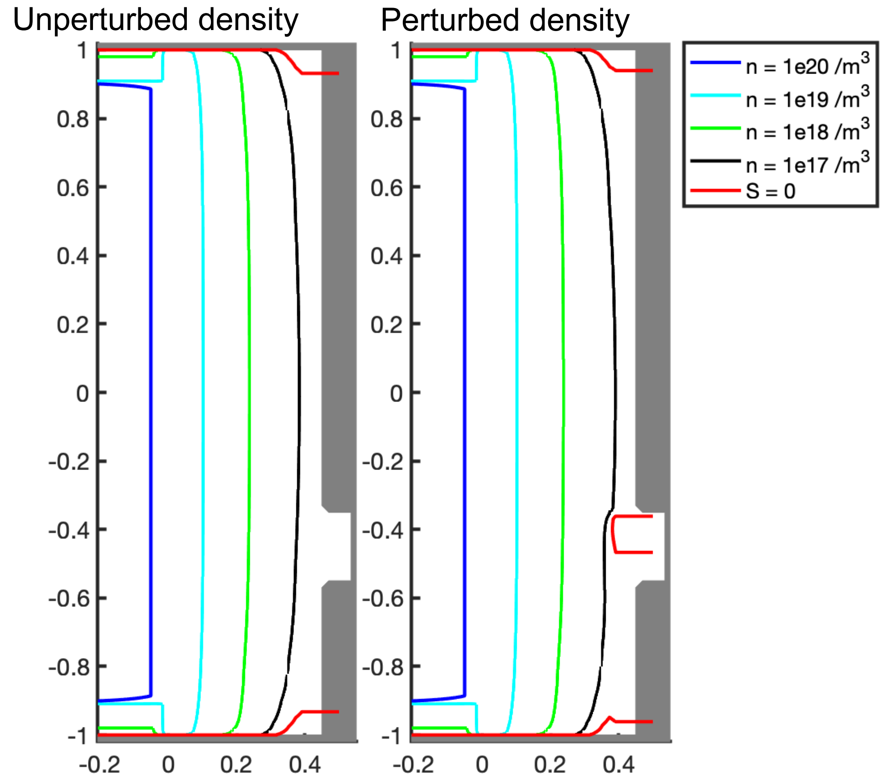
Input power flux 0.5 MW/m^2, NO converged solution



UEDGE/Vorpal iteration terminated for this case, low density prevents further explicit iteration

What physics governs density transport as RF power is increased?

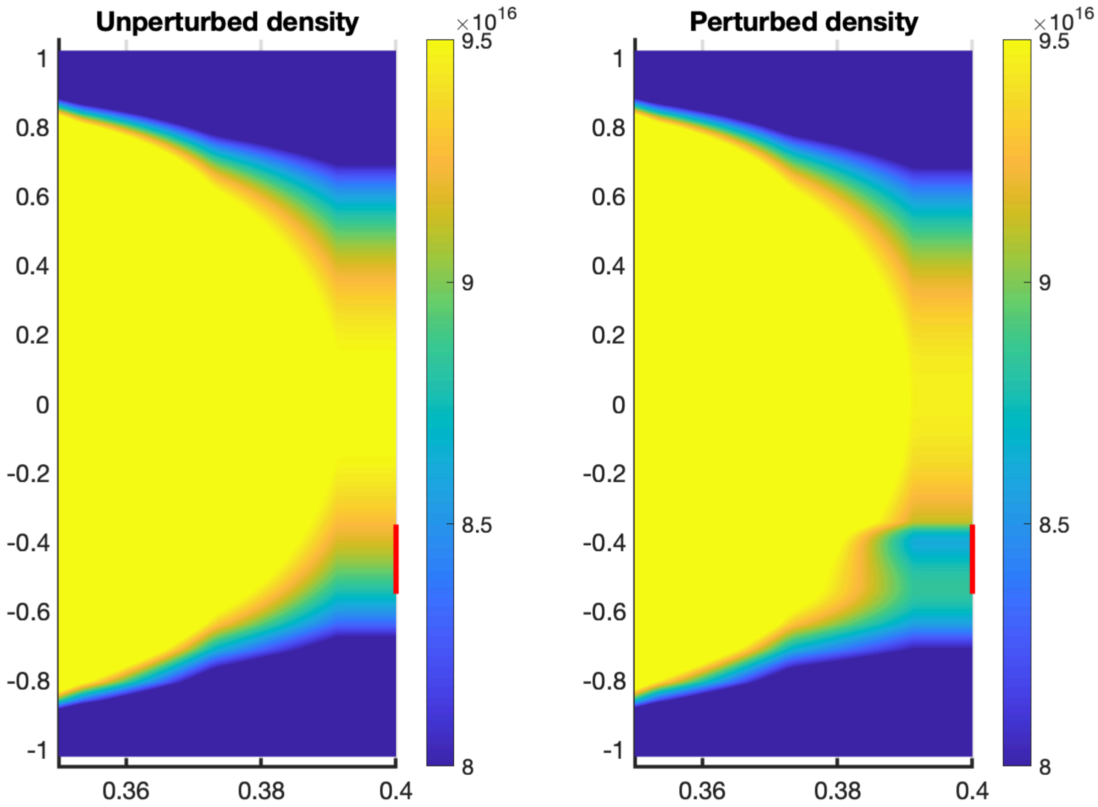




When density is low enough to cut off the fast wave near antenna:

- -PF tends to expel density further from this region
- -Low-density region where FW is cut off is enlarged
  - -more density expulsion!
- -Lower-hybrid cutoff can be pulled into plasma region, further affecting coupling

The density depression persists a few centimeters into the plasma, and will nonlinearly influence subsequent wave propagation



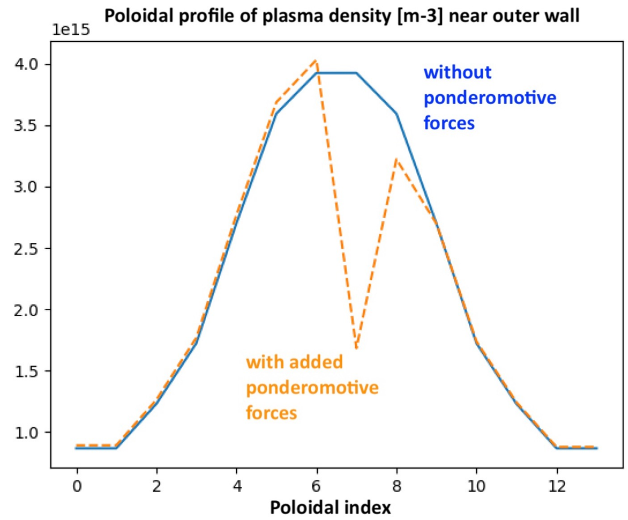
Coupling of antenna power to plasma will potentially be strongly affected by the induced density reduction, especially if it is self-reinforcing

Converged case, 0.3 MW/m^2 RF power

Results are consistent with heuristic estimates and UEDGE predictions: PF effects likely dominate edge transport at low densities

- Electron $PF_{||}$ – localized near antenna, of magnitude O(1) [N/m^3].
- Compare with thermal pressure gradient $dP/dx_{||} \sim nT/L$: for representative parameters [$n = 10^{16}$ m^{-3}, T = 10 eV, L ~ 1 m], we find $dP/dx_{||} \sim O(10^{-2})$ [N/m^3].
- Conclusion: PF terms >> other important edge transport effects, near a representative high-power RF antenna.

- UEDGE computations with ad hoc source terms confirm this hypothesis; forces of this magnitude significantly modify edge densities.



Propagating and evanescent waves contribute very differently to PF-induced edge/SOL transport

- Fast Wave (FW): evanescent at low densities, propagating at higher densities
- Main observations –
  - PF induced by evanescent FW nonlinearly reduces density (potentially strongly)
  - PF induced by propagating FW nonlinearly enhances density (not as strongly)
- Potentially yields volatile RF coupling behavior due to bifurcation:

high n: PF further increases n ↑

low n: PF further decreases n ↓

- Density depletion by ponderomotive forces may pull the LH cutoff in from behind the antenna and cause further interference with core coupling.
- Exploration of these ideas under the RF-SciDAC effort will continue throughout FY23.

Implications for ITER: ponderomotive effects are relevant and important

- ITER ICRF antennas will transmit 20 MW of power through a low-density evanescent region (fluxes ~1 MW/m^2)
  - Ponderomotive forces will be substantial due to the high power flux
  - To achieve optimal power coupling, their effects need to be well understood
- Vorpal can simulate the ITER antenna with full geometric fidelity; associated PF effects on edge/SOL transport can also be modeled using the toolset we've built
  - Upcoming RF-SciDAC efforts will focus on this modeling
  - Extension of present work to 3D is of interest (BOUT++)
  - PF effects on impurity species are also of interest (Dudson, Umansky)
- Work thus far strongly validates the concept of gas puffing near the ITER ICRF antennas, to prop up density locally and promote good RF power coupling
  - See, e.g., W. Zhang et al., "Scrape-off layer density tailoring with local gas puffing to maximize ICRF power coupling in ITER", Nucl. Mater. Energy **19**, 364 (2019).

Summary/Conclusions

- Ponderomotive forces become increasingly consequential at the RF power fluxes needed for burning plasma experiments (ITER, SPARC, etc.).
- We have successfully coupled Vorpal and UEDGE to model ponderomotive effects on edge/SOL transport.
- RF waves transitioning from evanescence to propagation drive PF-induced edge/SOL transport in quite different ways on opposite sides of the transition point, possibly leading to bifurcation.

This talk will be available on my website: https://nucleus.txcorp.com/~tgjenkins