Problems to solve:

1. When a reactor is running in a steady state, the density in the divertor region would be very high,
   - Prohibitive to study the whole plasma and neutral interaction in realistic tokamaks,
   - e.g., ‘sheath-dissipation’ closure in 2D simulations, related to the ‘sheath-limited’ low recycling regime in 3D geometry,
Overview

The effects of divertor particle sources on turbulence:

1. When a reactor is running in a steady state, the density in the divertor region would be very high,
   - Prohibitive to study the whole plasma and neutral interaction in realistic tokamaks,

2. Numerical advantages,
   - Speed up the calculation,
     - A prescribed, non-self-consistent particle source serves the same purpose?
     - How sensitive?,
   - Numerical stability,
Simulation methods - STORM

• **Scrape-off layer Transport ORiented Module**
  • Code for modelling SOL plasma physics,
  • Start: [L. Easy, et al. 2014 PoP 21, 122515]
  • Current on GitLab: [https://gitlab.com/CCFE_SOL_Transport/STORM](https://gitlab.com/CCFE_SOL_Transport/STORM)
STORM module

- Scrape-off layer Transport ORiented Module
- Cold ion limits \( T_i = 0 \),
  - \( T_i = T_e \) for dissipation coefficients,
  - Assuming \( n_i = n_e \),
  - Not physical but makes equations easier,
  - For current MAST relevant parameter space, the influence of hot ions on the stability of filaments may be negligible,
    [F. Militello, et al., 2016 PPCF 58.10 p. 105002]
- Electrostatic,
  - Current version,
STORM module

- Scrape-off layer Transport ORiented Module
- Cold ion limits,
- Electrostatic,
- It solves drift-reduced Braginskii equations on staggered grids,
  - Drift-reduced:
    - Slower than ion gyrofrequency timescale, $\Omega_i^{-1}$
    - $v_i = U \hat{b} + v_{EB} + v_{polarisation}$
    - $v_e = V \hat{b} + v_{EB} + v_{diamagnetic}$
  - High collisionality,
    - $\Rightarrow$ Braginskii closures for viscosity, heat flux, resistivity,
    - $n_{sep} \approx 6 \times 10^{18} \text{ m}^{-3}$, $T_{sep} \approx 40 \text{ eV}$,
**STORM module**

**Electron density**

\[
\partial_t \log(n) = -[\phi, \log(n)] - V \nabla_\parallel \log(n) - B \nabla_\parallel \left( \frac{V}{B} \right) \\
+ \frac{C(p)}{n} - C(\phi) + \frac{\nabla \cdot (\mu_n \nabla_\perp n)}{n} - \frac{S_n}{n},
\]

(1)

**Vorticity**

\[
\partial_t \Omega = -[\phi, \Omega] - U \partial_\parallel \Omega + 0.5 \left[ |\mathbf{v}_E \times B|^2, n \right] \\
+ n \left[ B \nabla_\parallel \left( \frac{U - V}{B} \right) + (U - V) \nabla_\parallel \log(n) \right] \\
+ C(p) + \nabla \cdot (\mu_\Omega \nabla_\perp \Omega),
\]

(2)

**Ion parallel momentum**

\[
\partial_t U = -[\phi, U] - U \nabla_\parallel U - \nabla_\parallel \phi - \frac{V}{\mu} (U - V) \\
+ 0.71 \nabla_\parallel T - U \frac{S_n}{n},
\]

(3)

**Electron parallel momentum**

\[
\partial_t V = -[\phi, V] - V \nabla_\parallel V - \mu T \nabla_\parallel \log(n) + \mu \nabla_\parallel \phi \\
+ V (U - V) - 1.71 \mu \nabla_\parallel T + V \frac{S_n}{n},
\]

(4)

**Electron pressure (p=n*T)**

\[
\partial_t \log(p) = -[\phi, \log(p)] - V \nabla_\parallel \log(p) - \frac{5}{3} B \nabla_\parallel \left( \frac{V}{B} \right) \\
- \frac{2}{3p} B \nabla_\parallel \left( \frac{a_\parallel}{B} \right) - \frac{5}{3} C(\phi) + \frac{5}{3} \left[ C(T) + \frac{C(p)}{n} \right] \\
- 0.71 \frac{2}{3} (U - V) \nabla_\parallel \log(T) + \frac{2}{3 \mu T^{5/2}} \nu_0 n (U - V)^2 \\
+ \frac{2}{3p} \nabla \cdot (\chi_\perp e \nabla_\perp T) + \frac{2}{3p} S_E - \frac{V^2}{3 \mu p} C(p),
\]

(after Bohn normalization)

(5)
STORM module

Poisson bracket $[\phi, A] = \nabla E \times B \cdot \nabla A$

$C(A) = \nabla \times (b/B) \cdot \nabla A$

In slab: $C(A) = g \frac{\partial}{\partial z} A,$

g = 2\rho_s/R_c$

$x$ – radial

$y$ – parallel

$z$ – binormal in slab

Dissipation parameters depend on $n, T$

$$q_\parallel = -(2/7)\kappa_\parallel \nabla \parallel T^{7/2} - 0.71TJ_\parallel; \quad \kappa_\parallel = 3.16T_0/\nu_{ei,0}m_e c_s \rho_s$$

$$\mu_n = \mu_{n,0} n/T^{1/2}; \quad \mu_\Omega = \mu_{\Omega,0} n/T^{1/2}; \quad \kappa_\perp = \kappa_{\perp,0} n^2/T^{1/2}$$
3D simulation domain

Double-Null

Simulation domain
STORM module

(1) Bohm sheath boundary conditions near targets in the parallel direction [N. Walkden, et al, 2016]:

\[ |U| \geq \sqrt{T}, \]

\[ |V| = \begin{cases} \sqrt{T} \exp\left(-V_{fl} - \phi/T\right) & \text{if } \phi > 0, \\ \sqrt{T} \exp(-V_{fl}) & \text{otherwise}, \end{cases} \]

\[ Q_\parallel = \gamma pV, \]

where \( V_{fl} = 0.5 \ln\left[\frac{2\pi}{\mu} (1 + 1/\mu)\right], \gamma = 2 - V_{fl}, \mu = m_i/m_e. \)

The parallel electron energy flux density \( Q_\parallel = 2.5pV + 0.5nV^3/\mu + q_\parallel \)

(2) In binormal (z-) direction: Periodic boundary condition,

(3) In radial (x-) direction: Neumann boundary condition,
3D simulation domain

Parameters:
(taken from MAST)

- \( L_\parallel = 10 \text{ m} \),
- \( \rho_s = 1.8 \times 10^{-3} \text{ m} \),
- \( R_c = 1.5 \text{ m} \Rightarrow g = 0.00243 \)

Resolution:

\( N_x \times N_y \times N_z = 192 \times 64 \times 128 \)
Sources (fixed)

In the core:
Particle source: $S_{p,\text{core}}$
Energy source: $S_E$

No extra energy source/sink in the divertors. e.g., impurity radiation
Sources (fixed)

Particle source in the divertors

The distributions of divertor particle sources depends on:

1. Recycling process of ions, neutral pathways,
2. Input power, fueling regime,
3. The geometry of the divertor and magnetic field lines,

Loarte, et al., 2001
sources

Particle source in the divertors

$S_{p,\text{div}} @ \text{target}$

EDGE2D-EIRENE simulations of JET
Vertical- and horizontal- target

D. Moulton et al 2018 Nucl. Fusion 58 096029
sources

Particle source in the divertors

\[ S_{p,\text{div}} \atop \text{at target} \]

No extra energy sink in the divertors e.g., impurity radiation

\[
\begin{array}{c|c|c|c|c}
 & FLAT_{\text{cond}} & FLAT_{hr} & PEAK_{\text{cond}} & PEAK_{hr} \\
\int S_{p,\text{div}} dV / \int S_{\text{core}} dV & 1 & 5 & 1 & 5 \\
S_{p,\text{div}}(x) & \text{step function} & \text{step function} & \text{decaying Gaussian} & \text{decaying Gaussian} \\
\end{array}
\]

From SOLPS-ITER simulations (Fil et al., 2020)
Results – in the saturation stage

- Transient phase lasts ~ 2 ms,
- Averaged over > 21 ms,
Results – fluxes on targets

Particle fluxes ($\Gamma_\parallel$) vs. flux-tube-integrated $S_{p,\text{div}}$

- The source dominates in $hr$ cases,
- Transport in the radial direction,
Fluxes on targets

Energy flux density, \( Q_{\parallel}(x) = \frac{1}{dA} \int_0^{L_z} Q_{\parallel}(x, z)dz \), from the same \( S_{E,\text{core}} \)

- Weakly depends on \( S_{p,\text{div}} \) in group FLAT
- Only in PEAK\(_{hr} \): \( Q_{\parallel} \) strongly damped @separatrix
Plasma profiles on diff poloidal planes

SOL midplane

Midplane

Divertor entrances
$T_e(x)$ on diff poloidal planes

- Temperature profiles are similar
\( n_e(x) \) on diff poloidal planes

- Behaviors on the midplane are similar,
- Relatively, fluctuations are enhanced in the divertors in group PEAK,
2D contours

The perpendicular transport

- In the divertor:
  - larger perpendicular transport,
  - larger radial gradient of $P$,
Effect on temperature profiles

Stangeby (2000):

\[ T_u^{7/2} = T_{\text{tar}}^{7/2} + \frac{7 q^* L}{2 \kappa_{e,0}} \]

- We can get \( Q_{||}, Q_{\perp} \) in simulations

\( T_e \) drops from the upstream (midplane) to targets

- Larger particle source leads to more \( T_e \) drops

![Graphs showing temperature profiles](image-url)
Effect on upstream temperature

Stangeby (2000):

\[ T_u^{7/2} = T_{tar}^{7/2} + \frac{7q^*L}{2\kappa_{e,0}} \]

Here upstream @ midplane

1. Heat flux density:

\[ q_{||} = -\kappa_{e,0} T^{(5/2)} \frac{dT}{dy}, \quad (A) \]

2. Also consider including the convection part,

energy flux density: \[ Q_{||} = q_{||} + q_{\text{conv}} \quad (B) \]

(This is what OB measured)
Effect on upstream temperature

- Both $q//, Q//$ overestimated $T_{mid}$.
- Ignore $T_{tar}$ is fine…
- $|T_{est}/T_{sim}| < 2$, 

\[
T_u^{7/2} = T_{tar}^{7/2} + \frac{7 q^* L}{2 \kappa_{e,0}}
\]

\[
FLAT_{cond}
\]

\[
PEAK_{cond}
\]

\[
FLAT_{hr}
\]

\[
PEAK_{hr}
\]
Effect on target temperature

Here upstream @ divertor entrance

Combine the particle and heat balance equation:
[D. Moulton et al 2018 Nucl. Fusion 58 096029]

$$T_{eT} = \frac{q_{\theta u}(B_u/B_{\theta u}) + \frac{1}{dA_u} \int_t^u S_q dV}{\gamma \left( \Gamma_i \theta u(B_u/B_{\theta u}) + \frac{1}{dA_i} \int_t^u S_i dV \right)} = \frac{Q_{tar}}{\gamma \Gamma_{tar}}$$ (1)

For our simulations

$$T_{et}(x) = \frac{Q_{up} - \frac{1}{dA_u} \int_{t_{up}}^u \nabla \cdot Q_{\perp} dV}{\gamma \left[ \Gamma_{i,up} + \frac{1}{dA_u} \int_t^{up} \left( S_{p,div} - \nabla \cdot \Gamma_{\perp} \right) dV \right]}$$, (17)
Effect on target temperature

\[ T_{et}(x) = \frac{Q_{up} - \frac{1}{dA} \int_{up} \nabla \cdot Q_{\perp} dV}{\gamma \left[ \Gamma_{i,up} + \frac{1}{dA} \int_t^{up} \left( S_{p,div} - \nabla \cdot F_{\perp} \right) dV \right]} \]
Conclusions

- When the divertor particle source is not much larger than the source in the core and not heavily peaked near the separatrix, the heat flux on the target is insensitive to the particle source profile,
  - Benefits for numerical studies,
- If a large amount of particle source peaks near the separatrix, turbulence in the divertor volume could be enhanced locally,
  - It can redistribute the energy flux on the target and reduce the maximum amplitude. E.g., an advantage of vertical targets,
  - Meanwhile, it leaves the plasma profiles measured at the outboard midplane marginally changed,
- The prediction of the temperature difference between the outboard midplane and the target would be underestimated, if the calculation only considers the conductive heat flux and ignores this enhanced cross-field transport in the divertor.
Thank you & Happy new year!
abstract

Tokamak edge turbulence is crucial for the cross-field transport of particles and energy away from the separatrix. One potentially important factor is the ion particle source in the divertor, as the neutral pathways and the ionisation source distributions are different depending on the divertor geometry, e.g., vertical- and horizontal-target configurations.

Numerically, how to represent the sources and mimic the effects on the SOL in the simulations is still an open question. Here we use the STORM module (which uses a staggered grid in the field-aligned coordinate) to study this problem in a simplified 3D slab geometry.

The results show that if the divertor particle source is not much larger than the source in the core and not heavily peaked near the separatrix, the heat flux on the target is insensitive to the particle source profile. But if a large amount of particle source peaks near the separatrix, turbulence in the divertor volume could be enhanced locally. It can redistribute the energy flux on the target and reduce the maximum amplitude. Meanwhile, it leaves the plasma profiles measured at the outboard midplane marginally changed. Our results also indicate that the prediction of the temperature difference between the outboard midplane and the target would be underestimated, if the calculation only considers the conductive heat flux and ignores this enhanced cross-field transport in the divertor.
sources

Particle source in the divertors

\[ S_{p,\text{div}} \text{ @ target} \]

\[ S_{FLAT} = S_{p,\text{div}} = H(x - x_{\text{sep}} - 0.03L_x) \cdot \left(1 - e^{-\frac{x-x_{\text{sep}}-0.03L_x}{\Delta_{\text{wid}}}}\right) \]

\[ \cdot \left[ \frac{25.6(y-L_y/2)}{2L_y^{3/2}} + e^{\frac{25.6(L_y/2-y)}{2L_y^{3/2}}} \right] \]

\[ \cdot \frac{A_r}{e^{25.6} - 1} \]

\[ (14) \]

\[ S_{PEAK} = S_{p,\text{div}} = H(x - x_{\text{sep}} - 0.03) \cdot \left(1 - e^{-\frac{x-x_{\text{sep}}-0.03}{\Delta_{\text{wid}}}}\right) \]

\[ \cdot \left[ \frac{25.6(y-\pi)}{4\pi^{3/2}} + e^{\frac{25.6(\pi-y)}{4\pi^{3/2}}} \right] \]

\[ \cdot \frac{A_r}{e^{25.6} - 1} \]

\[ \cdot \frac{1}{\sqrt{2\pi\Delta_w}} \cdot e^{-\frac{(x-x_{\text{sep}}-0.03-\Delta_{\text{shift}})^2}{\Delta_w^2}} \]

\[ (15) \]