Heat Load Broadening and Turbulence Spreading: Some Issues and a Look Ahead

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Background: The SOL Width Problem

- Long history, Key: Open field lines
- H-mode → HD Model (Goldston +)
  \[ \lambda_q \sim \varepsilon \rho_{\theta i} \] - pathetically small, unfavorable \( B_\theta \) scaling
- Why? → ExB shear quenches SOL modes
- Calculate SOL width for turbulent pedestal but locally stable SOL
  - Penetration depth of turbulence spreading ?!
  - See Chu, PD, Guo ‘22 NF
- N.B.: Many results from simulation now available, beyond color pictures
  See Nami Li, Zeyu Li, this meeting
Turbulent scattering broadens stable SOL

\[ \lambda = \left( \lambda_{HD}^2 + \varepsilon \tau_\parallel^2 \right)^{1/2} \]

\( \varepsilon \approx \) Turbulence energy intensity, in SOL

Separatrix turbulence energy flux specifies SOL turbulence drive

Spreading Calculation for \( \varepsilon \):

\[ \Gamma_{0,e} = \lambda_e |\gamma|\varepsilon + \lambda_e \sigma \varepsilon^{1+\kappa} \]

Relates \( \varepsilon \) to influx from pedestal

Broadening increases with \( \Gamma_{0,e} \)

Non-trivial dependence

\( \Gamma_{0,e} \) must overcome shear layer barrier
Summary: cont’d

- Critical: The Cost-Benefit Question

  → Can sufficient SOL broadening be achieved for tolerable pedestal Turbulence levels?

  → Require $\lambda/\lambda_{HD}$ vs. Pedestal fluctuation level

  ← Spreading through Shear Layer.

  Interesting levels of $\lambda/\lambda_{HD}$ for modest pedestal fluctuation $e\tilde{\phi}/T$
Fundamental Physics of Turbulence Spreading

- Structure of the intensity flux-gradient relation?
- Experiments: Ancient and Modern
- Pulsation Model of Spreading (New)

Comment:
- Turbulence spreading is seen as a ‘Deus ex Machina’
- Fundamental Physics poorly understood

Review:
Hahm, P.D.
J. Kor. Phys. Soc.
2018
On Spreading: A Familiar Phenomenon

• Turbulence spreading underpins turbulent wake → central example in high $Re$ fluids

Mixing length model
Similarity theory
Spreading
Wake expansion

$\begin{align*}
\frac{\partial \varepsilon}{\partial t} + \nabla \cdot (\vec{V}\varepsilon) + \cdots &= 0 \\
F_d &\sim \rho U^2 S C_D; \\
C_D &\rightarrow \text{indep } \nu
\end{align*}$

cf: Landau & Lifshitz

cf: Hahm, P.D. + 2004 et seq.

• Spreading fundamental to $k - \varepsilon$ type models, as $\varepsilon$ evolved as unresolved energy field → subgrid models

How render tractable?
2D Fluid Patch
How will spread?
Random walk vortices, under mutual induction?

- but:
vs. Dipole pair
good for spreading

Patch expansion is poorly understood……..

Illustration:

contrast: $k$-space
dual cascade etc.

rotate?! $V_{pair}$
On Spreading: cont’d

• What you get (usually):

\[
\frac{\partial \varepsilon}{\partial t} + \vec{V}_D \cdot \nabla \varepsilon + \langle \vec{V}_E (r) \rangle \cdot \nabla \varepsilon - \nabla \cdot D(\varepsilon) \nabla \varepsilon = P(\varepsilon) - P_{\text{damp}}(\varepsilon) \rightarrow \gamma (\vec{x}) \varepsilon
\]

\begin{align*}
&\text{drift} & \text{shear} & \text{turbulent mixing via closure} \\
&D(\varepsilon) \approx D_0 \varepsilon, \text{ et. seq. } \rightarrow \text{nonlinear diffusion} \\
&\gamma = \gamma (\text{gradients, etc})
\end{align*}

\[\varepsilon\] evolution as nonlinear Reaction-Diffusion Problem!

(P.D., Garbet, Hahm, Gurcan, Sarazin, Singh, Naulin…)

• Used also in:

– Multi-scale style layering models (Ashourvan +)

– 1D L→H models (Miki +)
On Spreading: cont’d

• Spreading as Front ➔ Fast Propagation
  i.e. \( V_f \sim (\gamma D)^{1/2} \), etc i.e. Fisher

• Key component:
  \[ \nabla \cdot \langle \vec{V} \epsilon \rangle \rightarrow -\nabla \cdot D(\epsilon) \cdot \nabla \epsilon \quad \text{[Fickian Model]} \]
  Expectation: \( D(\epsilon) \sim \chi \), \( D_n \) etc. for electrostatic turbulence

• Copious simulations: Z. Lin, W.X. Wang, S. Yi, Jae-Min Kwon, Y. Sarazin, …
  ➔ Observations of front tracking but critical analysis of model absent
  No test of Fickian flux-gradient model
Experiments: Ancient

• Not exactly a new idea … See Townsend ‘49 and book

Momentum and energy diffusion in the turbulent wake of a cylinder

By A. A. Townsend, Emmanuel College, University of Cambridge
(Communicated by Sir Geoffrey Taylor, F.R.S.—Received 6 October 1948)

A detailed experimental investigation of the turbulent motion in the wake of a circular cylinder, 0.013 cm. diameter placed in an air-stream of velocity 1290 cm. sec.\(^{-1}\), has been carried out with particular reference to those quantities determining the transport of turbulent energy and mean stream momentum. At distances of 80, 120 and 160 diameters downstream from the cylinder, direct measurements have been made of mean flow velocity, turbulent intensity, viscous dissipation, energy diffusion, scale, and form factors of the velocity components and their spatial derivatives. These observations show that, except close to the wake centre, the flow at a point fixed with respect to the cylinder is only intermittently turbulent, due to the passage of the point of observation through jets or billows of turbulent fluid emitted from the inner whirly turbulent core of the wake. Further consideration of the results indicates that the turbulent motion within the jets is solely responsible for the turbulent transfer of momentum, while diffusion of turbulent energy and of heat is carried out by the bulk movement of the jets. Most probably, the jets are initiated by local fluctuations of pressure inside the turbulent core, and in the later stages of their development that are slowed down by adverse pressure gradients. The existence of pressure-velocity correlations of sufficient magnitude is demonstrated by using the equation for the conservation of kinetic energy in the wake, all terms of which are known excepting the one involving the pressure-velocity correlation, which is then obtained by difference. While the conception of jets of turbulent fluid is more convenient for following the physical processes in the wake, the alternative but equivalent description that the turbulent motion consists of a motion of scale small compared with the mean flow superimposed on a slower turbulent motion whose scale is large compared with the mean flow may be used. A formal explanation of this two-stage turbulent structure in terms of the Fourier representation of the velocity field is suggested, which relates the structure to the presence of a quasi-constant source of energy of nearly fixed wave-number, and to the free boundary which allows an unlimited range of wave-numbers. It is expected that this type of motion will occur in all systems of turbulent shear flow with a free boundary, such as wakes, jets and boundary layers.

➔ Wake flow intermittently turbulent

➔ Compare transport of momentum and energy (spreading)
Experiments: Ancient, cont’d

The product $we$ may be regarded as the rate of transport of momentum (per unit mass), and similarly the rate of transport of turbulent energy is

$$\frac{1}{2}(u\overline{w} + v\overline{w} + w\overline{w}),$$

and, in principle, it is possible to calculate an energy diffusion coefficient $\delta$, analogous with $\varepsilon$, by use of the defining equation

$$u\overline{w} + v\overline{w} + w\overline{w} = -\frac{\partial}{\partial y}(u^2 + v^2 + w^2).$$

When this is attempted (figure 5), no simple behaviour is found either for $\delta$, or for the corresponding mixing length. Negative values occur near the wake centre, and, even where the turbulence gradient is fairly uniform, $\delta$ remains large compared with $\varepsilon$,

and decreases rapidly with distance from the wake centre. It must be concluded that the use of a diffusion coefficient to describe the transport of turbulent energy is not justified, and that energy diffusion is a process independent of momentum diffusion.

To remove this difficulty, it is not sufficient to consider the effects of intermittency. If the intermittency factor is known, then the mean intensity in the turbulent regions is

$$I_s = \frac{u^2 + v^2 + w^2}{\gamma},$$

and $I_s$ is found to vary only slightly over the greater part of the wake (figure 6). So a considerable transport of energy is found in the almost complete absence of a real intensity gradient, and it is difficult to see how energy flow can take place by turbulent movements inside the jets. For the transport mechanism, there is only left the bulk movement of the jets, which is naturally outwards and away from the wake centre. The compensating inflow will consist of non-turbulent fluid transporting no turbulent energy. Consequently, the flow of energy is not dependent on the local intensity gradient (if any), but only on the mean jet velocity and the jet turbulent intensity, which in turn are determined by conditions in the turbulent core.

- Fickian model for turbulent energy transport

- “It must be concluded that the use of a diffusion coefficient to describe the transport of turbulent energy is not justified and that energy diffusion is a process independent of momentum diffusion”
  
  i.e. the usual model of spreading is crap……

- Wake consists of ‘jets’ of turbulence energy
Experiments: Modern (c.f. Ting Long) 1

- HL-2A
- Aims:
  - Exploration of intensity flux – intensity gradient relation in edge turbulence (exploits spreading, shear layer collapse and density limit studies Long + NF’21)
  - Physics of “Jet Velocity” profile
    \[ V_I = \frac{\langle \tilde{V}_r \tilde{n}^2 \rangle}{\langle \tilde{n}^2 \rangle} \]
    \( \langle \tilde{V}_r \tilde{n}^2 \rangle \to \) spreading flux element
    N.B. Identified by Townsend
    See Manz+2015 for spreading ↔ blob connection.
There exits a region in plasma edge, where the turbulence spreading flux \( \langle \mathbf{v}_r \mathbf{n}^2 \rangle / 2 \) is large, but the turbulence intensity gradient \( \partial_r \langle \mathbf{n}^2 \rangle \) is near zero.

For close \( \bar{n}_e \):
- Lower current, width of region is \( \sim 5 \text{ mm} \) \((l_{cr} \sim 4.5 \text{ mm})\)
- Higher current, width of region is \( < 1 \text{ mm} \) \((\rho_i \sim 0.25 \text{ mm})\)

Notice: spreading diffusivity

\[
\chi_i = - \frac{\langle \mathbf{v}_r \mathbf{n}^2 \rangle}{\partial_r \langle \mathbf{n}^2 \rangle}
\]

Fickian model bombs……..
• The "mean jet velocity" of turbulence spreading $V_I = \frac{\langle \bar{v}_r \bar{n}^2 \rangle}{\langle \bar{n}^2 \rangle}$ and skewness of density fluctuations show strong correlation

\[ \begin{align*}
V_I & \quad (\text{m}\cdot\text{s}^{-1}) \\
I_{\text{sat}} & \quad \text{Skewness}
\end{align*} \]

- Their trends and signs are consistent
- More work is being done on the correlation between "blobs/holes" and turbulence spreading
- $V_I$ - skewness trend follows joint reflection symmetry relation

$x \rightarrow -x$

$\delta n \rightarrow -\delta n$

cf. P.D., Hahm' 95

Experiments: Modern 3
Spreading Pulses

- Avalanches, pulses are natural description
  \[ \delta P \equiv \text{deviation of profile from criticality} \]
  \[ \delta P \leftrightarrow (\nabla P - \nabla P_{\text{crit}})/P \]
  \[ \delta P \sim \delta \epsilon \sim \delta T \]

→ Spreading as intensity pulses dynamics
  (after PD, Hahm ‘95)

- New:
  - Order parameter not conserved → finite SOL dwell time
  - \( V_D \) - mean curvature drift
  - \( \Gamma_{0,e}\big|_{sep} \) drives system
Fluctuation Energy Pulses, cont’d

• Pulse model:

\[ \frac{\partial}{\partial t} \tilde{\varepsilon} + V_D \frac{\partial}{\partial x} \tilde{\varepsilon} + \alpha \tilde{\varepsilon} \frac{\partial}{\partial x} \tilde{\varepsilon} - D_0 \frac{\partial^2}{\partial x^2} \tilde{\varepsilon} + \frac{\tilde{\varepsilon}}{\tau} = 0 \]

\[ \tilde{\varepsilon}(0,t) \leftrightarrow \tilde{\Gamma}_{sep}(t) \]

• Some limits:

- \( \varepsilon \to 0 \), \( V_D \frac{\partial}{\partial x} \tilde{\varepsilon} \sim \frac{\tilde{\varepsilon}}{\tau} \to \lambda \sim \lambda_{HD} \) scale (1 vs 2)

- For \( \varepsilon \) to “matter” – i.e. broadening significant:

\[ \alpha \tilde{\varepsilon} > V_D \to \text{amplitude vs neo drift comparison} \quad (1 \text{ vs } 3) \]

• Structure is Burgers + Krook \( \rightarrow \) ‘Crooked Burgers’

• Excitation by boundary (separatrix) flux.

Dwell rate damping regulates Pdf

\( \tau \equiv \text{SOL dwell time} \)

Pulse energy not conserved in SOL
Fluctuation Energy Pulses, cont’d

- Predictions ? → Goal Pdf($l \mid \tilde{I}_{0,e}$)

→ Pulse equation characteristics: $\frac{dx}{dt} = \alpha \varepsilon$, $\frac{d\varepsilon}{dt} \approx -\frac{\varepsilon}{\tau}$

Solution: Shock for $f'(z) < -1/\tau$

Initial slope steep enough to shock before damping by $1/\tau$

→ $\alpha \frac{\partial \varepsilon}{\partial x} < -\frac{1}{\tau}$ → separatrix intensity gradient defines pulse formation criterion

→ pulse evolution → penetration depth

↔ evaporation

∴ Pulse penetration depth Pdf is the output
Broader Messages

- Turbulence spreading is viable mechanism for broadening the stable SOL. Turbulent pedestal states attractive for heat load management.
- Theory indicates that can achieve $\lambda/\lambda_{HD} > 1$ for acceptable pedestal fluctuation levels. Trade-off analysis is critical.
- Spreading dynamics best treated statistically. $\text{Pdf}(l_{pene})$ is goal.
- Simulations should stress calculation of spreading fluxes, and Pdfs over color visualizations.
Thoughts for BOUT World
(by request)
Heat Loads (1)

➢ Calculate spreading, fluxes, Pdfs.

   3D Brunner plot (Nami Li) is good start.

➢ High density is relevant regime.

   SOL is conductive. \( \tau_{dwell} \sim (Rq)^2/\chi_\parallel \)

   \( \lambda_q \) increases but \( \rightarrow \text{density limit?!} \) (Goldston)

   \( \therefore \) study high \( n \) regime where SOL instability restored.

   SOL \( \rightarrow \) pedestal spreading? \( \quad \) H \( \rightarrow \) L back transition?

➢ Pdf heat load distribution——\( \lambda_q \)?!
Turbulence Spreading (2)

➢ Need improve fundamental understanding of turbulence spreading.

   Present theory at “Fickian Diffusion” level ↔ primitive.

➢ Investment needed:

   ➢ BOUT++ 6-field model is “火锅”
   ➢ Try detailed study of simpler problems:
     ➢ 3-field model \((\phi, A_\parallel, p)\) ➢ 2D fluid
     ➢ Hasegawa-Wakatani (3D0)

➢ Consider fundamental experiment (Frontiers → LAPD) collaboration.
   cf. Compernolle +
Bout Code (3)

- Flux driven/source-sink version desperately needed. Critical on many fronts.
  If GYSELA, why not BOUT++?
- Related: Avoid studies of states from marginal → likely unphysical
- Self-consistent hyper-resistivity evolution ↔
  Multi-scale Ohm’s Law (cf. P.W. Xi +)
- Implement toroidal rotation and its effect on $V_E$. Study physically meaningful $E_r$ scans, explore intrinsic rotation, etc.
Thank you!

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