Global Gyrofluid Simulations of Turbulence in Tokamak Plasmas

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A brief history of GLF simulations at KFE

Introduction

3+1 Global Gyro-Landau Fluid model

Neoclassical Equilibrium

Ion Temperature Gradient-driven Linear/Nonlinear Simulations

Flux-driven Nonlinear ITG Simulations

Internal Transport Barrier Formation by An External Vorticity Source

Summary
A brief history of GLF simulations at KFE

- Fluid simulations of turbulent transport initiated from 2010
  - Main tool: TRB code [X Garbet et al PoP 2001]
    → Problem of non-resonant modes: a motivation switching to BOUT++

- BOUT++ simulations have been applied various areas
  - NL physics of edge pedestal collapse and transport
    - Elucidate the magnetic field stochastization mechanism and its impact on transport [T Rhee et al NF 2015; JW Kim et al PoP 2018]
    - RMP [JH Kim et al NF 2019]/ZF [H Jhang et al NF 2017] effects on pedestal collapse
  - Electromagnetic effects on enhanced $k_{||}$ symmetry breaking [H Kaang et al PoP 2018]
  - Fundamental physics of plasma turbulence [T Tran et al PoP 2019; T Tran et al PPCF 2020]
  - Development of a working global flux-driven GF code → focus on this talk
Progress in recent theoretical development

● A conservative formulation of a system of GF equations and an in-depth analysis of the effect of parallel closures on GF energetics [SS Kim & H Jhang PoP 2020]
  ➢ Fluid closure affects directly to field energy → a source of spurious field energy
  ➢ A closure necessitates additional FLR closure to be consistent with the field energy conservation
  ➢ Zonal flow closure consistent with 4+2 energetics under investigation

● Impact of magnetic field inhomogeneity on gyro-averaging operator [H Jhang & SS Kim PoP 2022]
  → Modification of GK Poisson and Ampere law under construction

● Heat source effects on RH residual zonal flow [SS Kim, S Ku, H Jhang, NF 2022]
  Development and implementation of a closure including the heat source effect on-going
Introduction

- To study long-time qualitative physics with low computational cost, we have been developing a 3+1 Global Gyro-Landau Fluid code using BOUT++ framework.

- We match linear ITG mode growth rates with gyrokinetic and gyrofluid results adjusting hyper-viscosity and neoclassical poloidal flow and Pfirsch-Schulter return flow at an equilibrium are satisfied.

- We achieve a nonlinear steady state of gradient-driven ITG turbulence in the global simulation. A simulation without an external heating is carried and a final state of $R_0/L_{Ti} \sim 5$ is obtained and similar to that of gyrokinetic code, gKPSP.

- In flux-driven simulations with various heating powers, heat transport shows characteristic of self-organized criticality (SOC)-like avalanche.

- We simulate internal transport barrier induced by an external vorticity source. After vorticity source injection, avalanche heat transport is reduced and stationary zonal flow pattern is formed in a global region.
3+1 Global Gyro-Landau Fluid model

- Gyrocenter density equation
\[
\frac{\partial \bar{n}}{\partial t} + \mathbf{V}_\Phi \cdot \nabla n + n_0 B \partial_{||} \frac{\mathbf{V}_{||}}{B} + n_0 K(\Phi) + K(\bar{P}_{||} + \bar{P}_{\perp}) = -\mu_{h,v} \nabla^4 \bar{n}
\]

- Gyrocenter parallel velocity equation
\[
\frac{\partial \bar{V}_{||}}{\partial t} + \mathbf{V}_\Phi \cdot \nabla \bar{V}_{||} + \partial_{||} \Phi + \frac{1}{n_0} \partial_{||} \bar{P}_{||} + (\bar{T}_{\perp} - \bar{T}_{||}) \partial_{||} \ln B + 4T_0 K(\bar{V}_{||}) = -\mu_{h,v} \nabla^4 \bar{V}_{||} + S_V(\bar{V}_{||})
\]

- Gyrocenter parallel temperature equation
\[
\frac{\partial \bar{T}_{||}}{\partial t} + \mathbf{V}_\Phi \cdot \nabla \bar{T}_{||} + \frac{B}{n_0} \partial_{||} \bar{q}_{||} + 2T_0 B \partial_{||} \frac{\bar{V}_{||}}{B} + \frac{2}{n_0} (\bar{q}_{\perp} + P_0 \bar{V}_{||} - P_0 V_{||}^{\text{neo}}) \partial_{||} \ln B + 2n_0 K(\Phi) + 3 \frac{T_0}{n_0} K(\bar{P}_{||}) - \frac{T_0}{n_0} K(\bar{P}_{\perp}) + T_0 K(3\bar{T}_{\perp} + \bar{T}_{||}) = -\mu_{h,v} \nabla^4 \bar{T}_{||} + S_T(\bar{T})
\]

- Gyrocenter perpendicular temperature equation
\[
\frac{\partial \bar{T}_{\perp}}{\partial t} + \mathbf{V}_\Phi \cdot \nabla \bar{T}_{\perp} + \frac{B}{n_0} \partial_{||} \bar{q}_{\perp} - \frac{1}{n_0} (\bar{q}_{\perp} + P_0 \bar{V}_{||}) \partial_{||} \ln B + n_0 K(\Phi) + 2 \frac{T_0}{n_0} K(\bar{P}_{\perp}) - \frac{T_0}{n_0} K(\bar{P}_{||}) + T_0 K(\bar{T}_{\perp} + 2\bar{T}_{\perp}) = -\mu_{h,v} \nabla^4 \bar{T}_{\perp} + S_T(\bar{T})
\]

where \( f = f_0 + \tilde{f} \), \( \Phi = \langle \phi \rangle = \frac{1}{1 - 0.5 \rho_i^2 \mathbf{v}_{\perp}^2} \phi \), \( \mathbf{V}_\Phi = \frac{1}{B} \mathbf{b} \times \nabla \Phi \),
\[
K(f) = \frac{1}{2B} (\mathbf{b} \times \mathbf{k} + \mathbf{b} \times \nabla \ln B) \cdot \nabla f, S_T(\bar{T})(S_V(\bar{V}_{||})): \text{sum of heat (momentum) source, sink and core diffusion}
\]
Some features for the model

- Developed under BOUT++ framework [B. Dudson et al. CPC’09].
- Gyrokinetic Poisson equation for n=0 potential
  \[-\nabla_\perp \cdot \frac{\rho_i^2}{\lambda_{Di}^2} \nabla_\perp \phi + \frac{1}{\lambda_{De}^2} (\phi - \langle \phi \rangle_f) = 4\pi e \langle \tilde{n} \rangle\]
- To construct gyrokinetic Poisson matrix in (r,\theta) plane, 4th order finite difference method and Simpson’s rule are adopted.
  - Direct matrix inversion (UMFPACK)
- Finite Larmor Radius (FLR) terms are partially considered in the model for numerical stability.
- Hyper-viscous damping terms are introduced for numerical stability and to reduce high k-modes
- Parallel Landau damping is taken into account.
- Residual zonal flow closure is not included but neoclassical poloidal damping and PS return flows are considered.
Neoclassical poloidal flow & Pfirsch-Schluter return flow at equilibrium

\[ V_{\theta}^{\text{neo}} = 1.17 \frac{\hat{\theta} \cdot \vec{b} \times \nabla T_i}{eB} \]

We confirm that neoclassical equilibrium is achieved.

- Poloidal flow is generated at neoclassical level.
- Parallel flow (i.e. P-S return flow) is developed to match \( \nabla \cdot (n\vec{V}) = 0 \).
Ion temperature gradient driven linear/nonlinear simulations

Simulations are carried using cyclone base parameters \( q = 1.4, \dot{s} = \frac{r dq}{q dr} = 0.772, R_0/L_{T_i} = 6.92, R_0/L_{n_e} = 2.22 \).

- As hyper-viscosity increases, linear ITG growth rate reduces to those of gyrokinetic simulation and gyrofluid simulation of Beer and Hammett and turbulence power density of high k-modes also decreases and k value of maximum turbulent power intensity moves to lower k value in nonlinear simulation.
Ion heat conductivity of gradient-driven nonlinear ITG turbulent simulations

- As hyper-viscosity increases, potential fluctuations are reduced (mostly high k-modes) and ion heat conductivity decreases correspondingly.
  - For $\mu_{hv}=1.0$, ion heat conductivity is comparable to that of local gyrokinetic simulation.
  - $\mu_{hv}=1.0$ is chosen for hyper-viscosity hereafter.

- A simulation using ‘3+1’ gyrofluid model by Beer and Hammett shows that ion heat conductivity bursts irregularly and it may come from non-conservative model or/and numerical issues.

- For $\mu_{hv}=1.0$, poloidal ExB velocity shows a zonal flow pattern and a potential fluctuation is regulated by the zonal flow.
Linear/Non-linear ITG turbulence threshold with $\mu_{h.v.}=1.0$

- Maximum growth rate of linear ITG turbulence, $\gamma_{max}$ increases as $R_0/L_{T_i}$ increases.
  - The threshold is $R_0/L_{T_i,crit} \sim 4.4$, similar to $R_0/L_{T_i,crit} \sim 4$ of gyrokinetic result.
- Nonlinear ITG turbulence simulation without an external heating source is carried for comparison of gyrokinetic simulation, gKPSP, nullifying n=0 potential.
  - Nonlinear critical gradient of the simulation, $R_0/L_{T_i,crit} \sim 5.03$, similar to $R_0/L_{T_i,crit} \sim 5.25$ of gKPSP result.
Flux-driven simulation setting

\[ S(T) = H + \chi_H \frac{d^2 \langle T \rangle}{dr^2} - \nu_{\text{sink}} \langle T \rangle \]

- \( H = H_0 \hat{H} \), \( \chi_H = 5m/s^2 \), and \( \nu_{\text{sink}} = 1 \) in code unit \((c_s/a)\).

- We obtain nonlinear steady state of flux-driven ITG simulations with various heating powers.
Flux driven simulation with 2MW heating power (1)

Radially extended heat fluxes are observed and $R_0/L_{T_i}$ varies correspondingly in spatio-temporal plots.

- Prolonged corrugation of $R_0/L_{T_i}$ is not clearly observed. ➔ Absence of residual ZF closure in GF model?

- Plot, spatially averaged from 0.4 r/a to 0.6 r/a, shows that $R_0/L_{T_i}$ grows and collapse after each heat bursts out.
Flux driven simulation with 2MW heating power (2)
- Probability distribution function and power spectral density

• Non-Gaussian PDF of heat flux with skewness, S=0.768, and kurtosis, K=4.325.
• Power spectral density shows 1/f dependency from low to intermediate frequency range.
  ➔ Self-organized criticality (SOC)-like heat avalanche
• PDF and PSD are evaluated using heat flux from 0.4 r/a to 0.6 r/a for 1000 steps after 1000 a/c_s.
Internal transport barrier formation by an external vorticity source (1)

- Additional equations

\[
\frac{\partial W}{\partial t} = S_{ni} = \rho^* \nabla^2 \nabla S_w \quad \text{Vorticity equation}
\]

\[
-\nabla \cdot \left( \frac{\rho_i^2}{\lambda_{di}^2} \nabla \phi + \frac{1}{\lambda_{de}^2} (\phi - \langle \phi \rangle) \right) = 4\pi e (\langle n \rangle + W) \quad \text{gyrokinetic Poisson equation}
\]

\[\Rightarrow\text{ solved simultaneously with the gyrofluid equations.}\]
After flux-driven ITG steady state with 4MW is achieved, three different vorticity sources \((4S_0, S_0, 0)\) are injected, where \(S_0 = 0.1n_0c_s/a\).

For strong vorticity injection \((S_{w0} = 4S_0)\), ExB flow shear develops strongly, \(R_0/L_{Ti}\) increases as time goes by and finally internal transport barrier of ion temperature is formed at 1500 \(a/c_s\).

\[ \begin{align*}
\text{ITB builds up from 0.49 } r/a \text{ to 0.56 } r/a \text{ in a monotonic q profile by strong vorticity injection.} \\
\text{ExB flow shear and } R_0/L_{Ti} \text{ are averaged in the ITB region.}
\end{align*} \]
Internal transport barrier formation by an external vorticity source – Strong vorticity source

- After strong vorticity injection, stationary zonal flow pattern is formed in a global region due to the vorticity transport by the vorticity injection.
- Avalanche heat transport is quenched after strong vorticity injection.
  ➔ The enhanced and persistent ExB flow shear after vorticity injection regulates heat transport.
The strength and the frequency of heat avalanches is reduced in the high confinement regime.

- VS reduces non-Gaussianity, i.e. \((S,K)\) change from \((1.04, 4.72)\) to \((0.56, 3.07)\) and non-Gaussian tail is located at about three times smaller \(Q_i\) after VS.
- Even though both PSD’s show broad 1/f region, the PSD at low frequency is about three times higher for the ITB case.

PDF’s and PSD’s are calculated using 500 steps in the ITB region before and after vorticity source injection.
Summary

• We developed a 3+1 Global Gyro-Landau Fluid code and showed linear and nonlinear results for global ITG mode.
  ➢ Neoclassical equilibrium is confirmed.
  ➢ Linear and nonlinear ITG thresholds are similar to those of gyrokinetic simulations.
  ➢ Linear ITG threshold $R/L_{T_{i,ctit}} \sim 4$, Nonlinear ITG threshold $R/L_{T_{i,ctit}} \sim 5$ without n=0 potentials.

• In flux-driven simulation, PDF of heat flux is non-Gaussian with (S,K)=(0.768, 4.325) and PSD shows 1/f dependency from low to intermediate frequency range.
  ➢ Self-organized criticality (SOC)-like heat avalanche

• An external vorticity source is considered by solving external vorticity equation separately and coupling with gyrokinetic Poisson equation.
  ➢ ITB is formed by a strong vorticity source injection.
  ➢ The enhanced and persistent ExB flow shear after vorticity injection regulates heat transport.
  ➢ Non-Gaussianity is reduced i.e. (S,K) change from (1.04, 4.72) to (0.56,3.07) and non-Gaussian tail is located at about three times smaller Qi after VS.
  ➢ Even though both PSD’s show broad 1/f region, the PSD at low frequency is about three times higher for the ITB case.
Thank you for attention.