Connecting Collisionless Landau Fluid Closures to Collisional Plasma Physics Models

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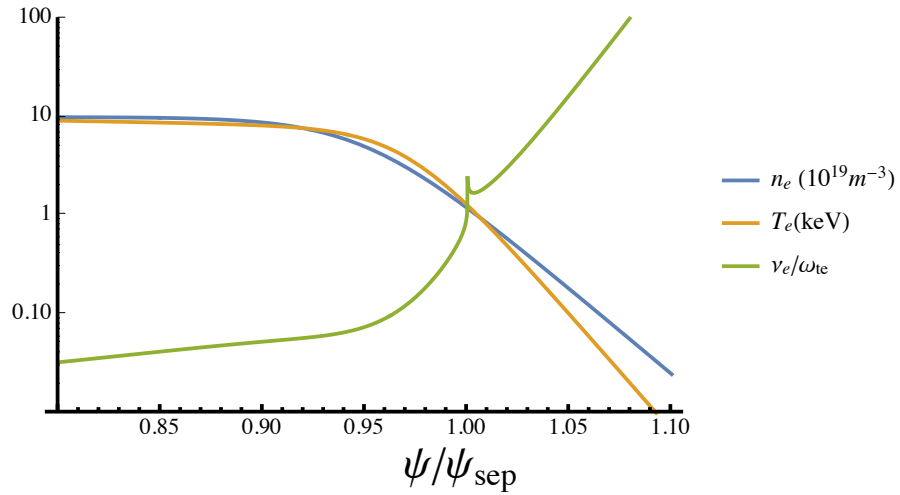
Outline

- **Motivation: Edge Plasma Physics**
- **Moments of the Gyrokinetic Equation**
- **Review of Collisionless Landau Fluid Closures**
- **Connecting the Collisionality Limits**
- **Conclusion**

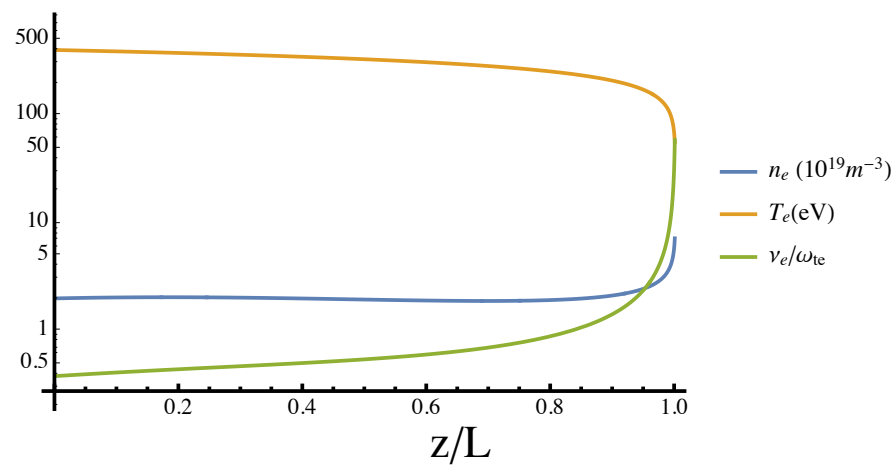
Motivation: Edge Plasma Physics

- **Collisionality varies by $\times 1000$ across the edge of a tokamak fusion reactor**

$$\frac{\nu}{k_{\|} v_T} \sim \frac{qR}{\lambda}$$



- **Collisionality varies by $\times 100$ from midplane to target plate**



- **Traditional fluid theory is invalid in the collisionless region**

- **Need quantitative theory to connect collisionless and collisional limits**

A Brief History of Fluid Closure Theory

- **Fluid equations have been studied for over 200 years**
  - **Closure of moment hierarchy rests on strong collisionality assumption:** $\omega \ll \nu$ **and** $k\lambda \ll 1$

    Navier-Stokes (1800's) $\longrightarrow$ Chapman-Enskog (1939)
- **Magnetized plasmas break these assumptions**
  - **Large gyro-frequency** $\Omega = eB/m \gg \nu$
  - **Small perpendicular scales** $\rho = v_\perp/\Omega \lll \lambda$
  - **Large Knudsen number** $k_\parallel \lambda \gg 1$
- **Gyro-fluid closures attempt to extend to** $k_\perp \rho \leq 1$
  - **Magnetized fluid equations for** $k_\perp \rho \ll 1$ **solved by late 1950s**

    L. Spitzer, H. Grad, S. I. Braginskii
  - **Gyro-fluid closures extend fluid theory to** $k_\perp \rho \sim 1$

    A. Brizard (1989)
    G. W. Hammett, W. Dorland, M. Beer, P. B. Snyder (1992-. . .)
    R. E. Waltz, G. D. Kerbel, J. Milovich, G. W. Hammett (1995-. . .)

Claude-Louis Navier

Sir George Gabriel Stokes

Some Modern Developments in Fluid Closure Theory

- **Linear fluid theory extended to large Knudsen number $k_{\|}\lambda \gg 1$**
  - **Laser-plasma interactions (LPI) drove research on closures for weakly collisional regime:**

    J. F. Luciani, et al. (1983), J. R. Albritton PRL (1983)
    E. M. Epperlein, R. W. Short, A. Simon, PRL (1992)
  - **Collisionless limit (i.e. Landau damping) first clearly understood:**

    E. Ott & R. N. Sudan (1969) and G. W. Hammett & F. W. Perkins (1990)
  - **Collisional closures begin to connect the collisional and collisionless limits:**

    Z. Y. Chang & J. D. Callen (1992), M. Beer, et al. (1996), P. B. Snyder, et al. (1997)

- **Recently, high accuracy linear collisional Landau-fluid closures have been developed**
  - **Extended Chapman-Enskog theory:** J.-Y. Ji & E. Held, POP (2014)
  - **Extended 3+1 theory:** I. Joseph & A. M. Dimits, CPP (2016), J.-Y. Ji & I. Joseph POP (2014)
  - **Trapped Particles/Neoclassical:** G. Staebler, J. E. Kinsey, R. E. Waltz (2007)

- **Extensions to nonlinear regime is nontrivial**
  - **LPI**: G. Schurtz, Ph. D. Nicolai, M. Busquet, POP (2000), Ph. D. Nicolai, et al., POP (2006)
  - **Trapped Particle Nonlinearity:** N. Mattor and S. E. Parker, PRL (1997), ...
    I. Holod, J. Weiland, A. Zagorodny, POP (2002)
  - **Comparison of nonlinear models:** J. P. Broderick, et al., POP (2018)

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Gyrokinetic equation is the starting point

- **Rapid gyro-averaging allows us to consider gyro-averaged distribution** $f(v_\parallel, \mu)$
  - **Here we will assume** $k_\perp \rho_\perp \sim k_\parallel \rho_\parallel \ll 1 \longrightarrow$ **drift kinetic regime**
- **The gyro-kinetic equation in a constant magnetic field** $\vec{B}$ **coordinates** $\{x, v_\parallel, \mu\}$ **with Jacobian** $\mathcal{J} = B$

$$\partial_t \mathcal{J} f + \vec{\partial}_x \cdot \mathcal{J} f \vec{v} + \partial_{m v_\parallel} \mathcal{J} f F_\parallel = \mathcal{J} \mathcal{C}[f]$$

  - **Guiding center velocity** $\vec{v} = v_\parallel \hat{b} + \vec{E} \times \hat{b}/B + \ldots$
  - **Parallel force** $F_\parallel = eE_\parallel - \mu \partial_\parallel B$
  - **Collision operator** $\mathcal{C}[f]$
- **For simplicity, here, we neglect** ∇B **effects in closure**

Fluid theory takes moments to obtain a finite set of equations

- **Full Moments (uncentered)**

$$M_{j,2l} = \int {v_\parallel}^j (\mu B/m)^l f 2\pi dv_\parallel d\mu B$$

- **Centered Moments w.r.t. mean flow velocity** $u_\parallel$

$$m_{j,2l} = \int (v_\parallel - u_\parallel)^j (\mu B/m)^l f 2\pi dv_\parallel d\mu B$$

Moment Definitions: Parallel Hierarchy

upper case for full moments, lower case for centered moments

- **The first moments are density n and velocity $u_\parallel$**

$$n = \int f d^3v$$

$$nu_\parallel = \int v_\parallel f d^3v$$

$$\dots$$

- **The next moments define the rest of the parallel series**

$$P_\parallel = m \int {v_\parallel}^2 f d^3v = p_\parallel + \varrho {u_\parallel}^2$$

$$Q_\parallel = m \int {v_\parallel}^3 f d^3v = q_\parallel + 3p_\parallel u_\parallel + \varrho {u_\parallel}^3$$

$$R_\parallel = m \int {v_\parallel}^4 f d^3v = r_\parallel + 4q_\parallel u_\parallel + 6p_\parallel {u_\parallel}^2 + \varrho {u_\parallel}^4$$

$$\dots$$

Moment Definitions: Perpendicular Hierarchy

upper case for full moments, lower case for centered moments

- **Adding a weight of μB defines the perpendicular series**

$$P_\perp = \int \mu B f d^3v = p_\perp$$

$$Q_\perp = \int v_\parallel \mu B f d^3v = q_\perp + p_\perp u_\parallel$$

$$R_\perp = \int {v_\parallel}^2 \mu B f d^3v = r_\perp + 2q_\perp u_\parallel + p_\perp {u_\parallel}^2$$

$$\ldots$$

- **And so on ...**

$$R_{\perp\perp} = \int (\mu B)^2 f d^3v/m = r_{\perp\perp}$$

$$\ldots$$

Moments of Collision Operator yield friction forces

- **Parallel Series:**

$$\mathrm{C}_{u\|} = m \int v_\| \mathcal{C}[f] d^3v$$

$$\mathrm{C}_{p\|} = m \int (v_\| - u_\|)^2 \mathcal{C}[f] d^3v$$

$$\mathrm{C}_{q\|} = \int (v_\| - u_\|) \mu B \mathcal{C}[f] d^3v$$

$$\ldots$$

- **Perpendicular Series:**

$$\mathrm{C}_{p\perp} = \int \mu B \mathcal{C}[f] d^3v$$

$$\mathrm{C}_{q\perp} = \int v_\| \mu B \mathcal{C}[f] d^3v$$

$$\ldots$$

- **And so on ...**

$$\mathrm{C}_{r\perp\perp} = \int (\mu B)^2 \mathcal{C}[f] d^3v/m$$

$$\ldots$$

Full moment equations are naturally written in conservative form

- **Conservation of Density and Momentum:** $\vec{U}_\perp = \vec{E} \times \hat{b}/B$

$$\partial_t \varrho + \nabla \cdot \varrho \left(\vec{U}_\perp + u_\parallel \hat{b} \right) = 0$$

$$\partial_t \varrho u_\parallel + \nabla \cdot \left(\varrho u_\parallel \vec{U}_\perp + P_\parallel \hat{b} \right) - P_\perp \nabla \cdot \hat{b} = neE_\parallel + \mathrm{C}_{u\parallel}$$

- **Parallel Series:**

$$\partial_t P_\parallel + \nabla \cdot \left(P_\parallel \vec{U}_\perp + Q_\parallel \hat{b} \right) - 2Q_\perp \nabla \cdot \hat{b} = 2nu_\parallel eE_\parallel + \mathrm{C}_{P\parallel}$$

$$\partial_t Q_\parallel + \nabla \cdot \left(Q_\parallel \vec{U}_\perp + R_\parallel \hat{b} \right) - 3R_\perp \nabla \cdot \hat{b} = 3P_\parallel eE_\parallel/m + \mathrm{C}_{Q\parallel}$$

- **Perpendicular Series:**

$$B\partial_t P_\perp/B + B\nabla \cdot \left(P_\perp \vec{U}_\perp + Q_\perp \hat{b}/B \right) = \mathrm{C}_{p\perp}$$

$$B\partial_t Q_\perp/B + B\nabla \cdot \left(Q_\perp \vec{U}_\perp + R_\perp \hat{b} \right)/B - R_{\perp\perp} \nabla \cdot \hat{b} = P_\perp eE_\parallel/m + \mathrm{C}_{Q\perp}$$

- **and so on** . . .

Centered moment equations have a number of cancellations

- **For a Maxwellian with anisotropic temperatures:**

$$v_{T\parallel}^2 = T_\parallel/m \qquad v_{T\perp}^2 = T_\perp/m$$

$$r_\parallel = 3p_\parallel v_{T\parallel}^2 \qquad r_\perp = p_\perp v_{T\parallel}^2 \qquad r_{\perp\perp} = 2p_\perp v_{T\perp}^2$$

- **Parallel Series:**

$$\partial_t p_\parallel + \nabla\cdot\left(p_\parallel \vec{U} + q_\parallel \hat{b}\right) + 2p_\parallel \partial_\parallel u_\parallel - 2q_\perp \nabla\cdot\hat{b} = \mathrm{C}_{p\parallel}$$

$$\partial_t q_\parallel + \nabla\cdot\left(q_\parallel \vec{U} + r_\parallel \hat{b}\right) + 3q_\parallel \partial_\parallel u_\parallel = 3v_{T\parallel}^2 \partial_\parallel p_\parallel + 3\left(r_\perp + p_\parallel({v_{T\parallel}}^2 - {v_{T\perp}}^2)\right)\nabla\cdot\hat{b} + \mathrm{C}_{q\parallel} - 3v_{T\parallel}^2 \mathrm{C}_{u\parallel}$$

- **Perpendicular Series:**

$$B\partial_t B^{-1} p_\perp + \nabla\cdot B^{-1}\left(p_\perp \vec{U} + q_\perp \hat{b}\right) = \mathrm{C}_{p\perp}$$

$$B\partial_t B^{-1} q_\perp + B\nabla\cdot B^{-1}\left(q_\perp \vec{U} + r_\perp \hat{b}\right) + q_\perp \partial_\parallel u_\parallel = v_{T\perp}^2 \partial_\parallel p_\parallel + \left(r_{\perp\perp} + p_\perp({v_{T\parallel}}^2 - {v_{T\perp}}^2)\right)\nabla\cdot\hat{b} + \mathrm{C}_{q\perp} - v_{T\parallel}^2 \mathrm{C}_{u\parallel}$$

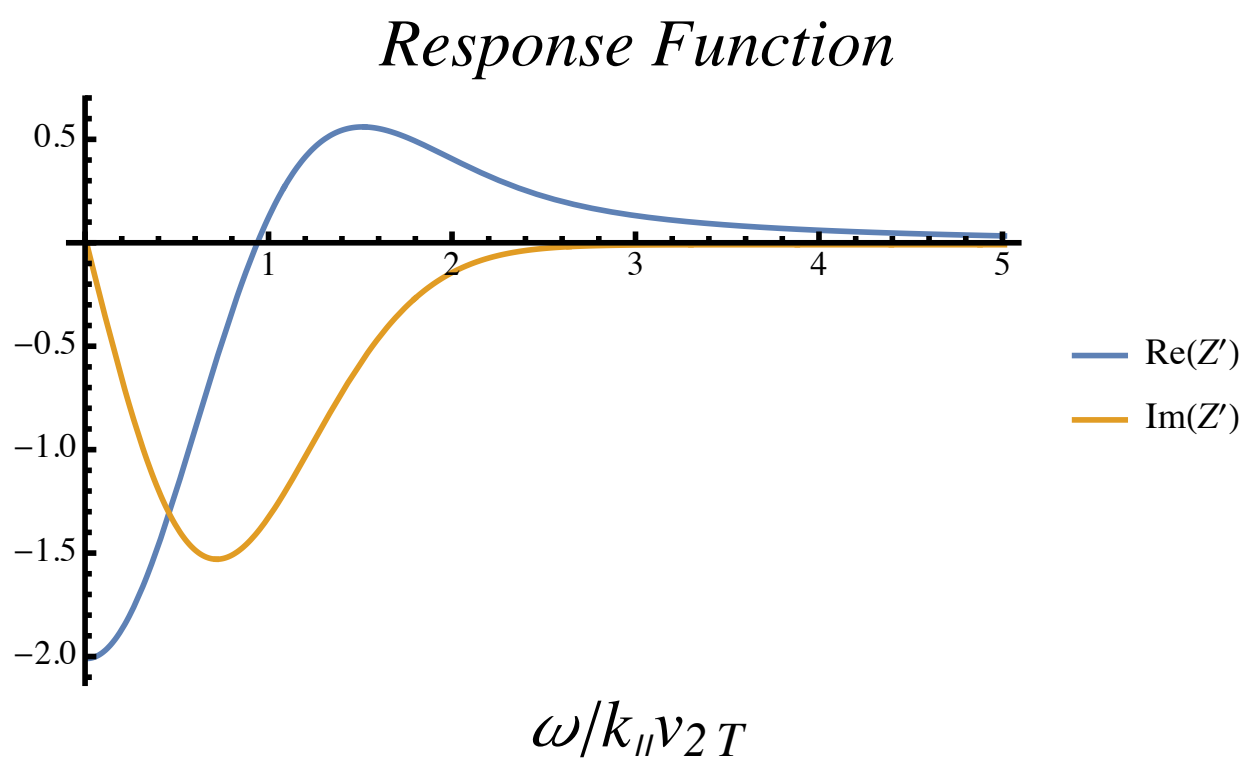
- **and so on** . . .

Linear density response function is a useful metric for comparison

- **Response function**

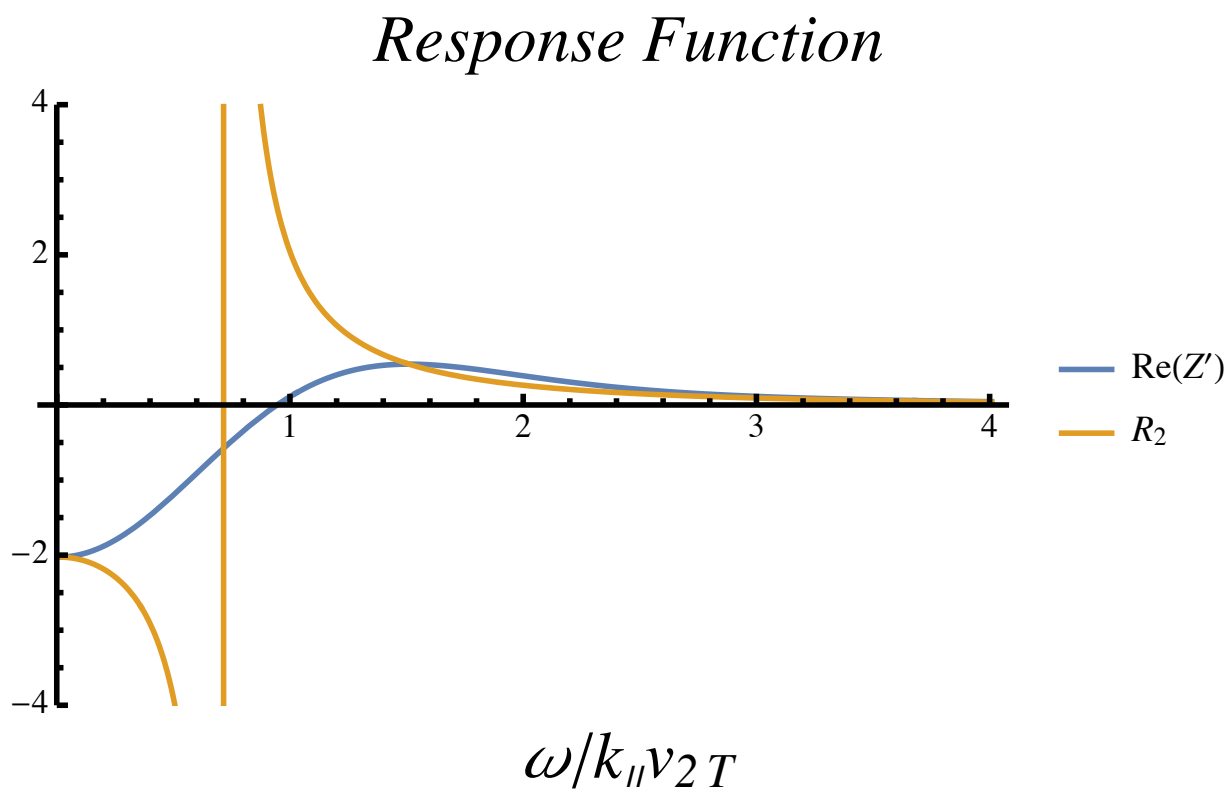
$$\frac{\delta n}{n} = R_{\omega,k_\parallel} \frac{q\delta\phi}{2T_\parallel}$$

- **For an electrostatic wave,** $R = Z'(\zeta)$ **where** $\zeta = \omega/2^{1/2}k_\parallel v_T$ **and** $Z(\zeta)$ **is the plasma dispersion function**



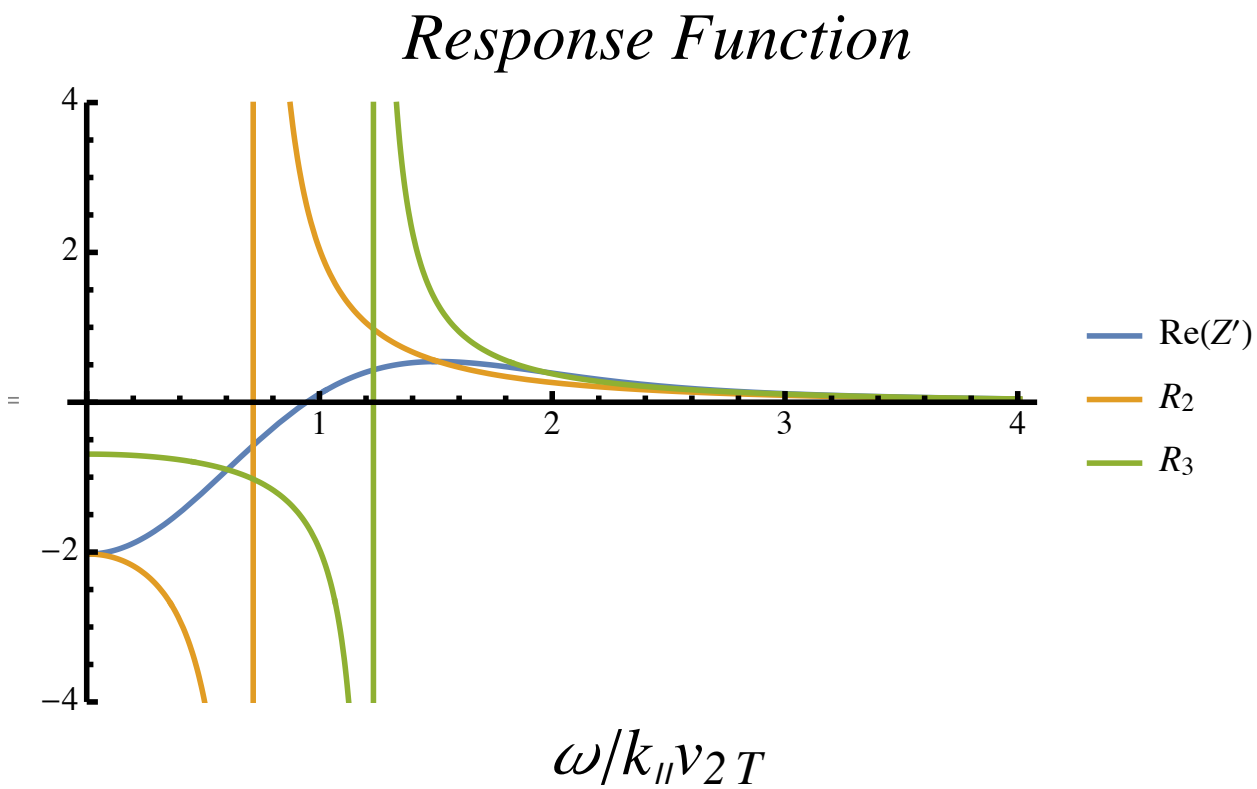
Ideal Moment Closure

- **2 moments** $(n, u_{\|})$
- **Sound wave resonance for specific real frequencies**
- **No damping**



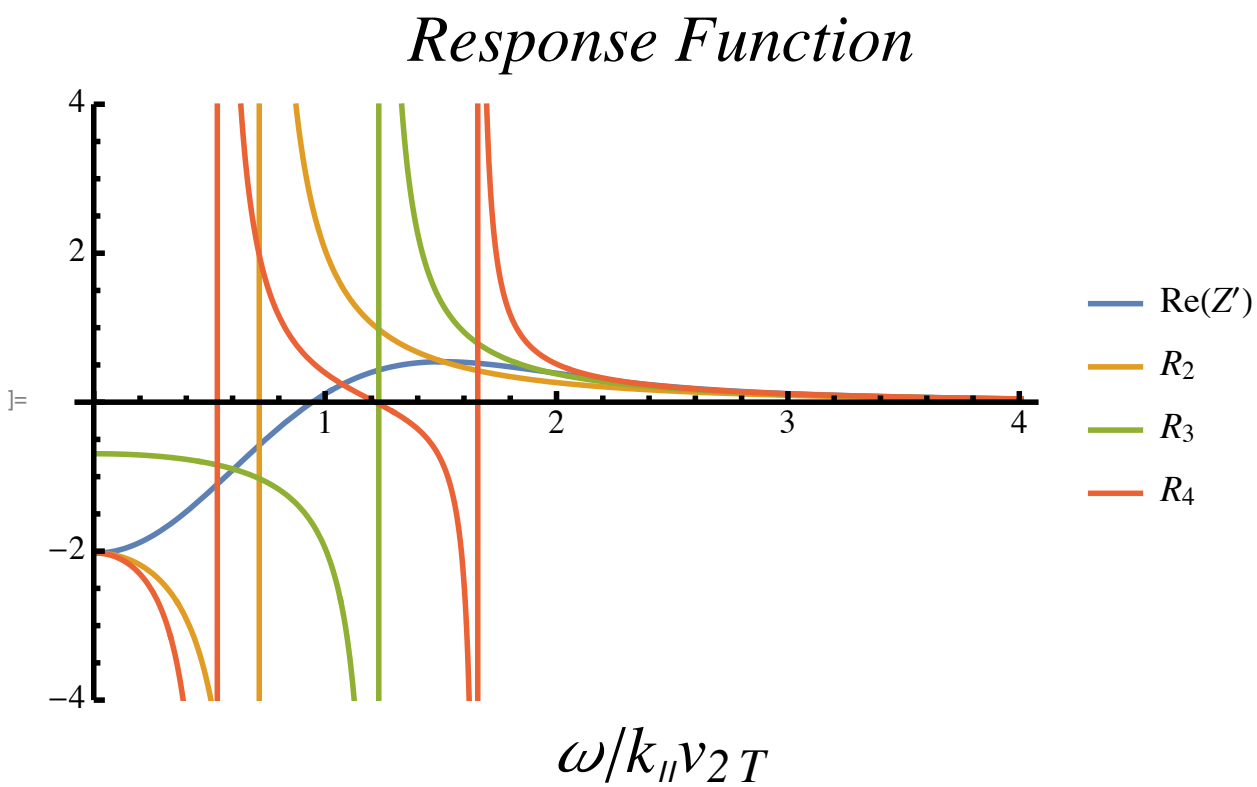
Does Adding More Moments Help?

- **3 moments** $(n, u_{\|}, p_{\|})$
- **Sound wave resonance for specific real frequencies**
- **No damping**



Does Adding More Moments Help? No!

- **4 moments** $(n, u_{\|}, p_{\|}, q_{\|})$
- **Adding moments improves accuracy for** $\omega \to \infty$
- **Doesn't help for** $\omega \to 0$



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Chapman-Enskog approach to Landau fluid closure

- **Sketch of Derivation: Starting from linearized Drift Kinetic Equation**

$$\partial_t f_1 + v_\parallel \hat{b} \cdot \nabla f_1 + \nu f_1 = S_1$$

- **Solution in Fourier space** $f(x,t) = \int \hat{f}(\omega, k) e^{ikx - i\omega t} dk d\omega$

$$\hat{f}_1 = \frac{\hat{S}_1}{i(kv_\parallel - \omega) + \nu}$$

- **Collisionless limit yields a dissipative term á la Landau**

$$\lim_{\nu \to 0} \hat{f}_1 = \mathrm{PV} \frac{\hat{S}_1}{i(kv_\parallel - \omega)} + \pi \hat{S}_1 \delta(kv_\parallel - \omega) \longrightarrow \frac{\pi}{|k|} \hat{S}_1 \delta(v_\parallel - \omega/k)$$

Collisionless Landau fluid closures can be chosen to yield high-fidelity linear response

- **Since the remainder is driven by gradients $S_1 \propto -\nabla_{\|} m_j + \ldots$, the moment closure must have the leading form**

$$\hat{m}_{j+1} = -D_j v_T \frac{\nabla_{\|}}{|\nabla_{\|}|} \hat{m}_j \qquad \longrightarrow \qquad \nabla_{\|} m_{j+1} = D_j v_T |\nabla_{\|}| m_j$$

- **Padé approximation can be used to choose coefficients D_j that optimize accuracy of response function**
  - Fit to $\omega \to 0$ density response function and use standard fluid theory to get $\omega \to \infty$ response
  - G. W. Hammett & F. W. Perkins, Phys. Rev. Lett. (1990)

- **Use linear response to "remainder" of kinetic equation (analogous to Chapman-Enskogg approach)**
  - Z. Y. Chang & J. D. Callen, Phys. Fluids B (1992)
  - J. Y. Ji & E. Held, Phys. Plasmas (2013)

Collisionless closures adds non-local anomalous transport

- **Anomalous sub-diffusion operator $v_T|\nabla|$ yields ballistic transport scaling $\Delta x \sim v_T \Delta t$**

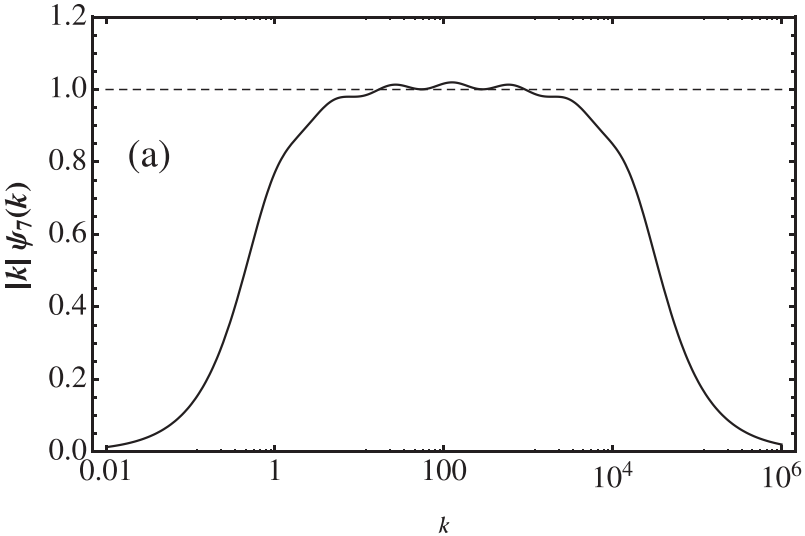
$$\boxed{|\nabla| \equiv |\nabla^2|^{1/2} \sim |k|}$$

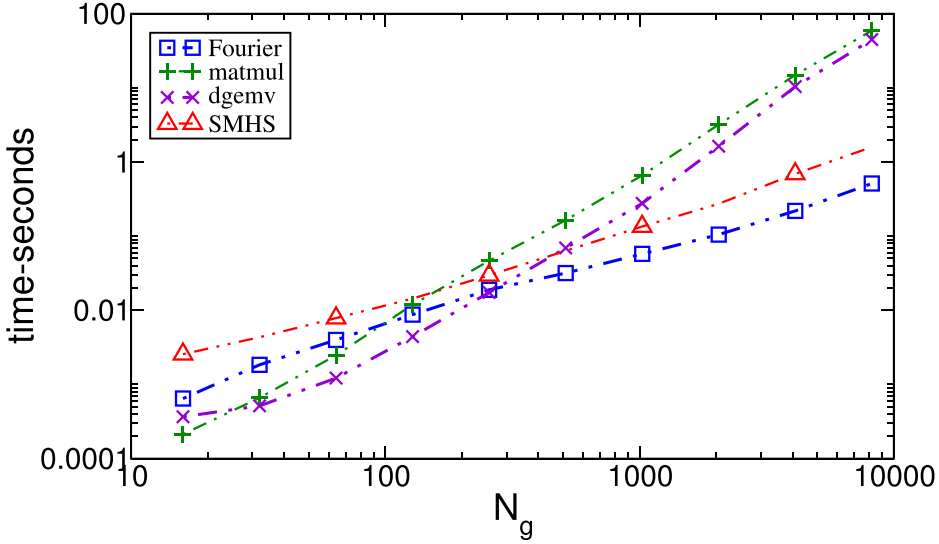
- **The Hilbert transform $\mathcal{H} \sim -ik/|k|$ is the building block for $|\nabla| = \mathcal{H}\nabla = \nabla\mathcal{H}$**

$$\mathcal{H}f(x) = -\mathsf{PV}\int_0^\infty dx' \frac{[f(x'+x) - f(x'-x)]}{\pi x'} = -\int \frac{ik}{|k|}\hat{f}(k)e^{ikx}dk$$

Efficient finite difference Hilbert transform has been derived, implemented and used for gyro-Landau fluid simulations

- **A. M. Dimits, I. Joseph and M. V. Umansky, Phys. Plasmas 21, 055907 (2014)(2014)**



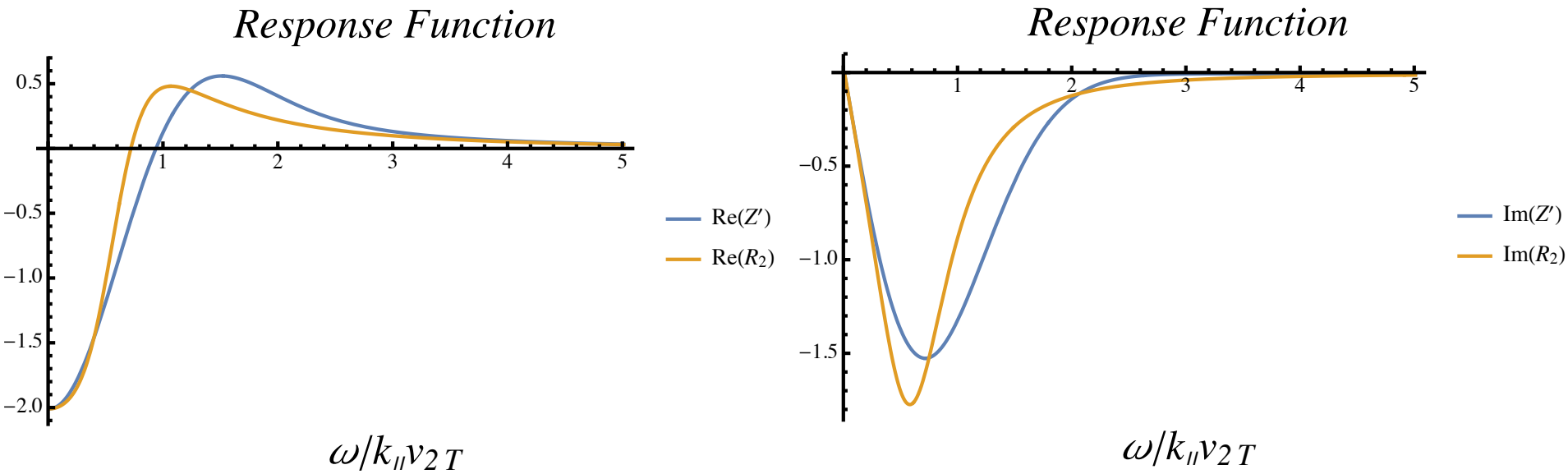


- **X. Q. Xu, A. M. Dimits, P. W. Xi, et al., Phys. Plasmas 20, 056113 (2013)**
- **M. V. Umansky, A. M. Dimits, I. Joseph, et al., Nucl. Fusion 463, 506 (2015)**

Collisionless Closure Adds Landau Damping

- **2 moment equations** $(n, u_{\|})$
- **Closure equation for pressure**

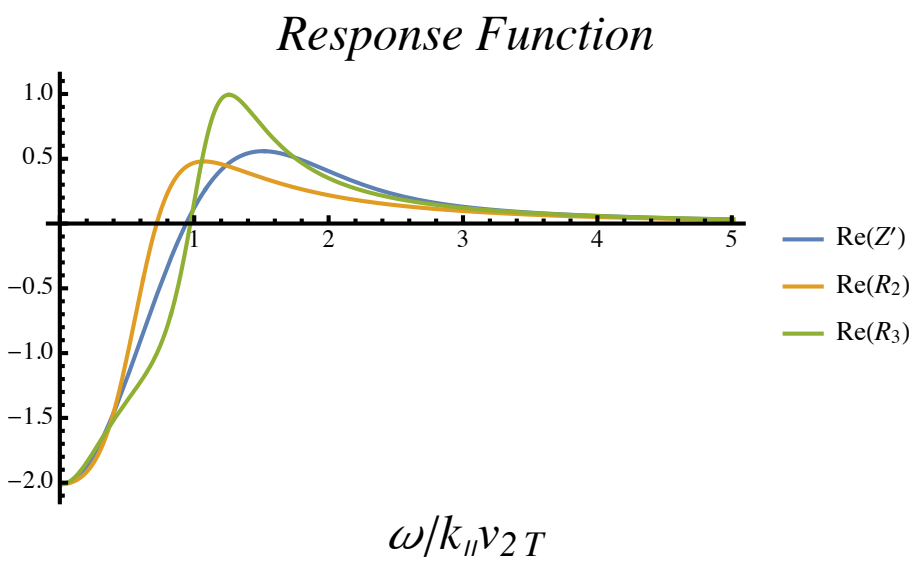
$$p_{\|} = n v_{T\|} \sqrt{\frac{\pi}{2}} \mathcal{H} u_{\|}$$

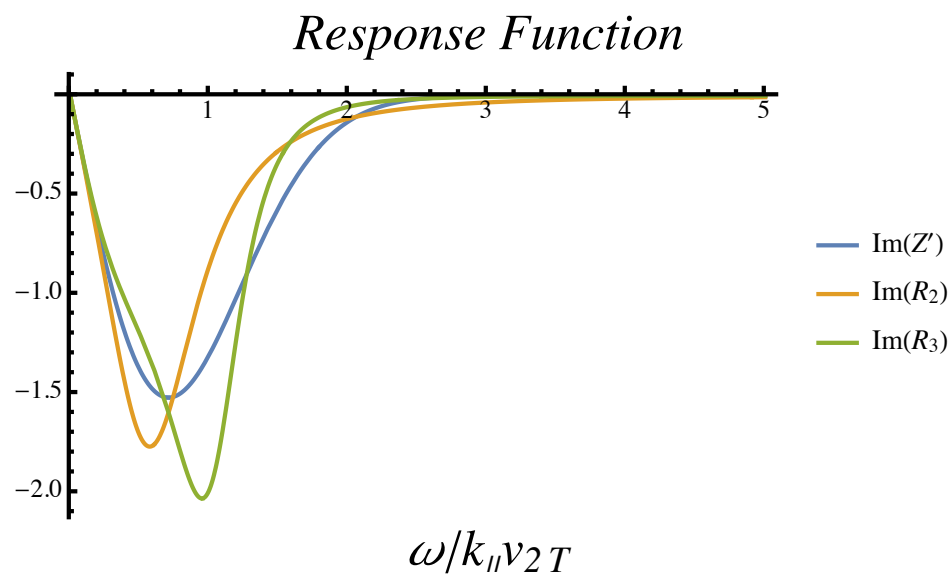


Collisionless Closure: More Moments Improve Accuracy

- **3 moment equations** $(n, u_{\|}, p_{\|})$
- **Closure equation for heat flux**

$$q_{\|} = n v_{T\|} 2\sqrt{\frac{2}{\pi}} \mathcal{H} T_{\|}$$

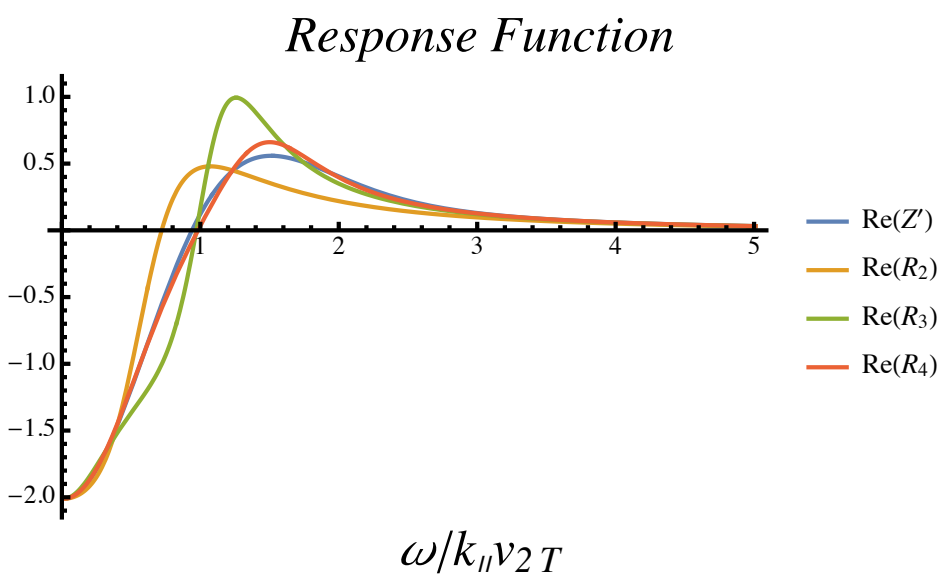


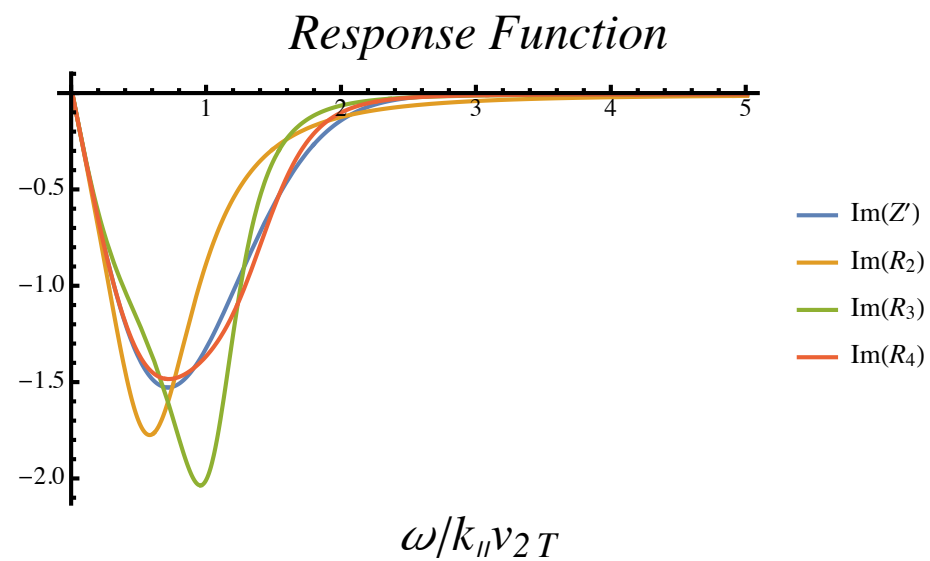


Collisionless Closure: More Moments Improve Accuracy

- **4 moment equations** $(n, u_{\|}, p_{\|}, q_{\|})$
- **Closure equations begin to include more moments**

$$r_{\|} = n v_{T\|} \frac{2\sqrt{2\pi}}{3\pi - 8} \mathcal{H} \frac{q_{\|}}{n} - \frac{32 - 9\pi}{3\pi - 8} \frac{p_{\|}}{m} (T_{\|} - \bar{T}_{\|}) + 3 \frac{p_{\|}}{m} T_{\|}$$

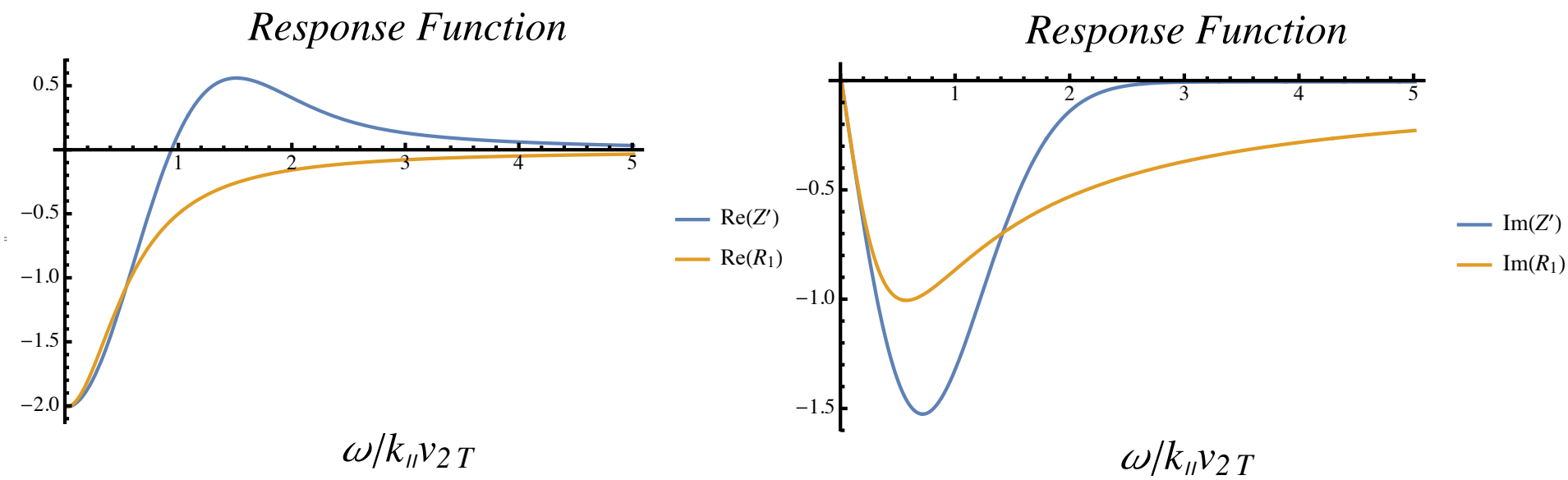




Even a single moment can produce sensible results for $\omega \to 0$

- **1 moment equation** (n)
- **Closure equation for velocity**

$$nu_{\parallel} = v_{T\parallel}\sqrt{\frac{2}{\pi}}\left(\mathcal{H}n + n\mathcal{H}e\phi\right)$$



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A Collisional Extension of Collisionless Landau Fluid Closures

- **In the collisional limit, the solution of the DKE with $S_1 \propto -\nabla_{\|} m_j + \ldots$ has the form**

$$\lim_{\nu \to \infty} f_1 = \frac{S_1}{\nu} \qquad \longrightarrow \qquad m_{j+1} = -\chi_j \nabla_{\|} m_j$$

- **Padé Approximation for combined closure must have the form**

$$\boxed{m_{j+1} = -\frac{1}{(|\lambda_j \nabla_{\|}| + 1)} \chi_j \nabla_{\|} m_j}$$

- **The diffusivity χ_j is determined by the collisional limit**

$$\chi_j = j v_T^2 / \nu_j$$

- **The effective mean free path λ_j is determined by both limits**

$$\lambda_j = j v_T / \nu_j D_j$$

Collision operator needed to specify collisional friction forces & collision frequencies

- **In collisional regions, the PDF is nearly isotropic**

- **Coulomb collision operator matrix elements tabulated in orthogonal polynomial basis**
  - Laguerre-Legendre polynomials
  - Defines infinite series of damping frequencies ν_j^{mn}
  - Onsager symmetric in kinetic energy weighting m, n (diagonal in anisotropy order j)

- **Friction forces needed to complete moment equations:**

$$
\begin{array}{lcllll}
\mathrm{C}_{u\|} & = & \nu_1^{00} & \varrho(\bar{u}_1 - u_\|) & +\nu_1^{01} & \left(q_\| + 2q_\perp\right) m/5T \\
\mathrm{C}_{p\|} & = & \nu_0^{11} & (\bar{p} - p) & -2\nu_2^{11} & \left(p_\| - p_\perp\right)/3 \\
\mathrm{C}_{p\perp} & = & \nu_0^{11} & (\bar{p} - p) & +\nu_2^{11} & \left(p_\| - p_\perp\right)/3 \\
\mathrm{C}_{q\|} & = & -3\nu_1^{10} & p(\bar{u} - u_\|) & -3\nu_1^{11} & \left(q_\| + 2q_\perp\right)/5 \qquad -2\nu_3^{00} \quad \left(q_\| - 3q_\perp\right)/5 \\
\mathrm{C}_{q\perp} & = & -\nu_1^{10} & p(\bar{u} - u_\|) & -\nu_1^{11} & \left(q_\| + 2q_\perp\right)/5 \qquad +\nu_3^{00} \quad \left(q_\| - 3q_\perp\right)/5.
\end{array}
$$

Collisional closure is complicated by pitch-angle scattering

- **Higher moments have many equations coupled by friction forces**
- **Pitch-angle scattering couples** $\|, \perp, \perp\perp, \ldots$ **hierarchy**

$$\begin{aligned} \mathrm{C}_{q\|} &= \ldots \quad -3\nu_1^{11} \quad \left(q_\| + 2q_\perp\right)/5 \quad -2\nu_3^{00} \quad \left(q_\| - 3q_\perp\right)/5 \\ \mathrm{C}_{q\perp} &= \ldots \quad -\nu_1^{11} \quad \left(q_\| + 2q_\perp\right)/5 \quad +\nu_3^{00} \quad \left(q_\| - 3q_\perp\right)/5. \end{aligned}$$

- **Full closure is complicated & potentially expensive to implement**
- **Need to invert a matrix of Landau fluid closure operators with multiple mean free paths** λ_j **for each moment**

A surprisingly simple closure scheme has correct collisional and collisionless limits

- **Higher order anisotropies must become small in collisional limit**
- **If the anistropy of the closure moment $q_{\|} \simeq 3q_{\perp}$ is neglected in the friction force, only energy scattering remains**

$$\begin{aligned} \mathrm{C}_{q\|} &= \ldots \quad -3\nu_1^{11} \quad \left(q_{\|} + 2q_{\perp}\right)/5 \quad + \ldots \quad \simeq \quad -\nu_1^{11} q_{\|} \\ \mathrm{C}_{q\perp} &= \ldots \quad -\nu_1^{11} \quad \left(q_{\|} + 2q_{\perp}\right)/5 \quad + \ldots \quad \simeq \quad -\nu_1^{11} q_{\perp} \end{aligned}$$

- **Thus, we can identify ν_j with an energy scattering frequency**

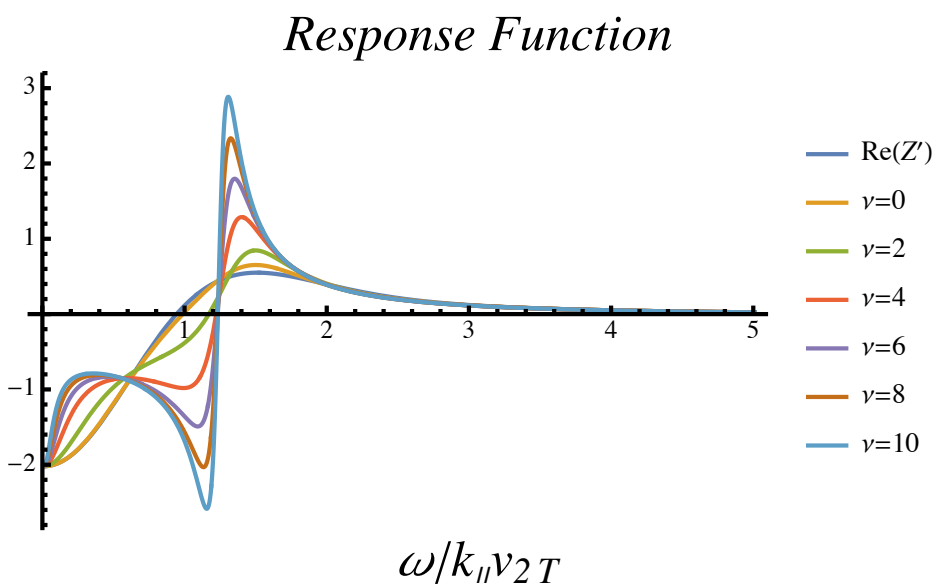
Combined Collisional-Collisionless Closure for heat flux $q_{\|}, q_{\perp}$

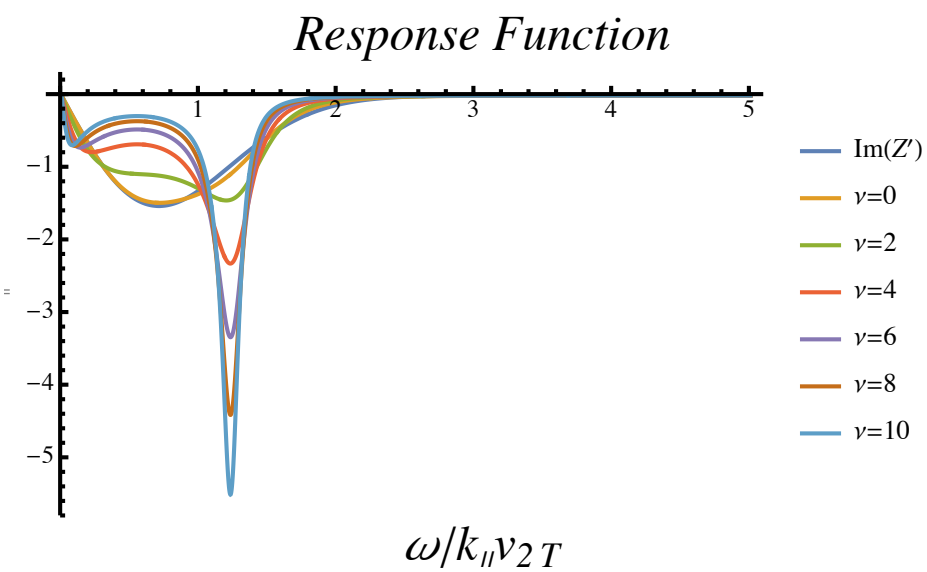
- $3_{\|} + 1_{\perp}$ **moment equations** $(n, u_{\|}, p_{\|}; p_{\perp})$
- **Closure for anisotropic heat flux**

$$q_{\|} = -\frac{3}{|\lambda_{\|}\nabla_{\|}| + 1} n\chi\nabla_{\|}T_{\|} \qquad \lambda_{\|} = \frac{3v_T}{\nu_1^{11} 2\sqrt{2\pi}} \qquad \chi = \frac{v_T^2}{\nu_1^{11}}$$

$$q_{\perp} = -\frac{1}{|\lambda_{\perp}\nabla_{\|}| + 1} n\chi\nabla_{\|}T_{\perp} \qquad \lambda_{\perp} = \frac{v_T}{\nu_1^{11}\sqrt{\pi/2}}$$

- **Closure smoothly connects collisionless and collisional limits**

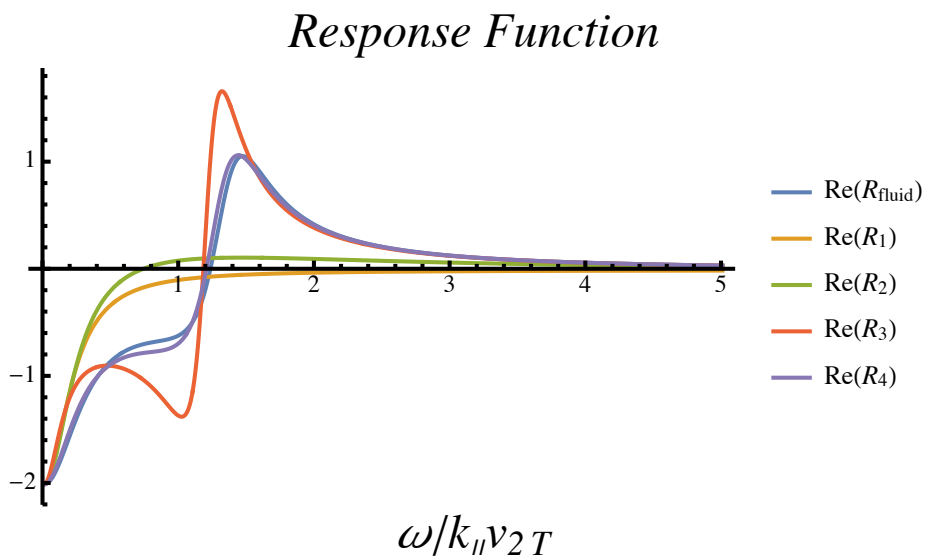


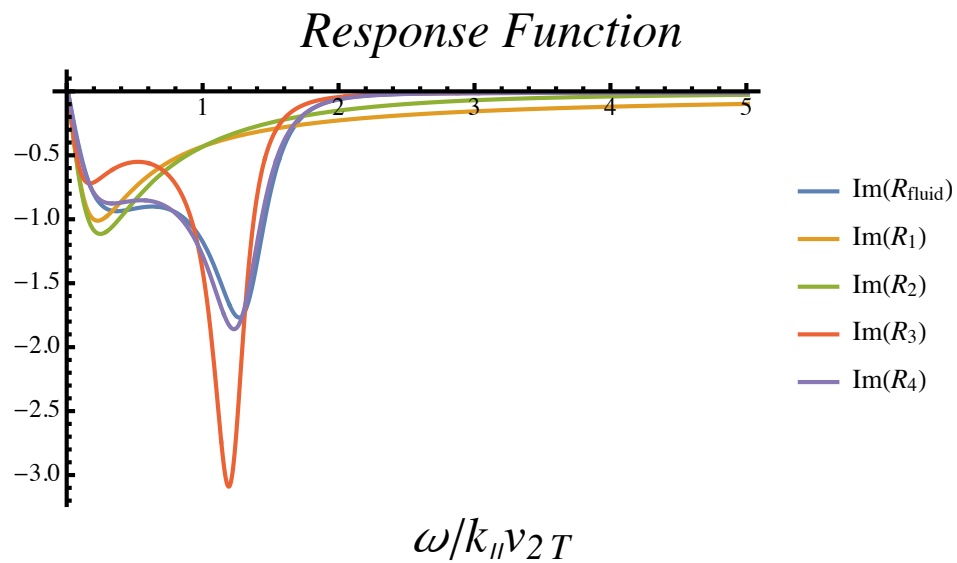


Collisional Landau fluid closure procedure can be generalized to both higher and lower moment equations

- **For higher moments: use energy scattering frequency**

- **For lower moments: can still identify a correct Padé approximation**
  - Even though self-collisions conserve particles, momentum and energy





Friction forces yield additional terms: frictional heat flux, thermal force, anisotropic electric conductivity

- **Frictional Heat Fluxes: coefficient** $\alpha = \nu_1^{10}/\nu_1^{11}$

$$q_{u\parallel} = \frac{3/5}{(|\lambda_\parallel \nabla_\parallel| + 1)} \alpha p_\parallel (\bar{u} - u_\parallel) \qquad q_{u\perp} = \frac{2/5}{(|\lambda_\perp \nabla_\parallel| + 1)} \alpha p_\perp (\bar{u} - u_\parallel)$$

- **Thermal force has same coefficient** α **due to Onsager symmetry** $\nu_1^{10} = \nu_1^{01}$

$$F_{u\parallel} = -\frac{\alpha \nu_1^{11}}{T} \left[\frac{3/5}{(|\lambda_\parallel \nabla_\parallel| + 1)} n\chi \nabla T_\parallel + \frac{2/5}{(|\lambda_\perp \nabla_\parallel| + 1)} n\chi \nabla T_\perp - \bar{q}_1 \right]$$

- **Anisotropic electrical resistivity operator**

$$\frac{\eta_\parallel}{\eta_\perp} = 1 - \frac{\alpha}{T} \frac{\nu_1^{11}}{\nu_1^{00}} \left(\frac{3/5}{|\lambda_\parallel \nabla_\parallel| + 1} + \frac{2/5}{|\lambda_\perp \nabla_\parallel| + 1} \right) \alpha T$$

- **These collisional effects all vanish in collisionless limit!**

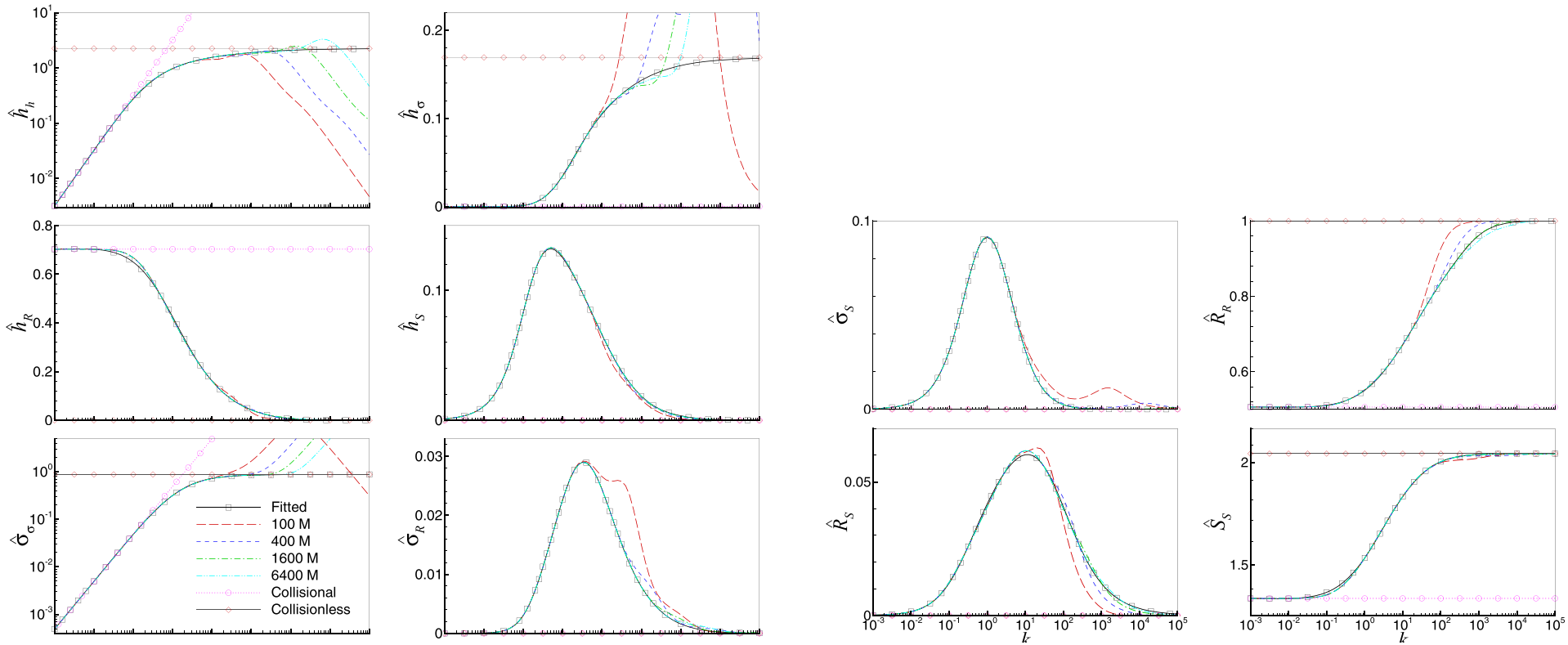
Collaboration with Jeong-Young Ji has led to highly accurate linear collisional 3+1 electron Landau-fluid closures

- **More advanced formulation:**
  - For $e - e$ scattering alone, including collisional moments within closure leads to 4 closure functions: $q_\parallel, q_\perp, C_{u\parallel}, C_{\pi\parallel} = C_{p\parallel} - C_{p\perp}$
  - Symmetric closure matrix has $n(n+1)/2 = 10$ matrix elements
- **Results are fit to**
  - Exact linear kinetic model with Coulomb collisions (6400 moments)
  - Asymptotic results for collisionless response function
- **A simple analytic fitting formula was found to capture coefficients within a few % accuracy for all collisionality regimes**

$$\tilde{K}_{AB} = \frac{ak^\alpha}{1 + d_1 k^\delta + d_2 k^{2\delta} + d_3 k^{3\delta} + d_4 k^{4\delta} + d_5 k^{5\delta} + d_6 k^{6\delta}}$$

Collisional closures as functions of $k\lambda$

- **Coefficients given in spherical harmonic form**



- **SHS method can be applied to each coefficient**

Outline

- Motivation: Edge Plasma Physics
- Moments of the Gyrokinetic Equation
- Review of Collisionless Landau Fluid Closures
- Connecting the Collisionality Limits
- **Conclusion**

Conclusion

- **Landau fluid closures can connect collisionless & collisional limits**
  - Collisionless regime has ballistic $\Delta x \sim v_T \Delta t$ anomalous sub-diffusive transport
  - Collisional regime has usual diffusive transport $(\Delta x)^2 \sim \chi \Delta t$
  - Smooth connection between regimes
- **Collisional form derived from closure moment equations** $(q_\parallel, q_\perp)$
  - Correct collisional limit is only obtained when using the collisional Landau closure
  - Neglecting pitch-angle scattering in closure itself yields simple form
  - Includes frictional heat flux, parallel thermal force, and anisotropic resistivity
  - Explicit Onsager symmetry found to hold
- **Approach easily generalized to higher & lower moment closures**