Progress in simulating scrape-off layer plasma dynamics with STORM

F. Riva, D. Hoare, F. Militello, S. Newton, T. Nicholas, J.T. Omotani, D. Schwörer, N.R. Walkden, and B.D. Dudson
The scrape-off layer (SOL) region

- Scrape-off layer
- Roles of the SOL:
  - Heat exhaust
  - Plasma fueling and ash removal
  - Plasma confinement
  - Impurity control

STORM module of BOUT++

- Need to understand transport mechanisms in the SOL:
  - Filaments (generation, motion,…)
  - Turbulence dynamics

Scrape-off layer

Plasma outflowing from the core

Parallel flows

Sheath losses

What are the heat and particle loads on vessel components?

➔ Need to understand transport mechanisms in the SOL:

- Filaments (generation, motion,…)
- Turbulence dynamics
STORM: the workhorse of our projects

https://github.com/boutproject/STORM

- The STORM isothermal model
- Verification and validation
- Comparison with 2D simulations
- Effects of filaments’ amplitude
- The STORM thermal model
- Validation

Past

Present

Future

- What’s next?
- Plasma turbulence

April 2013
Plasma properties in the SOL

- Large fluctuations
- Fairly cold $T \lesssim 100$ eV
- Losses at the sheath
- Low frequencies $\omega \ll \Omega_{ci}$

[Walkden et al., NME (2017)]
A plasma model for the SOL

- High collisionality $\nu^* \gg 1$ → Braginskii equations

- Slow timescales, $\rho_s \ll L_\perp$ → Drift reduction
  
  $\mathbf{v}_i \approx U \mathbf{b} + \mathbf{v}_{E \times B} + \frac{\mathbf{b}}{\omega_{ci}} \times \frac{d}{dt} \mathbf{v}_{E \times B}$

- Slab geometry → Magnetic curvature and gradients added artificially

- Isothermal electrons → $T = 1$

- Boussinesq approximation →
  
  $\nabla \cdot \left( \frac{n}{d} \frac{d \nabla \perp \phi}{dt} \right) \approx n \frac{d \nabla \perp^2 \phi}{dt}$

- Electrostatic → $\mathbf{E} = - \nabla \phi$
A plasma model for the SOL

- Continuity equation
  \[ \frac{\partial n}{\partial t} = -v_{E \times B} \cdot \nabla n - \nabla || (nV) + g \left( \frac{\partial n}{\partial z} - n \frac{\partial \phi}{\partial z} \right) + \nabla \cdot (\mu_n \nabla n) + S_n \]

- Ohm’s law
  \[ \frac{\partial V}{\partial t} = -v_{E \times B} \cdot \nabla V - V \partial || V + \frac{m_i}{m_e} \left[ \partial || \phi + \nu || (U - V) - \frac{1}{n} \partial || n \right] - V \frac{S_n}{n} \]

- Parallel ion momentum equation
  \[ \frac{\partial U}{\partial t} = -v_{E \times B} \cdot \nabla U - U \partial || U - \partial || \phi - \nu || (U - V) - U \frac{S_n}{n} \]

- Poisson’s equation
  \[ \Omega = \nabla^2 \phi \]

- Bohm-Chodura boundary conditions
  \[ \left. U \right|_{\text{target}} \gtrless \pm 1, \left. V \right|_{\text{target}} = \pm \exp(\phi_{\text{wall}} - \phi) \]
The STORM module of BOUT++

- Implemented within BOUT++, solved with pvode/cvode
- Arakawa scheme for \( E \times B \) terms
- \( U \) and \( V \) staggered in \( y \)
- Upwind schemes for parallel advection terms
- Central finite difference schemes for other terms

Is the code bug free?
Code verification, order-of-accuracy test

Method of manufactured solutions
[Roache et al., AIAA J. (1984)]

1) Choose arbitrary function \( g \)
2) Define \( S = M(g) \)
3) Solve \( M_h(g_h) - S = 0 \)
4) Compute (see also BO)

STORM is verified!
The scrape-off layer (SOL) region

Roles of the SOL

• Heat exhaust

What are the heat and particle loads on vessel components?

How to reduce them?

➔ Need to understand transport mechanisms in the SOL

- Filaments (generation, motion, ...)
- Turbulence dynamics
Plasma background & seeded filaments

\[ S_n \propto \exp(\alpha y) \]

Are we capturing the filament dynamics correctly?
Validation against TORPEX experiment

TORPEX [Fasoli et al., PoP (2006)]

Provide
- Initial condition for simulations (density, …)
- Observables (radial velocity, …)

[Furno et al., PPCF (2011)]
Validation against TORPEX experiment

Satisfactory agreement

[Riva et al., PPCF (2016)]
2D closures

Used in the past to investigate plasma turbulence and filament dynamics [Krasheninnikov et al., JPP(2008)]

Sheath dissipation closure

\[ k_\parallel = 0 \]

\[ (nU)_{\text{target}} = \pm n_{\mid \text{midplane}} \]

\[ (nV)_{\text{target}} = \pm n \exp(\phi_{\text{wall}} - \phi)_{\mid \text{midplane}} \]

Vorticity advection closure

\[ U \nabla_\parallel = V \nabla_\parallel = \frac{1}{L_\parallel} \]

How do they compare to 3D models?
2D-3D comparison

Inertial regime

Sheath current regime

[Easy et al., PoP (2014)]
Effects of filaments’ amplitude

Implemented multigrid within BOUT++ \( \rightarrow \) Relaxed Boussinesq approximation (project coordinated by J.T. Omotani)

\[ \Omega = \nabla \cdot (n \nabla_{\perp} \phi) \]

\[ \frac{\partial \Omega}{\partial t} = -v_{E \times B} \cdot \nabla \Omega - U \partial_{\parallel} \Omega + \nabla_{\parallel} [n(U - V)] + g \frac{\partial n}{\partial z} + \nabla \cdot (\mu \Omega \nabla \Omega) - \frac{1}{2} [v_{E \times B}^2, n] \]

Inertial regime \( \propto \sqrt{A} \)

Sheath current regime \( \propto \frac{A}{1 + \beta A} \)

[Omotani et al., PPCF (2015)]
A plasma model for the SOL

- High collisionality $\nu^* \gg 1$ → Braginskii equations

- Slow timescales, $\rho_s \ll L_\perp$ → Drift reduction

$$\mathbf{v}_i = U \mathbf{b} + \mathbf{v}_{E \times B} + \frac{\mathbf{b}}{\omega_{ci}} \times \frac{d}{dt} \mathbf{v}_{E \times B}$$

- Slab geometry → Magnetic curvature and gradients added artificially

- Isothermal electrons → $T = 1$

- Boussinesq approximation

- Electrostatic → $\mathbf{E} = -\nabla \phi$
STORM: the workhorse of our projects

https://github.com/boutproject/STORM

April 2013

• The STORM isothermal model
• Verification and validation
• Comparison with 2D simulations
• Effects of filaments' amplitude

Past

• The STORM thermal model
• Validation

• What's next?
• Plasma turbulence

April 2013
Thermal electrons

Filaments may carry significant temperature perturbations

- Extension of isothermal model
  
  \[ g \frac{\partial n}{\partial z} \rightarrow g \frac{\partial (nT)}{\partial z}, \ldots \]

- Energy equation
  
  \[ \frac{3}{2} n \frac{\partial T}{\partial t} = -\nabla \parallel q \parallel + \ldots \]

- Boundary condition for heat flux
  
  \[ Q_\parallel \big|_{\text{target}} = \gamma (nT^{3/2}) \big|_{\text{target}} \]

  \[ \gamma \approx 2 - 0.5 \ln \left( 2\pi \frac{m_e}{m_i} \right) \]

  \[ q_\parallel = Q_\parallel - \frac{5}{2} nTV - \frac{1}{2} m_e nV^3 \]

More details in [Walkden et al., PPCF (2016)]
A plasma model for the SOL

- High collisionality $\nu^* \gg 1$ → Braginskii equations
- Slow timescales, $\rho_s \ll L_\perp$ → Drift reduction
- Slab geometry, magnetic curvature and gradients added
- Isothermal electrons
- Boussinesq approximation
- Electrostatic $E = -\nabla \phi$

Did we improve our modeling capabilities?
Validation against MAST

[Militello et al., PPCF (2016)]

Observable: filament motion

Initial conditions
Validation against MAST

Other studies performed with thermal model:

- Thermal effects on filament motion [Walkden et al., PPCF (2016)]
- Interaction between filaments [Militello et al., PPCF (2017)]
- Impact of background [Schwörer et al., NME (2017)]
STORM: the workhorse of our projects

https://github.com/boutproject/STORM

- The STORM isothermal model
- Verification and validation
- Comparison with 2D simulations
- Effects of filaments' amplitude

- The STORM thermal model
- Validation

- What’s next?
  - Plasma turbulence

April 2013
The scrape-off layer (SOL) region

Roles of the SOL

- Heat exhaust

What are the heat and particle loads on vessel components?

How to reduce them?

➔ Need to understand transport mechanisms in the SOL

- Filaments (generation, motion,....)
- Turbulence dynamics
Turbulence in s-α geometry

Evolve plasma equilibrium \(\Rightarrow\) flux driven \(\Rightarrow\) particle and energy sources

Turbulence typically characterized by \(k_r L_{eq} \gtrsim 1\) \(\Rightarrow\) global simulations

\[
R(r, \theta) = R_0 + r \cos(\theta)
\]

\[
Z(r, \theta) = -r \sin(\theta)
\]
MAST simulations

Core
Upper PF
Outer SOL

Lower PF
A plasma model for the SOL

- High collisionality $\nu^* \gg 1$ $\rightarrow$ Braginskii equations

- Slow timescales, $\rho_s \ll L_\perp$ $\rightarrow$ Drift reduction

$$\mathbf{v}_i = U \mathbf{b} + \mathbf{v}_{E \times B} + \frac{\mathbf{b}}{\omega_{ci}} \times \frac{d}{dt} \mathbf{v}_{E \times B}$$

- Slab geometry

- Isothermal electrons

- Boussinesq approximation

- Electrostatic $\rightarrow$ $\mathbf{E} = -\nabla \phi$
Other activities

- Filaments
  - Electromagnetic effects [Hoare et al., in preparation]
  - Filament separation at the separatrix
  - Magnetic shear effects
  - Neutrals (diffusive model)

- Turbulence
  - Divertor leg [Walkden et al., NME (submitted)]
  - 2D/3D comparison
  - Validation against MAST
A plasma model for the SOL

- High collisionality $\nu^* \gg 1$ → Braginskii equations
- Slow timescales, $\rho_s \ll L_\perp$ → Drift reduction

$$v_i = U b + v_{E\times B} + \frac{b}{\omega_{ci}} \times \frac{d}{dt} v_{E\times B}$$

- Slab geometry
- Isothermal electrons
- Boussinesq approximation
- Electrostatic
STORM: the workhorse of our projects

https://github.com/boutproject/STORM

• The STORM isothermal model
• Verification and validation
• Comparison with 2D simulations
• Effects of filaments’ amplitude

• The STORM thermal model
• Validation

Future

• Plasma turbulence
• What’s next?

April 2013
Top priorities for future development

Include

• thermal ions
• kinetic neutrals
• alternative magnetic configurations
• nonlocal effects

to understand

• first wall erosion
• heat flux at the target.

Numerical necessities:

• Compatibility between shifted grids and shifted metric
• 3D multigrid solver (PETSc)
[Easy et al., “Three dimensional simulations of plasma filaments in the scrape off layer: A comparison with models of reduced dimensionality”, PoP (2014)]


[Riva et al., “Blob dynamics in the TORPEX experiment: a multi-code validation”, PPCF (2016)]

[Easy et al., “Investigation of the effect of resistivity on scrape off layer filaments using three-dimensional simulations”, PoP (2016)]

[Walkden et al., “Dynamics of 3D isolated thermal filaments”, PPCF (2016)]

[Militello et al., “Multi-code analysis of scrape-off layer filament dynamics in MAST”, PPCF (2016)]

[Militello et al., “On the interaction of scrape off layer filaments”, PPCF (2017)]

[Schwörer et al., “Influence of plasma background including neutrals on scrape-off layer filaments using 3D simulations”, NME (2017)]
A plasma model for the SOL

- High collisionality $\nu^* \gg 1 \rightarrow$ Braginskii equations
  \[
  \frac{\partial n}{\partial t} + \nabla \cdot (n\mathbf{v}_\alpha) = 0, \ ...
  \]

- Slow timescales, $\rho_s \ll L_\perp \rightarrow$ Drift reduction
  \[
  \mathbf{v}_i = U\mathbf{b} + \mathbf{v}_{E \times B} + \frac{\mathbf{b}}{\omega_{ci}} \times \frac{d}{dt} \mathbf{v}_{E \times B} \quad \mathbf{v}_e = V\mathbf{b} + \mathbf{v}_{E \times B} + \mathbf{v}_{de}
  \]

- Slab geometry $\rightarrow$ Magnetic curvature and gradients added artificially
  \[
  \left( \nabla \times \frac{\mathbf{b}}{B} \right) \cdot \nabla A \approx g \frac{\partial A}{\partial z} \\
  g = 2 \frac{\rho_s 0}{R}
  \]

- Isothermal electrons $\rightarrow$ $T = 1$

- Boussinesq approximation $\rightarrow$ \[
  \nabla \cdot \left( n \frac{d \nabla \perp \phi}{dt} \right) \approx n \frac{d \nabla^2 \phi}{dt}
  \]

- Electrostatic $\rightarrow$ $\mathbf{E} = -\nabla \phi$
A plasma model for the SOL

- **Continuity equation**
  \[
  \frac{\partial n}{\partial t} = -\mathbf{v}_{E\times B} \cdot \nabla n - \nabla_\parallel (nV) + n \left( \frac{\partial n}{\partial z} - n \frac{\partial \phi}{\partial z} \right) + \nabla \cdot (\mu_n \nabla n) + S_n
  \]

- **Ohm’s law**
  \[
  \frac{\partial \Omega}{\partial t} = -\mathbf{v}_{E\times B} \cdot \nabla \Omega - U \partial_\parallel \Omega + \frac{1}{n} \nabla_\parallel \left[ n(U - V) \right] + \frac{g}{n} \frac{\partial n}{\partial z} + \nabla \cdot (\mu_\Omega \nabla \Omega)
  \]

- **Parallel ion momentum equation**
  \[
  \frac{\partial V}{\partial t} = -\mathbf{v}_{E\times B} \cdot \nabla V - V \partial_\parallel V + \frac{m_i}{m_e} \left[ \partial_\parallel \phi + \nu_\parallel (U - V) - \frac{1}{n} \partial_\parallel n \right] - V \frac{S_n}{n}
  \]

- **Parallel ion momentum equation**
  \[
  \frac{\partial U}{\partial t} = -\mathbf{v}_{E\times B} \cdot \nabla U - U \partial_\parallel U - \partial_\parallel \phi - \nu_\parallel (U - V) - U \frac{S_n}{n}
  \]

- **Poisson’s equation**
  \[
  \Omega = \nabla^2_\perp \phi
  \]
A plasma model for the SOL

Drift approximation breaks at magnetic pre-sheath entrance

Boundary conditions to be applied at magnetic pre-sheath entrance

Assuming $B \perp$ wall

$U \gg \pm 1$

$V = \pm \exp(\phi_{\text{wall}} - \phi)$

Direction $\perp$ wall

Debye sheath

Magnetic pre-sheath entrance
Verification of the parallel dynamics

The shock tube problem

- Central scheme introduces strong oscillations
- Overall good agreement for high resolution

[Easy, Ph.D. Thesis (2016)]
Our model: $M(f) = 0$, $f$ unknown

Solve $M_h(f_h) = 0$ for $f_h$, but $\epsilon_h = \|f - f_h\| = ?$

IS $\epsilon_h = \|f - f_h\| = n^2 + O(n^2)$? [Roache et al., AIAA J. (1984), Riva et al., PoP (2014)]

Method of manufactured solutions

1) Choose arbitrary function $g$, compute $S = M(g)$

2) Solve $M_h(g_h) - S = 0 \Rightarrow \epsilon_h = \|g - g_h\|$ [Easy, Ph.D. Thesis (2016)]

---

**Code verification, order-of-accuracy test**

---

[Diagram showing plots of $\epsilon_h$ vs $h$ for different functions, indicating order $O(h^2)$ behavior.]
Validation against TORPEX experiment

[Riva et al., PPCF (2016)]
Effects of filaments’ perpendicular size

\[-g \frac{\partial n}{n \partial z} = -\frac{d\Omega}{dt} + \frac{1}{n} \nabla_{\parallel} [n(U - V)] + \nabla \cdot (\mu \Omega \nabla \Omega)\]

- Diamagnetic and polarization currents negligible for large filaments
- More complex current path for small filaments

[Easy et al., PoP (2014)]
Effects of filaments’ parallel extent

- Connected filaments display faster radial propagation
- Filaments spin if not connected with the sheath

[Easy et al., PoP (2014)]
Effects of filaments’ shape

Inertial regime $\propto \sqrt{\delta_x}$

Sheath current regime $\propto \delta_z^{-2}$

[Omotani et al., PPCF (2015)]
Effects of plasma resistivity

\[ \nu_{div} = 10000 \nu_{0} \]

\[ \nu_{\parallel} \] disconnects filaments from the target, shifting the transition between sheath current and inertial regime

Two-region model
[Myra et al., PoP (2006)]
Thermal effects

If $\frac{\delta T}{T_{bg}} \gg \frac{\delta n}{n_{bg}}$:

- Increased propagation in binormal direction
- Reduced propagation in radial direction
- Faster parallel pressure losses

[Walkden et al., PPCF (2016)]
Filament interaction

Filaments separated by more than 5 widths
⇒ like independent filaments

[Militello et al., PPCF (2017)]
Effects of plasma background

- Radial velocity decreases by increasing $n_0$
- Radial velocity increases by increasing $T_0$

[Schwörer et al., NME (2017)]
Electromagnetic effects

Do filaments affect field lines?

Do filaments behave differently when including electromagnetic effects?

\[ \Rightarrow \text{ include electromagnetic effects in STORM} \]

Semi-electrostatic limit:

\[ E_{\perp} = -\nabla_{\perp} \phi \]

\[ E_{\parallel} = -\nabla_{\parallel} \phi - \beta \frac{\partial \psi}{2 \partial t} \]

Code modifications:

\[ \frac{\partial U}{\partial t} + \frac{\beta}{2} \frac{\partial \psi}{\partial t} = \cdots \]

[Hoare et al., PPCF (submitted)]
Filament separation

Understand how filaments cross the separatrix
Turbulent mixing in divertor legs

Understand how turbulence spreads heat and particles in the divertor leg

[Walkden et al., PSI (2018)]
Simulating plasma turbulence

Source of particles and heat

• Evolve plasma equilibrium flux driven

• Seed perturbation ⇒ start from random noise

• Rewrite equations to ensure $n, T > 0$

\[
\frac{\partial n}{\partial t} = f_n(t, n, T, \ldots) \rightarrow \frac{\partial \log(n)}{\partial t} = \frac{f_n(t, n, T, \ldots)}{n}, \quad \frac{\partial T}{\partial t} = f_T(n, t) \rightarrow \frac{\partial \log(T)}{\partial t} = \frac{f_T(t, n, T, \ldots)}{T}
\]