Gyro-Landau fluid extension in BOUT++ framework

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2018 BOUT++ workshop, Livermore, CA

August 16, 2018

This work was performed under the auspices of the U.S. Deportment of Energy by Lawrence Livermore National Laboratory under Contract DE-AC52-07NA27344 and LLNL-LDRD project 12-ERD-022. Some materials are reused from Ma's 2015 presentation (LLNL-PRES-680185).

LLNL-PRES-756299

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    - Ion Temperature Gradient (ITG) mode
    - Kinetic Ballooning Mode (KBM) by

Fluid, Kinetic and Gyro-fluid

- Fluid model at edge
  - Braginskii model (strongly-collisional, magnetized plasma)

    collisional criterion $\nu \gg \tilde{v}_E/L \sim 10^5 s^{-1}$ (and $\lambda_{mfp} \ll L_{\|}$)

	Top of pedestal				SOL			
	T/eV	n $/10^{19}m^{-3}$	ν_{ee}/s^{-1}	ν_{ii}/s^{-1}	T/eV	n $/10^{19}m^{-3}$	ν_{ee}/s^{-1}	ν_{ii}/s^{-1}
DIII-D	800	5	3.3e5	5.9e3	50	1	4.0e6	6.0e4
C-Mod	600	30	2.8e6	5.0e4	50	10	3.3e7	5.5e5
EAST	500	3	4.0e5	7.0e3	50	0.8	2.9e6	4.9e4
ITER	4000	8	5.3e4	9.5e2	200	3	1.5e6	2.5e4

justified | marginal | violation

  - Anisotropy, trapped particles, Landau damping, FLR effects become important for weakly-collisional plasma.

Fluid, Kinetic and Gyro-fluid (cont.)

- Kinetic/gyro-kinetic model
  - Extremely expansive computationally
    - for both particle and distribution function approaches
    - vast span of spatial and temporal scale depends on interested problems
  - Complicate collisional operator

- Gyro-fluid model
  - A compromise of fluid and kinetic model
    - contains essential kinetic physics
    - is numerical feasible
  - Derived from moment expansion of gyro-kinetic equation, closed via various closures
  - BOUT++ implements the δf GF formation by Snyder and Hammett

Snyder and Hammett, PoP, 8, 3199 (2001)

GLF model in BOUT++

- For ions, retain FLR effects – 4 equations (3+1 model)

$$\frac{\partial \tilde{n}_i}{\partial t} = -\frac{b_0}{B_0}\times\nabla\Phi\cdot\nabla\tilde{n}_i - \frac{\mathbf{b}_0}{eB_0}\times\kappa\cdot\nabla\left(\tilde{p}_{\parallel i}+\tilde{p}_{\perp i}\right) - \frac{n_i\mathbf{b}_0}{B_0}\times\kappa\cdot\nabla\left(2+\frac{1}{2}\hat{\nabla}_\perp^2\right)\Phi$$
$$-n_0B_0\nabla_\parallel\frac{\tilde{u}_{\parallel i}}{B_0} - \frac{n_0\mathbf{b}_0}{2T_0B_0}\times\nabla\left(\hat{\nabla}_\perp^2\Phi\right)\cdot\nabla\tilde{T}_{\perp i}$$

$$\frac{\partial \tilde{u}_{\parallel i}}{\partial t} = -\frac{\mathbf{b}_0}{B_0}\times\nabla\Phi\cdot\nabla\tilde{u}_{\parallel i} - \frac{B_0}{n_0m_i}\nabla_\parallel\frac{\tilde{p}_\parallel}{B} - \frac{4\mathbf{b}_0}{eB_0}\times\kappa\cdot\nabla T_{i0}\tilde{u}_{\parallel i} - \left(\frac{\tilde{p}_{\perp i}}{n_0m_i}+\frac{e}{2m_i}\hat{\nabla}_\perp^2\Phi\right)\nabla\ln B_0 - \frac{i\omega_d}{p_{i0}}\left(\tilde{q}_{\parallel i}+\tilde{q}_{\perp i}\right)$$

$$\frac{\partial \tilde{p}_{\parallel i}}{\partial t} = -\frac{\mathbf{b}_0}{B_0}\times\nabla\Phi\cdot\nabla\tilde{p}_{\parallel i} - \frac{n_0\mathbf{b}_0}{2B_0}\times\nabla\left(\hat{\nabla}_\perp^2\Phi\right)\cdot\nabla\tilde{T}_{\perp i} - \frac{p_{\parallel i}\mathbf{b}_0}{B_0}\times\kappa\cdot\nabla\left(4+\frac{1}{2}\hat{\nabla}_\perp^2\right)\Phi$$
$$-3B_0\nabla_\parallel\frac{p_{i0}\tilde{u}_{\parallel i}}{B_0} - i\omega_d\left(\tilde{r}_{\parallel,\parallel}+\tilde{r}_{\parallel,\perp}\right) - B_0\nabla_{\parallel 0}\frac{\tilde{q}_{\parallel i}}{B_0}$$

$$\frac{\partial \tilde{p}_{\perp i}}{\partial t} = -\frac{\mathbf{b}_0}{B_0}\times\nabla\Phi_2\cdot\nabla\tilde{p}_{\perp i} - \frac{n_0\mathbf{b}_0}{B_0}\times\hat{\hat{\nabla}}_\perp^2\Phi\cdot\tilde{T}_{\perp i} - \frac{p_{\perp i}\mathbf{b}_0}{B_0}\times\kappa\cdot\nabla\left(3+\frac{3}{2}\hat{\nabla}_\perp^2+\hat{\hat{\nabla}}_\perp^2\right)\Phi$$
$$-B_0^2\nabla_\parallel\frac{p_{\perp i}\tilde{u}_{\parallel i}}{B_0^2} - i\omega_d\left(\tilde{r}_{\parallel,\perp}+\tilde{r}_{\perp,\perp}\right) - B_0^2\nabla_{\parallel 0}\frac{\tilde{q}_{\perp i}}{B_0^2}$$

with gyro-averaged electric potential $\Phi=\Gamma_0^{1/2}\phi$ and $\phi_f=\Phi-\phi$. Here $\Gamma_0^{1/2}\approx<J_0^2>^{1/2}\approx<J_0>$ and the modified Laplacian operators are $\hat{\nabla}_\perp^2\Phi=2b\frac{\partial\Gamma_0^{1/2}}{\partial b}\phi$, $\hat{\hat{\nabla}}_\perp^2\Phi=b\frac{\partial^2}{\partial b^2}b\Gamma_0^{1/2}\phi$ with argument $b=k_\perp^2\rho_i^2=-\rho_i^2\nabla_\perp^2$.

Ma and Xu, NF, 57, 016002 (2017)

GLF model in BOUT++

- For electrons, neglecting FLR effects and use isotropic assumption→ reduced equations

$$\frac{\partial \tilde{\omega}_G}{\partial t} = -\frac{\mathbf{b}_0}{B_0}\times\nabla\phi\cdot\nabla\tilde{\omega}_G + B_0\nabla_\parallel\frac{\tilde{J}_\parallel}{B_0} + \mathbf{b}_0\times\kappa\cdot\nabla\left(\tilde{p}_{\parallel i}+\tilde{p}_{\perp i}+\tilde{p}_{\parallel e}+\tilde{p}_{\perp e}\right) + eB_0\mathbf{b}_0\times\nabla\phi_f\cdot\nabla\tilde{n}_i$$
$$+\frac{eB_0 n_0\mathbf{b}_0}{2T_{i0}}\times\nabla\left(\hat{\tilde{\nabla}}_\perp^2\Phi\right)\cdot\nabla\tilde{T}_{\perp i} - eB^2\mathbf{b}_0\times\nabla A_f\cdot\nabla\frac{n_0 u_{\parallel i}}{B_0} + en_0\mathbf{b}_0\times\kappa\cdot\nabla\left(2\phi_f+\hat{\tilde{\nabla}}_\perp^2\Phi\right)$$
$$\frac{\partial A_\parallel}{\partial t} = -\nabla_\parallel\phi + \frac{B_0}{n_0 e}\nabla_\parallel\frac{\tilde{p}_{\parallel e}}{B_0} + \frac{\eta}{\mu_0}\nabla_\perp^2 A_\parallel - \frac{\eta_H}{\mu_0}\nabla_\perp^4 A_\parallel$$
$$\frac{\partial \tilde{T}_e}{\partial t} = -\frac{\mathbf{b}_0}{B_0}\times\nabla\phi\cdot\nabla\tilde{T}_e - \frac{2}{3}T_e\left[\left(\frac{2\mathbf{b}_0}{B_0}\times\kappa\right)\cdot\left(\nabla\phi - \frac{1}{en_{e0}}\nabla\tilde{p}_e - \frac{5}{2e}\nabla\tilde{T}_e\right) + B_0\nabla_\parallel\frac{\tilde{u}_{\parallel e}}{B_0}\right] + \frac{2}{3n_{i0}}\nabla_{\parallel 0}\tilde{q}_{\parallel e}$$

with vorticity and perturbed current defined as

$$\tilde{\omega}_G = eB\left[\bar{n}_i - \tilde{n}_i - n_0\left(1-\Gamma_0\right)\frac{e\phi}{T_0} + \frac{e\rho_i^2}{T_0}\nabla n_0\cdot\nabla\left(\Gamma_0-\Gamma_1\right)\phi\right],$$
$$\tilde{J}_\parallel = -\frac{1}{\mu_0}\nabla_\perp^2 A_\parallel = en_0\left(\bar{u}_{\parallel i} - \tilde{u}_{\parallel e}\right)$$

and gyro-averaged ion density and parallel velocity

$$\bar{n}_i = \Gamma_0^{1/2}\tilde{n}_i + \frac{n_0}{T_0}b\frac{\partial}{\partial b}\Gamma_0^{1/2}\tilde{T}_{\perp i}, \quad \bar{u}_{\parallel i} = \Gamma_0^{1/2}u_{\parallel i}.$$

Ma and Xu, NF, 57, 016002 (2017)

GLF Closures

- Landau closure

$$\tilde{q}_{\|i,e}=-n_0\sqrt{\frac{8}{\pi}}v_{thi,e}\frac{ik_\|\tilde{T}_{\|i,e}}{|k_\||+1/(2\lambda_{i,e}^{mfp})},\quad \tilde{q}_{\perp i}=-n_0\sqrt{\frac{2}{\pi}}v_{thi}\frac{ik_\|}{|k_\||+1/(2\lambda_i^{mfp})}\left(\tilde{T}_{\perp i}+\frac{e}{2}\hat{\nabla}_\perp^2\Phi\right)$$

Landau damping

collision

Hammett and Perkins, PRL, 64, 3019 (1990) Umansky et.al, JNM, 463, 506 (2015)

- Toroidal closure

$$i\omega_d\left(\tilde{r}_{\|,\|}+\tilde{r}_{\|,\perp}\right)=i\omega_d\left[7\tilde{p}_\|+\tilde{p}_\perp-4T_0\tilde{n}-2i\frac{|\omega_d|}{\omega}\left(\nu_1\tilde{T}_\|+\nu_2\tilde{T}_\perp\right)\right]$$

$$i\omega_d\left(\tilde{r}_{\|,\perp}+\tilde{r}_{\perp,\perp}\right)=i\omega_d\left[\tilde{p}_\|+5\tilde{p}_\perp-3T_0\tilde{n}-2i\frac{|\omega_d|}{\omega}\left(\nu_3\tilde{T}_\|+\nu_4\tilde{T}_\perp\right)\right]$$

$$i\omega_d\left(\tilde{q}_{\|i}+\tilde{q}_{\perp i}\right)=2n_0T_0\nu_5|\omega_d|\tilde{u}_{\|i}$$

where $\nu=\nu_r+i\nu_i\frac{|\omega_d|}{\omega_d} \implies \begin{cases}\nu_1=(1.232,0.437)\\ \nu_2=(-0.912,0.362)\\ \nu_3=(-1.164,0.294)\\ \nu_4=(0.478,-1.926)\\ \nu_5=(0.515,-0.958)\end{cases}$

Beer and Hammett, PoP, 3, 4046 (1996)

Additional Physics in GLF

- FLR effects (on ions)
  - $\rho_i(\rho_s)$ physics – FLR effects correct dispersion and linear growth rate of gradient driven instabilities
- Landau damping
  - Weakly-collisonal limit
  - Important to drift-type instability
- Ion anisotropy (optional for electrons)
  - $T_{\|i} \neq T_{\perp i}$, no trapped particle yet
- Toroidal drift resonance

Back to GLF Equations

Ma and Xu, NF, 57, 016002 (2017)

- All these physics are embedded in GLF equations:
  - First order FLR effects are retained via gyro-average operator $\Gamma_0^{1/2}$;
  - Landau damping is included in $\tilde{q}_{\parallel,\perp i}$;
  - Ions have anisotropic $\tilde{p}_{\parallel i}$ and $\tilde{p}_{\perp i}$;
  - Toroidal drift resonance is included in $\tilde{r}_{\parallel,\perp}$ and so on.

FLR
LD
A
TDR

$$\frac{\partial \tilde{n}_i}{\partial t} = -\frac{b_0}{B_0}\times\nabla\Phi\cdot\nabla\tilde{n}_i - \frac{\mathbf{b}_0}{eB_0}\times\kappa\cdot\nabla\left(\tilde{p}_{\parallel i}+\tilde{p}_{\perp i}\right) - \frac{n_i\mathbf{b}_0}{B_0}\times\kappa\cdot\nabla\left(2+\frac{1}{2}\hat{\nabla}_\perp^2\right)\Phi$$
$$-n_0B_0\nabla_\parallel\frac{\tilde{u}_{\parallel i}}{B_0} - \frac{n_0\mathbf{b}_0}{2T_0B_0}\times\nabla\left(\hat{\nabla}_\perp^2\Phi\right)\cdot\nabla\tilde{T}_{\perp i}$$

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$$\frac{\partial \tilde{p}_{\parallel i}}{\partial t} = -\frac{\mathbf{b}_0}{B_0}\times\nabla\Phi\cdot\nabla\tilde{p}_{\parallel i} - \frac{n_0\mathbf{b}_0}{2B_0}\times\nabla\left(\hat{\nabla}_\perp^2\Phi\right)\cdot\nabla\tilde{T}_{\perp i} - \frac{p_{\parallel i}\mathbf{b}_0}{B_0}\times\kappa\cdot\nabla\left(4+\frac{1}{2}\hat{\nabla}_\perp^2\right)\Phi$$
$$-3B_0\nabla_\parallel\frac{p_{i0}\tilde{u}_{\parallel i}}{B_0} - i\omega_d\left(\tilde{r}_{\parallel,\parallel}+\tilde{r}_{\parallel,\perp}\right) - B_0\nabla_{\parallel 0}\frac{\tilde{q}_{\parallel i}}{B_0}$$

$$\frac{\partial \tilde{p}_{\perp i}}{\partial t} = -\frac{\mathbf{b}_0}{B_0}\times\nabla\Phi_2\cdot\nabla\tilde{p}_{\perp i} - \frac{n_0\mathbf{b}_0}{B_0}\times\hat{\hat{\nabla}}_\perp^2\Phi\cdot\tilde{T}_{\perp i} - \frac{p_{\perp i}\mathbf{b}_0}{B_0}\times\kappa\cdot\nabla\left(3+\frac{3}{2}\hat{\nabla}_\perp^2+\hat{\hat{\nabla}}_\perp^2\right)\Phi$$
$$-B_0^2\nabla_\parallel\frac{p_{\perp i}\tilde{u}_{\parallel i}}{B_0^2} - i\omega_d\left(\tilde{r}_{\parallel,\perp}+\tilde{r}_{\perp,\perp}\right) - B_0^2\nabla_{\parallel 0}\frac{\tilde{q}_{\perp i}}{B_0^2}$$

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Ma and Xu, NF, 57, 016002 (2017)

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  - First order FLR effects are retained via gyro-average operator $\Gamma_0^{1/2}$;
  - Landau damping is included in $\tilde{q}_{\parallel,\perp i}$;
  - Ions have anisotropic $\tilde{p}_{\parallel i}$ and $\tilde{p}_{\perp i}$;
  - Toroidal drift resonance is included in $\tilde{r}_{\parallel,\perp}$ and so on.

FLR
LD
A
TDR

$$\frac{\partial \tilde{\omega}_G}{\partial t} = -\frac{\mathbf{b}_0}{B_0}\times\nabla\phi\cdot\nabla\tilde{\omega}_G + B_0\nabla_\parallel\frac{\tilde{J}_\parallel}{B_0} + \mathbf{b}_0\times\kappa\cdot\nabla\left(\tilde{p}_{\parallel i}+\tilde{p}_{\perp i}+\tilde{p}_{\parallel e}+\tilde{p}_{\perp e}\right) + eB_0\mathbf{b}_0\times\nabla\phi_f\cdot\nabla\tilde{n}_i$$

$$+\frac{eB_0 n_0\mathbf{b}_0}{2T_{i0}}\times\nabla\left(\hat{\tilde{\nabla}}_\perp^2\Phi\right)\cdot\nabla\tilde{T}_{\perp i} - eB^2\mathbf{b}_0\times\nabla A_f\cdot\nabla\frac{n_0 u_{\parallel i}}{B_0} + en_0\mathbf{b}_0\times\kappa\cdot\nabla\left(2\phi_f+\hat{\tilde{\nabla}}_\perp^2\Phi\right)$$

$$\frac{\partial A_\parallel}{\partial t} = -\nabla_\parallel\phi + \frac{B_0}{n_0 e}\nabla_\parallel\frac{\tilde{p}_{\parallel e}}{B_0} + \frac{\eta}{\mu_0}\nabla_\perp^2 A_\parallel - \frac{\eta_H}{\mu_0}\nabla_\perp^4 A_\parallel$$

$$\frac{\partial \tilde{T}_e}{\partial t} = -\frac{\mathbf{b}_0}{B_0}\times\nabla\phi\cdot\nabla\tilde{T}_e - \frac{2}{3}T_e\left[\left(\frac{2\mathbf{b}_0}{B_0}\times\kappa\right)\cdot\left(\nabla\phi - \frac{1}{en_{e0}}\nabla\tilde{p}_e - \frac{5}{2e}\nabla\tilde{T}_e\right) + B_0\nabla_\parallel\frac{\tilde{u}_{\parallel e}}{B_0}\right] + \frac{2}{3n_{i0}}\nabla_{\parallel 0}\tilde{q}_{\parallel e}$$

Similarly, for vorticity, perturbed current, gyro-averaged ion density and parallel velocity:

$$\tilde{\omega}_G = eB\left[\bar{n}_i - \tilde{n}_i - n_0\left(1-\Gamma_0\right)\frac{e\phi}{T_0} + \frac{e\rho_i^2}{T_0}\nabla n_0\cdot\nabla\left(\Gamma_0-\Gamma_1\right)\phi\right],$$

$$\tilde{J}_\parallel = -\frac{1}{\mu_0}\nabla_\perp^2 A_\parallel = en_0\left(\bar{u}_{\parallel i} - \tilde{u}_{\parallel e}\right), \quad \bar{n}_i = \Gamma_0^{1/2}\tilde{n}_i + \frac{n_0}{T_0}b\frac{\partial}{\partial b}\Gamma_0^{1/2}\tilde{T}_{\perp i}, \quad \bar{u}_{\parallel i} = \Gamma_0^{1/2}u_{\parallel i}.$$

Numerical implementation

- Most operators are already exist in BOUT++ framework

- Padé approximation for gyro-averaging operator

$$\Gamma_0^{1/2} = \frac{1}{1+b/2}, \quad \Gamma_0 = \frac{1}{1+b}, \quad \Gamma_1 - \Gamma_0 = 1.$$

As a result, $\hat{\nabla}_\perp^2 \Phi = 2(\Phi_2 - \Phi)$, $\hat{\hat{\nabla}}_\perp^2 \Phi = 2(\Phi_3 - \Phi_2)$ where $\Phi_2 = \Gamma_0^{1/2}\Phi$, $\Phi_3 = \Gamma_0^{1/2}\Phi_2$ and $\phi = (1 - \frac{1}{2}\rho_i^2 \nabla_\perp^2)\Phi$.

- Fast non-Fourier method for Landau closure Dimits et.al., PoP, 21, 055907 (2014)

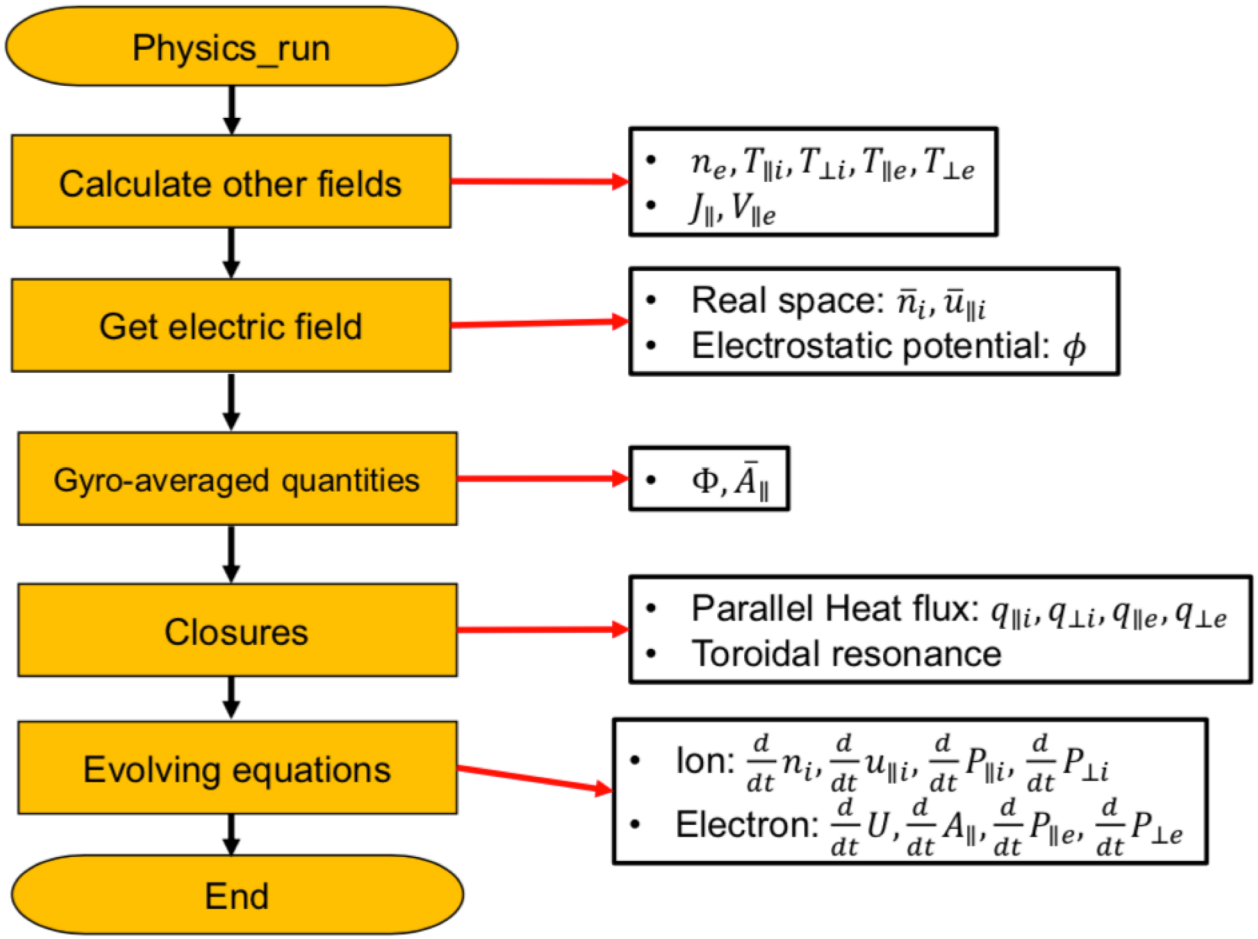
- Solve vorticity for electrostatic potential

$$\tilde{\omega}_G = eB\left[\bar{n}_i - \tilde{n}_i - n_0\left(1-\Gamma_0\right)\frac{e\phi}{T_0} + \frac{e\rho_i^2}{T_0}\nabla n_0 \cdot \nabla\left(\Gamma_0 - \Gamma_1\right)\phi\right]$$

is approximated by

$$\nabla_\perp^2 \phi - \frac{1}{n_0}\nabla n_0 \cdot \nabla\phi = \frac{-T_0}{e\rho_i^2}\left(1 - \rho_i^2 \nabla_\perp^2\right)\left(\bar{n}_i - \tilde{n}_i - \frac{\tilde{\omega}_G}{eB}\right).$$

Code flowchart



Hands-on Exercise

- Installation

- Two examples:
  - electrostatic ITG – credit to S. S. Kim
  - electromagnetic KBM – credit to C. H. Ma

Installation

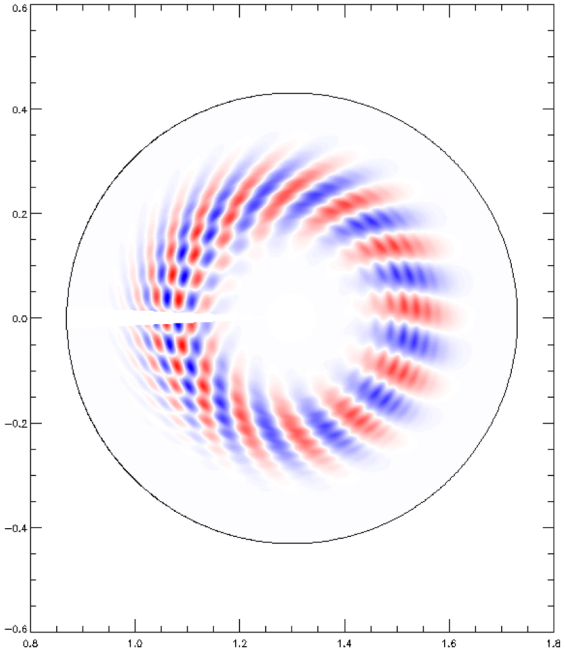
- login Edison
  - % *ssh –Y trainXX@edison.nersc.gov*
  - % *module load git*
- Get the bout_glf git repository:
  - % *git clone ssh://user@portalauth.nersc.gov/ project/projectdirs/bout_glf/www/git/bout_glf.git*
- Switch to the modomegad branch:
  - % *git checkout bout_modomegad*
- • Compile the code (on edison):
  - % *./configure.Edison*
  - % *make*
- Run examples
  - % *cd examples/glfkbm3–1 (or, examples/glf_itg3+1)*
  - % *make*
  - % *sbatch jobscription_edison*

Electrostatic, collisonless ITG

- For electrostatic, collisonless ITG case, we assume electrons are adiabatic

$$
\begin{aligned}
\frac{\partial \tilde{n}_i}{\partial t} = & -\frac{b_0}{B_0}\times\nabla\Phi\cdot\nabla\tilde{n}_i - \frac{\mathbf{b}_0}{eB_0}\times\kappa\cdot\nabla\left(\tilde{p}_{\parallel i}+\tilde{p}_{\perp i}\right) - \frac{n_i\mathbf{b}_0}{B_0}\times\kappa\cdot\nabla\left(2+\frac{1}{2}\hat{\nabla}_\perp^2\right)\Phi \\
& -n_0B_0\nabla_\parallel\frac{\tilde{u}_{\parallel i}}{B_0} - \frac{n_0\mathbf{b}_0}{2T_0B_0}\times\nabla\left(\hat{\nabla}_\perp^2\Phi\right)\cdot\nabla\tilde{T}_{\perp i} \\
\frac{\partial \tilde{u}_{\parallel i}}{\partial t} = & -\frac{\mathbf{b}_0}{B_0}\times\nabla\Phi\cdot\nabla\tilde{u}_{\parallel i} - \frac{B_0}{n_0m_i}\nabla_\parallel\frac{\tilde{p}_\parallel}{B} - \frac{4\mathbf{b}_0}{eB_0}\times\kappa\cdot\nabla T_{i0}\tilde{u}_{\parallel i} \\
& -\left(\frac{\tilde{p}_{\perp i}}{n_0m_i}+\frac{e}{2m_i}\hat{\nabla}_\perp^2\Phi\right)\nabla\ln B_0 - \frac{i\omega_d}{p_{i0}}\left(\tilde{q}_{\parallel i}+\tilde{q}_{\perp i}\right) \\
\frac{\partial \tilde{p}_{\parallel i}}{\partial t} = & -\frac{\mathbf{b}_0}{B_0}\times\nabla\Phi\cdot\nabla\tilde{p}_{\parallel i} - \frac{n_0\mathbf{b}_0}{2B_0}\times\nabla\left(\hat{\nabla}_\perp^2\Phi\right)\cdot\nabla\tilde{T}_{\perp i} - \frac{p_{\parallel i}\mathbf{b}_0}{B_0}\times\kappa\cdot\nabla\left(4+\frac{1}{2}\hat{\nabla}_\perp^2\right)\Phi \\
& -3B_0\nabla_\parallel\frac{p_{i0}\tilde{u}_{\parallel i}}{B_0} - i\omega_d\left(\tilde{r}_{\parallel,\parallel}+\tilde{r}_{\parallel,\perp}\right) - B_0\nabla_{\parallel 0}\frac{\tilde{q}_{\parallel i}}{B_0} \\
\frac{\partial \tilde{p}_{\perp i}}{\partial t} = & -\frac{\mathbf{b}_0}{B_0}\times\nabla\Phi_2\cdot\nabla\tilde{p}_{\perp i} - \frac{n_0\mathbf{b}_0}{B_0}\times\hat{\hat{\nabla}}_\perp^2\Phi\cdot\tilde{T}_{\perp i} - \frac{p_{\perp i}\mathbf{b}_0}{B_0}\times\kappa\cdot\nabla\left(3+\frac{3}{2}\hat{\nabla}_\perp^2+\hat{\hat{\nabla}}_\perp^2\right)\Phi \\
& -B_0^2\nabla_\parallel\frac{p_{\perp i}\tilde{u}_{\parallel i}}{B_0^2} - i\omega_d\left(\tilde{r}_{\parallel,\perp}+\tilde{r}_{\perp,\perp}\right) - B_0^2\nabla_{\parallel 0}\frac{\tilde{q}_{\perp i}}{B_0^2} \\
\bar{n}_i = & \frac{n_0\phi}{T_{e0}} + \frac{n_0}{T_{i0}}\left(1-\Gamma_0\right)\phi.
\end{aligned}
$$

Electrostatic, collisonless ITG (cont)



- Run examples
  - % *cd examples/glf_itg3+1*
  - % *make*
  - % *sbatch jobscription_edison*

- Analysis data
  - plot poloidal snapshot
    - open IDL
      - % *IDL*
    - Load data
      - *IDL> phi=collect(var="phi")*
    - Load grid information
      - *IDL> g=file_import("./data/cyclone_260x64.nc")*
    - Plot poloidal snapshot at t=50
      - *IDL> plotpolslice,phi[*,*,*,49],g,period=10,zangle=0*

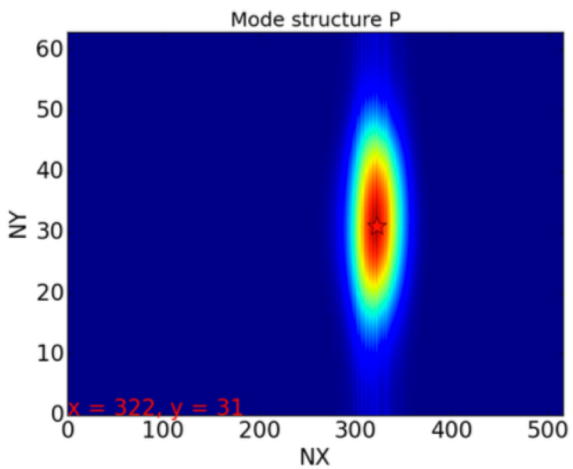
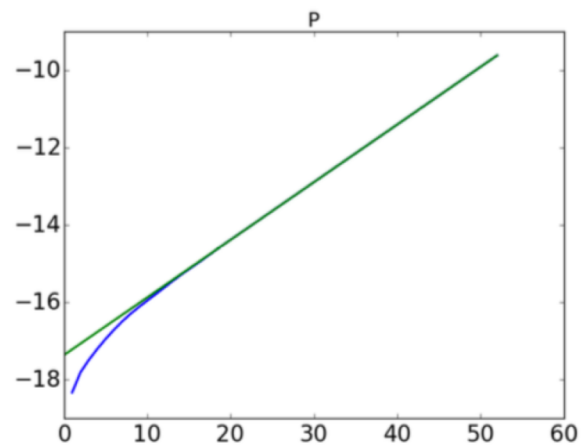
  - or, get the linear growth rate (see next page)

Electromagnetic KBM

- Run examples
  - % *cd examples/glfkbm3-1*
  - % *make*
  - % *sbatch jobscription_edison*

- Analysis data
  - get the linear growth rate

```
python growthrate.py -v -f P data
```

Example output:

```
Growth rate from linear fit from -20 to -1 is: 0.1491077
```