Implementation and Testing of a Model for SOL Turbulence and RF Effects

M.V. Umansky[1]

Acknowledgments: A.M. Dimits[1], I.Joseph[1], J.R. Myra[2], T.D. Rognlien[1], D.N. Smithe[2]
[1]LLNL, [2]Lodestar Inc., [3]Tech-X Corporation

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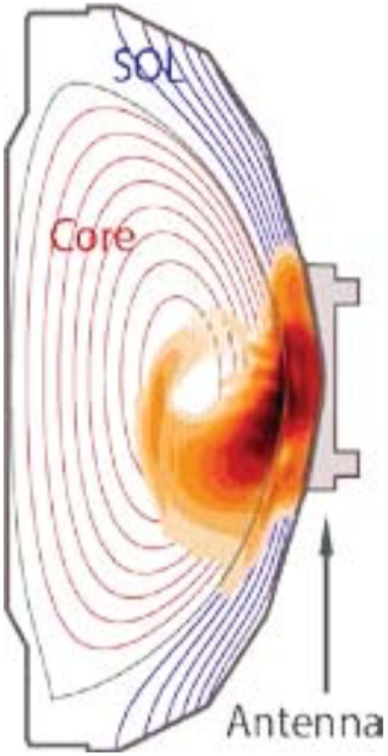
Interaction of boundary plasma turbulence with RF waves presents a challenging and important issue in tokamak edge physics

- **Heating and driving current in tokamak plasma by intense RF waves is critically important for for existing tokamaks and future fusion reactors**

- **Edge plasma affects penetration of RF waves into plasma (scattering, absorption**

- **RF waves have effect on edge plasma**
  - **RF sheath**
  - **Ponderomotive forces**



As a part of our ongoing RF SciDAC project, we are developing a BOUT++ based model for edge plasma turbulence, including RF effects

For existing tokamaks, electrostatic fluid models should be reasonable for SOL and divertor

	C-Mod	NSTX	DIII-D
a_{min} [m]	0.3	0.8	0.7
R_{maj} [m]	0.9	1.5	2.3
B_t [T]	4.0	0.3	1.5
B_p [T]	0.7	0.2	0.3
$N_{i,sepx}$ [m^{-3}]	3e19	6e18	1e19
$T_{e,sepx}$ [eV]	30	20	100
L_c/λ_{ei}	**4.**	**1.0**	**0.2**
$k_\perp \rho_i$	**0.1**	**0.9**	**0.5**
β	**0.2e-4**	**5e-4**	**2e-4**

- **For existing tokamaks, SOL is relatively collisional, low beta, small gyro-radius => fluid, electrostatic models should work well there**
- **For future tokamaks (ITER, DEMO, etc.) probably less collisional SOL**

What physics is important for tokamak SOL turbulence?

1. **Drift-wave instability** (known to be important for SOL-like parameters in laboratory plasma experiments)

2. **Resistive-ballooning instability** (curvature drive is important for tokamak in general, even if SOL plasmas are ideally stable)

3. **Sheath-driven instability** (predicted theoretically to play a role)

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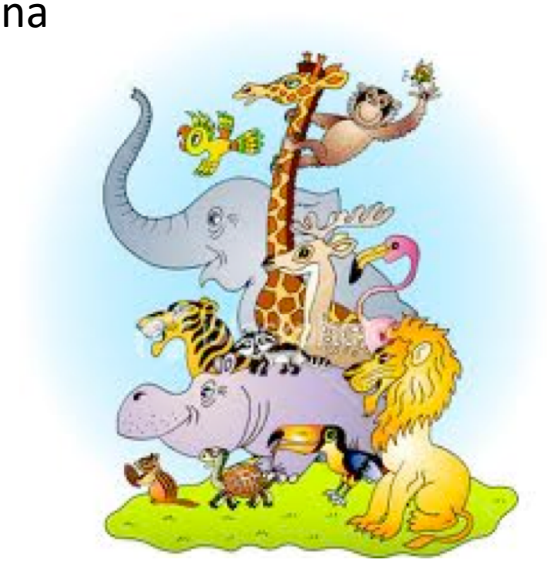
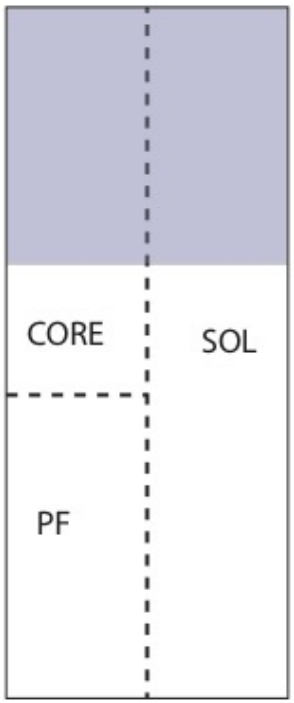
Beyond that, potentially a whole zoo of physics:

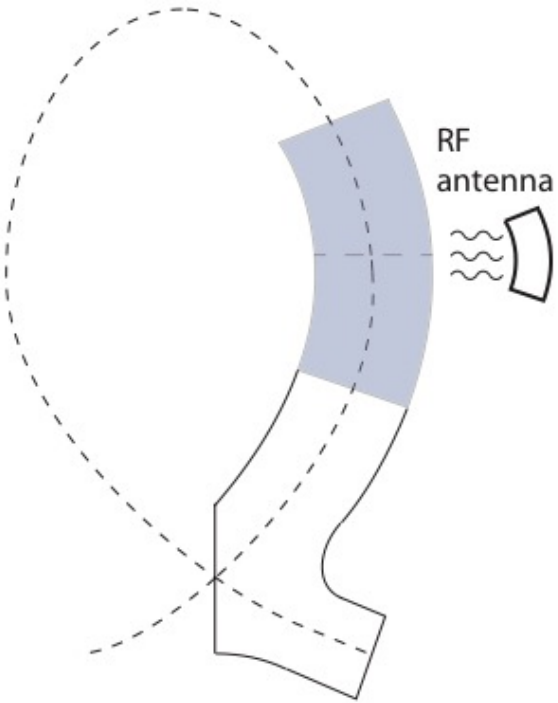
- various kinetic phenomena
- neoclassical orbits
- magnetic stochasticity
- neutrals
- impurities
- radiation
- sputtering
- materials
- etc.



Envisioning a hierarchy of models, increasing complexity of physics and geometry, for a plasma + RF and/or divertor model

- **Initially simplifying the geometry in favor of focusing on physics model**
- **Gradually we'll increase geometric complexity by adding realistic shaping and branch-cuts**
- **Eventually use full edge domain**
- **Avoiding known difficulties with including zonal flows in edge turbulence model in full tokamak geometry**



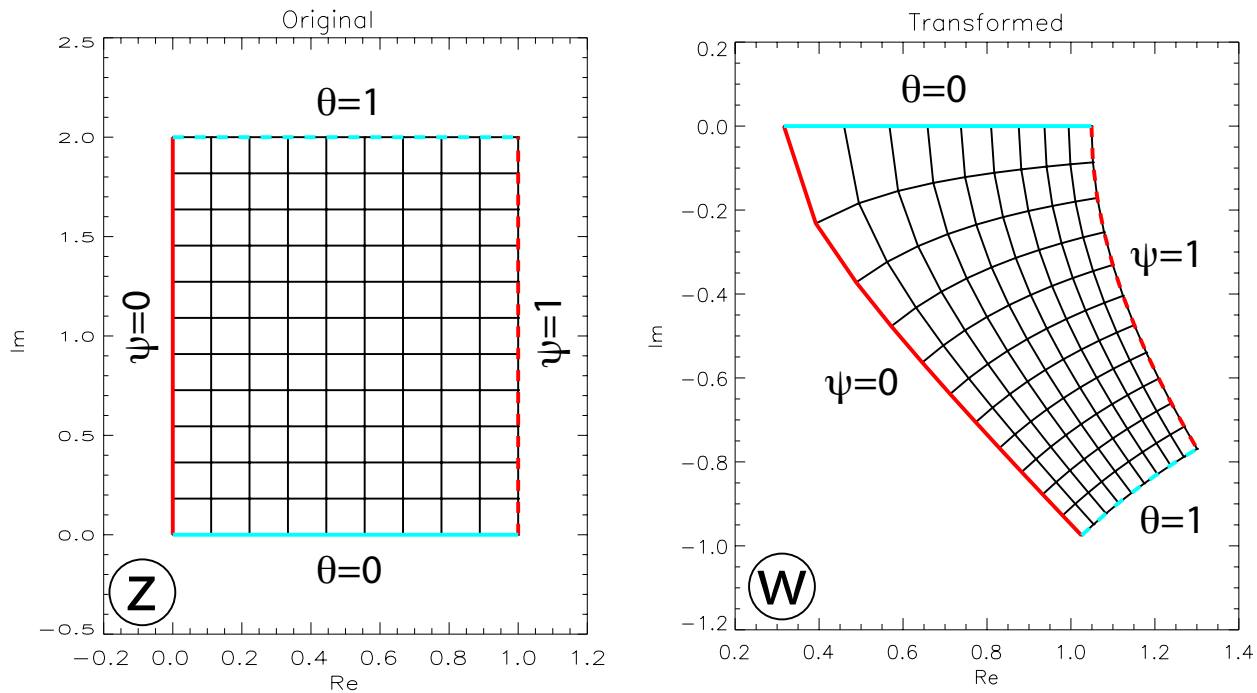


The model is motivated by Lodestar's SOLT model but extended to 3D hence dubbed **SOLT3D**

For including geometric shape features, an efficient method has been developed based on conformal mapping

$$w=(z^*+0.1)^{0.5}$$

"Simple slab"



"Divertor leg"

- **Range of geometric shapes produced by appropriate choice of complex function w(z)**
- **Potentially can implement Schwartz-Christoffel maps (polygon to polygon)**

Implementation and testing of physics models contained in SOLT3D

- **Drift-Resistive-Ballooning Mode**
  - Hybrids of drift-resistive and ballooning modes, driven by radial gradients of plasma pressure

- **Conducting wall mode instability**
  - Flute-like modes driven by radial gradient of plasma temperature and sheath boundary conditions

- **Blobs (nonlinear ballooning instability)**
  - Manifestation of nonlinear ballooning
  - Ideal ballooning drive contained also in resistive ballooning models

Drift-Resistive-Ballooning Mode (DRBM) 2-field physics model implemented in SOLT3D module

$$\frac{\partial N_i}{\partial t} = -V_E \bullet \nabla N_{i0}$$

$$\frac{\partial \varpi}{\partial t} = \boxed{2\omega_{ci} b_0 \times \kappa \bullet \nabla P} + N_{i0} Z_i e \frac{4\pi V_A^2}{c^2} \nabla_{\parallel} j_{\parallel}$$

$$j_{\parallel} = \frac{N_{i0}}{0.51\nu_{ei}} \left(-\frac{e}{m_e} \partial_{\parallel}\phi + \boxed{\frac{T_{e0}}{N_{i0} m_e} \partial_{\parallel} N_i} \right)$$

$$\varpi = N_{i0} \nabla_{\perp}^2 \phi$$

$$P = T_{e0} N_i$$

Ballooning-specific

Drift-specific

Radial gradients of P and N_i and magnetic curvature drive DRBM instability

Main nonlinear terms to be added, e.g.,

$$\frac{\partial N_i}{\partial t} = -V_E \bullet \nabla N_i + \ldots$$

DRBM is a small subset of plasma fluid equations that has produced relevant edge turbulence results (pioneered by Guzdar et al. at U. Maryland in early 1990s)

Dispersion relation produced with Mathematica

Extra terms can be added

$$\begin{pmatrix} -i\omega & ik_{\perp}\dfrac{c}{B}\dfrac{N_{i0}}{L_n} & ik_{\parallel}N_{i0} \\ ik_{\parallel}\dfrac{T_{e0}}{N_{i0}m_e} & -ik_{\parallel}\dfrac{e}{m_e} & 0.51\nu_{ei} \\ ik_{\perp}\dfrac{2T_{e0}\Omega_{ci}}{R_c} & i\omega e k_{\perp}^2 N_{i0} & ik_{\parallel}N_{i0}^2 e^2 \dfrac{4\pi V_A^2}{c^2} \end{pmatrix} \begin{pmatrix} \tilde{N}_i \\ \tilde{\phi} \\ \tilde{V}_{\parallel e} \end{pmatrix} = \vec{0}$$

- **Dispersion relation found from det(M)=0**
- **Can be done by hand but with Mathematica extends easily**

Dispersion relation produced with Mathematica

Mathematica gives coefficients (α=0.51) for: ω^0, ω^1 ,ω^2

$$\left\{\frac{T_{e0}\left(-\frac{i\,B^2\,e^2\,k_{par}^2}{\alpha\,k_{perp}\,m_e\,M_i\,\nu_{ei}}+\frac{2\,c^2\,\Omega_{ci}}{R_c}\right)}{B\,c\,e\,L_n},\ \frac{i\,B^2\,e^2\,k_{par}^2}{c^2\,\alpha\,k_{perp}^2\,m_e\,M_i\,\nu_{ei}},\ 1\right\}$$

$$\frac{iB^2e^2k_{\parallel}^2}{c^2k_{\perp}^2m_eM_i\ 0.51\nu_{ei}}\rightarrow i\sigma_{\parallel}$$

$$T_{e0}\frac{iB^2e^2k_{\parallel}^2}{k_{\perp}m_eM_i\ 0.51\nu_{ei}}\frac{1}{BceL_n}\rightarrow i\sigma_{\parallel}\omega_*$$

$$T_{e0}\frac{2c^2}{R_c}\frac{\Omega_{ci}}{BceL_n}\rightarrow\Omega_K^2$$

$$\boxed{\omega^2+i\sigma_{\parallel}\omega+\Omega_K^2-i\sigma_{\parallel}\omega_*=0}$$

DRBM linear dispersion relation is quadratic equation containing ballooning (interchange) and drift terms

Ballooning-specific

Drift-specific

$$\omega^2 + \Omega_K^2 + i\sigma_{||}(\omega - \omega_*) = 0$$

where

$$\sigma_{||} = \left(\frac{k_{||}}{k_\perp}\right)^2 \frac{\Omega_{ci}\omega_{ci}}{0.51\nu_{ei}}$$

$$\sigma_\perp = 0.51\nu_{ei}\left(\frac{ck_\perp}{\omega_{pe}}\right)^2$$

$$\omega_{pe} = \omega_* = k_\perp v_{pe} = k_\perp \frac{V_{te}^2}{\omega_{ce}L_N}$$

$$\Omega_K = \sqrt{\frac{2C_s^2}{R_C L_N}}$$

$$C_s = \sqrt{T_{e0}/M_i}$$

DRBM linear dispersion relation has one stable and one unstable root

Dispersion relation $\omega^2 + i\sigma_{||}\omega + \Omega_K^2 - i\sigma_{||}\omega_* = 0$

Using Vieta's formulas, infer locations of roots in complex ω plane

$\omega_1 + \omega_2 = -i\,\sigma_{||}$

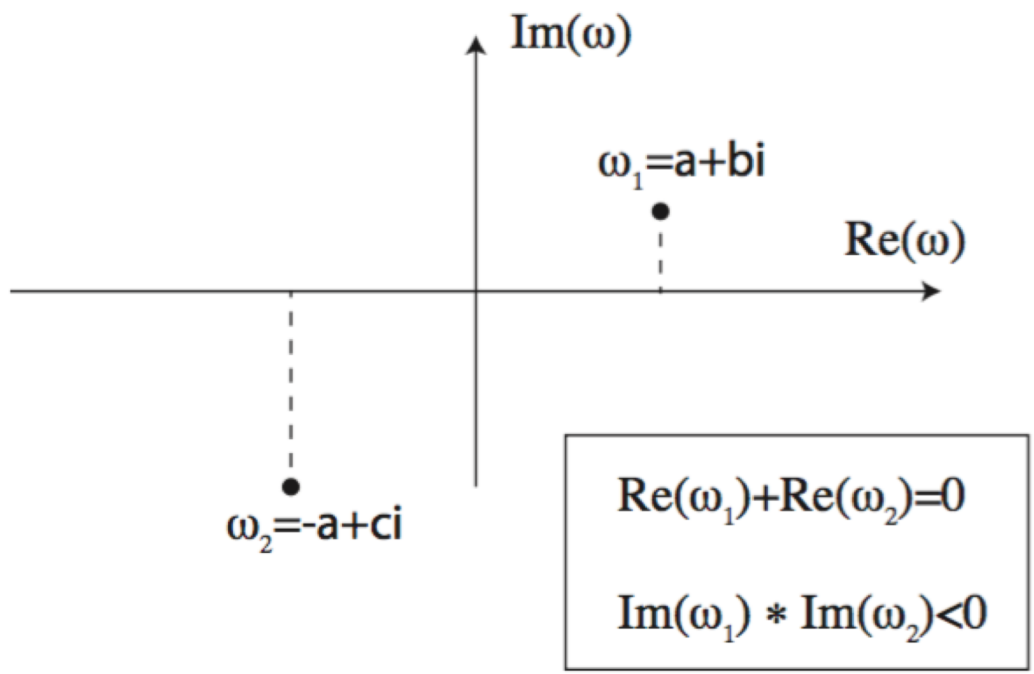
$\omega_1\,\omega_2 = -i\,\sigma_{||}\,\omega_* + \Omega_K^2$

=>

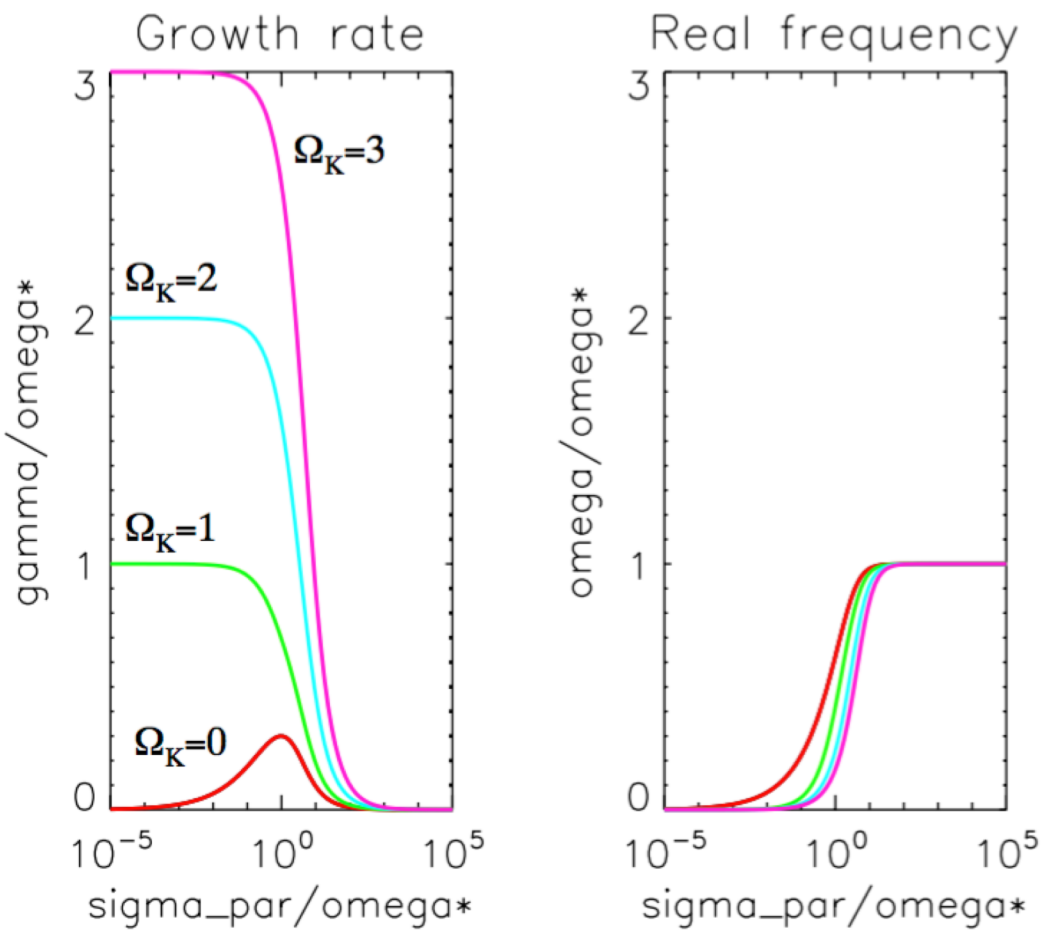
$\omega_1 = a + bi$

$\omega_2 = -a + ci$

$bc = -a^2 - \Omega_K^2 < 0$



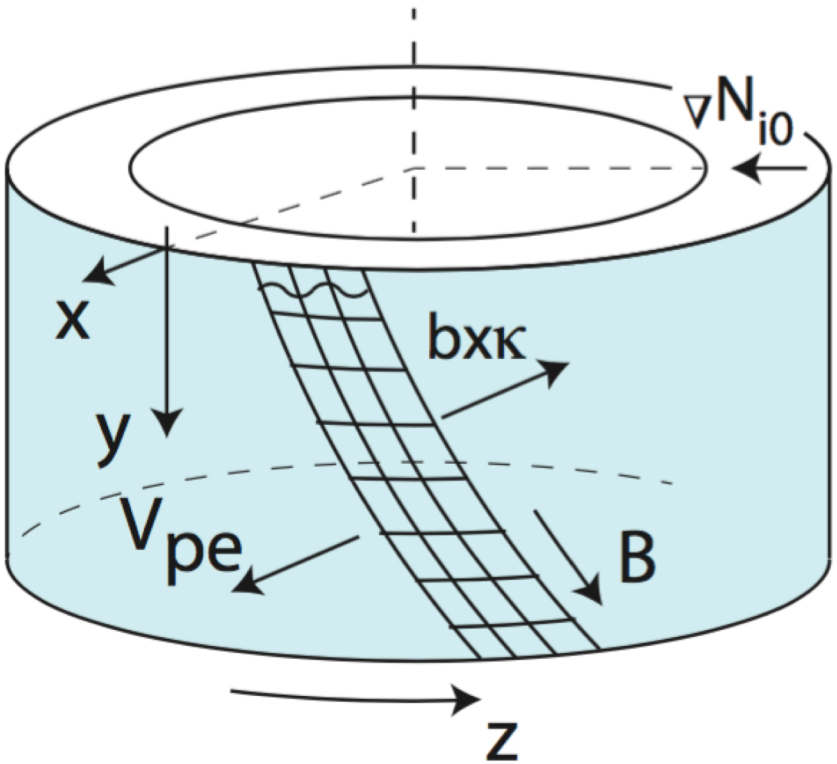
Showing growth rate and real frequency for unstable root of DRBM



For Ω_k=0 the standard drift-wave dispersion relation, e.g., in F. Chen's textbook

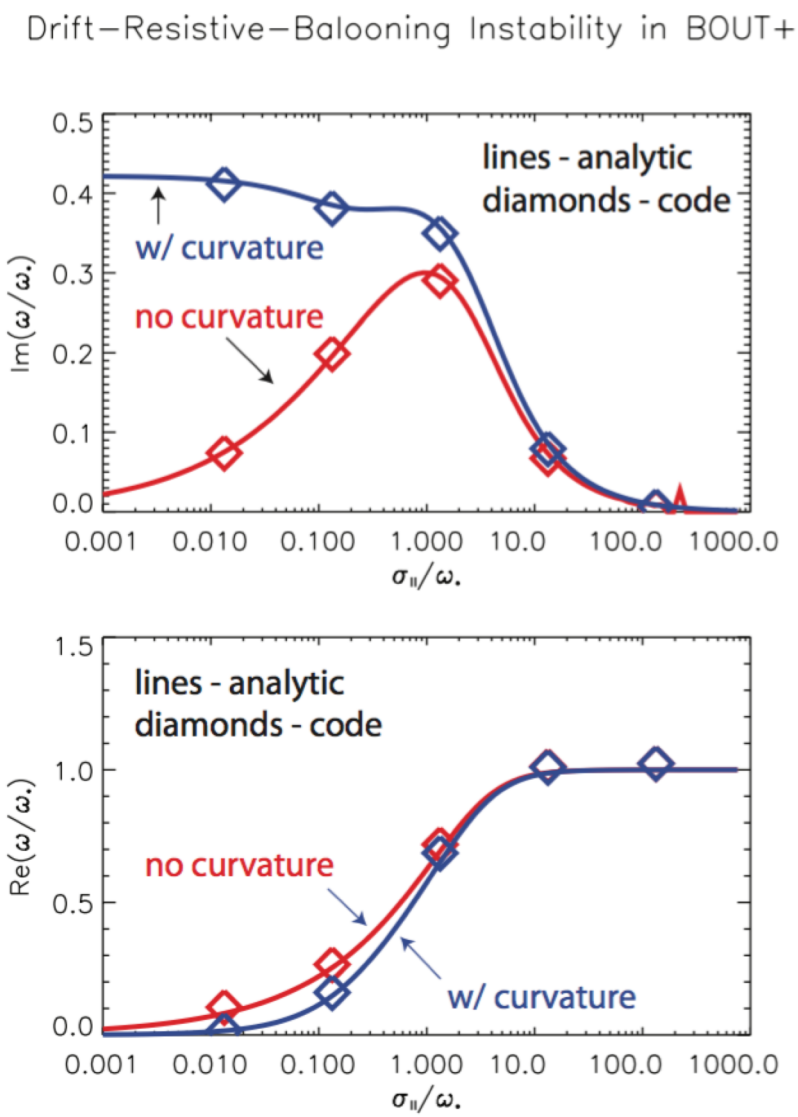
Cylindrical slab geometry used for verifying DRBM linear dispersion relation

$R \in [1.0,1.05]$ m,
$Z \in [0.0,1.0]$ m,
$B_t = 1$ T,
$B_p = 0.1$ T,
$T_{e0} = 100$ eV,
$N_{i0} = 0.5\times10^{20}$ m^{-3}
$L_n = 4.55$ m
$n_{tor} = 500$



Geometry, magnetic field, grid etc. constructed by a grid-generator built for SOLT3D

BOUT++ data points are right on analytic DRBM dispersion relation curves



Implementation and testing of physics models contained in SOLT3D

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  - Hybrids of drift-resistive and ballooning modes, driven by radial gradients of plasma pressure

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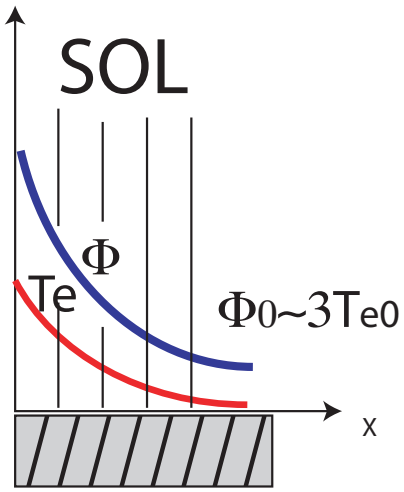
Conducting-Wall Mode (CWM) 2-field physics model is included in SOLT3D module

Radial gradient of T_e and sheath B.C. drive CWM instability

$$\frac{\partial T_e}{\partial t} = -\left(V_{E0} + V_E\right) \bullet \nabla T_{e0}$$

$$\frac{\partial \varpi}{\partial t} = -\left(V_{E0} + V_E\right) \bullet \nabla \varpi + N_{i0} Z_i e \frac{4\pi V_A^2}{c^2} \nabla_{\|} j_{\|}$$



$$j_{\|} = \frac{eN_{i0}}{0.51\nu_{ei}}\left(-\frac{e}{m_e}\partial_{\|}\phi\right)$$

$$\frac{j_{\|,BC}}{j_{\|0}} = \pm\left(\Lambda_1 \frac{e\phi}{T_{e0}} - \Lambda_1 \frac{T_e}{T_{e0}}\right)$$

Linearized sheath B.C.

$$j_{\|0} = C_{s0} N_{i0} e$$

$$\varpi = N_{i0} \nabla_{\perp}^2 \phi$$

- CWM was studied theoretically by Berk, Ryutov, et al. in early 1990s
- Expected to exist in tokamak edge but still not confirmed experimentally

CWM analytic linear dispersion relation leads to transcendental complex equation

- **Consider symmetric parallel domain $-L \le x \le L$**
- **Use fastest growing mode ~$\cos(k_{||}\, x)$**
- **Assume for sheath B.C. $\Lambda_1=0$, $\Lambda_2=1$**

Dispersion relation

$$\eta^3 \tan(\eta) = i\Lambda_2 \frac{\Omega_T}{\Omega_{ci}} \xi^3 \left[\frac{\Omega_{pi}^2}{4\pi\Omega_{ci}} \right]^2$$

$$\omega = -i4\pi\sigma_{||} \left(\frac{\Omega_{ci}}{\Omega_{pi}} \right)^2 \left(\frac{\eta}{\xi} \right)^2$$

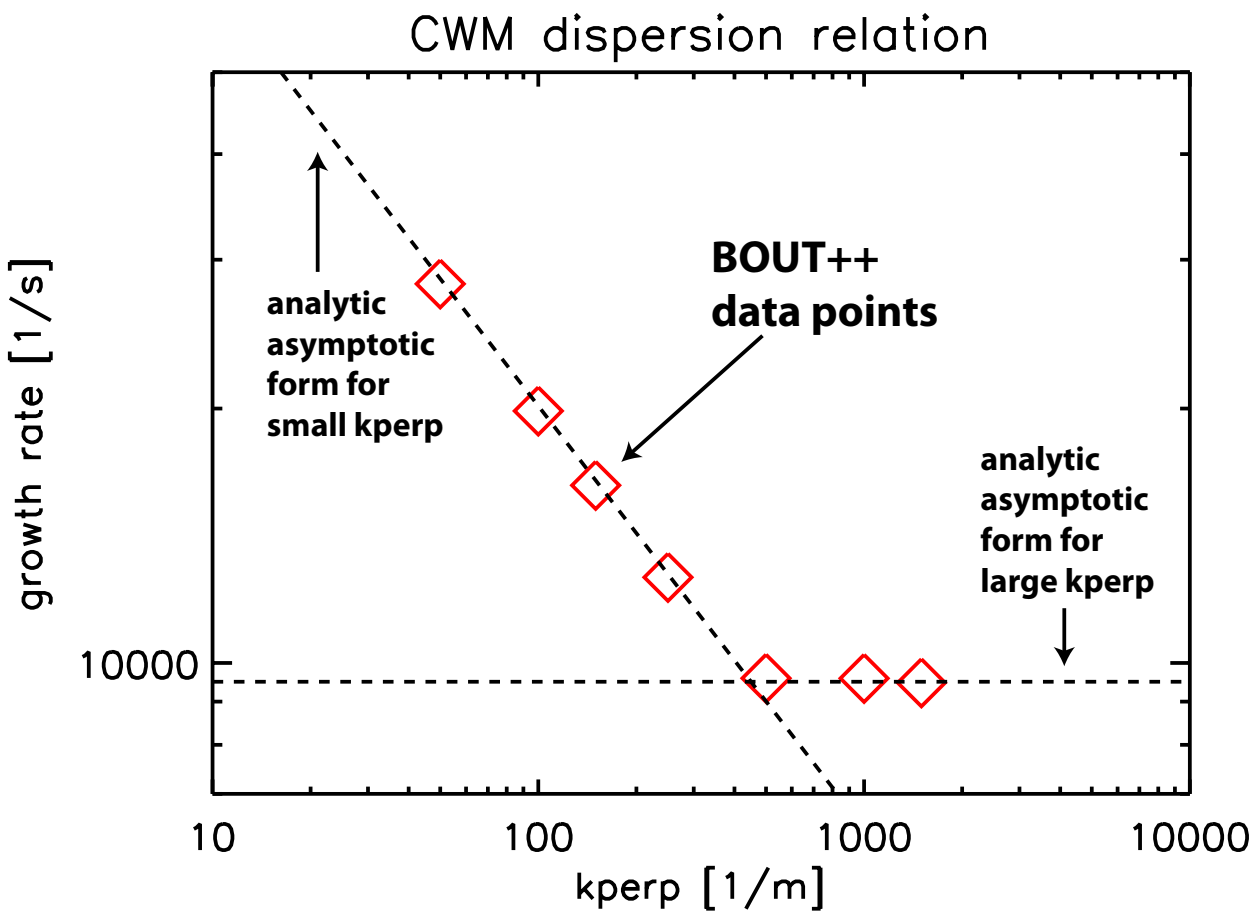
$\eta = k_{||}L$, $\xi = k_{\perp}L$

$L_T = -T_{e0}/(dT_{e0}/dr)$

$\Omega_T = C_{s0}/L_T$

- **Two asymptotic limits:**
  - **$\xi \ll 1 \Rightarrow \eta \ll 1 \Rightarrow$** $\omega^2 = -i\Lambda_2 \dfrac{\Omega_{ci}\Omega_T}{\xi}$
  - **$\xi \gg 1 \Rightarrow \tan(\eta) \approx i \Rightarrow$** ω=const

Numerical results from BOUT++ match analytic asymptotic CWM growth rate



Calculations are done in a slab geometry,
for a particular choice of magnetic field and plasma profiles

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  - Manifestation of nonlinear ballooning
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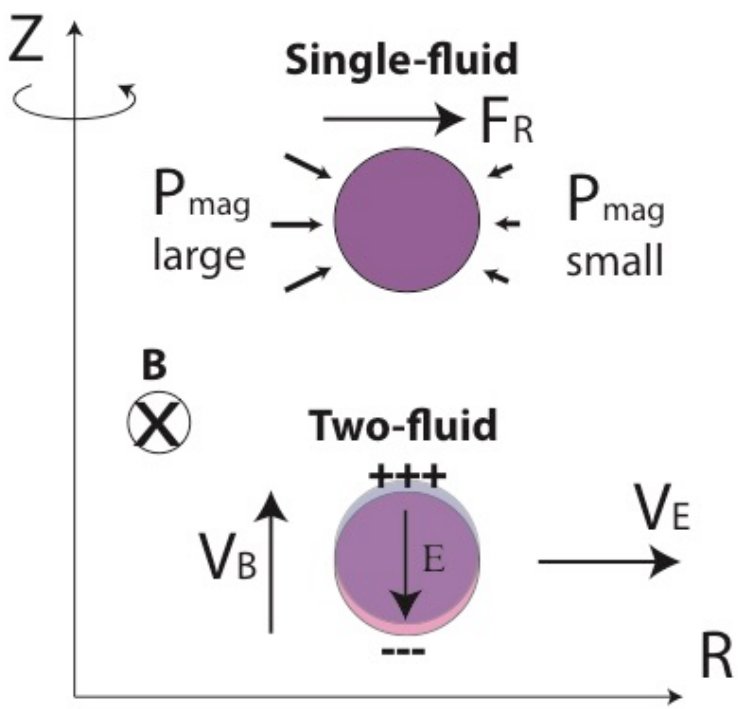
Blob are field-aligned density filaments driven radially outward by magnetic curvature

Can be physically understood in either single-fluid or two-fluid picture



$$\frac{\partial N_i}{\partial t} = -V_E \bullet \nabla N_{i0}$$

$$\frac{\partial \varpi}{\partial t} = 2\omega_{ci} b_0 \times \kappa \bullet \nabla P \quad + \quad N_{i0} Z_i e \frac{4\pi V_A^2}{c^2} \nabla_{\parallel} j_{\parallel}$$

$$j_{\parallel} = \frac{N_{i0}}{0.51\nu_{ei}} \left(-\frac{e}{m_e} \partial_{\parallel} \phi \right)$$

$$\frac{j_{\parallel,BC}}{j_{\parallel 0}} = \pm \left(\Lambda_1 \frac{e\phi}{T_{e0}} - \Lambda_1 \frac{T_e}{T_{e0}} \right)$$

Linearized sheath B.C.

$$j_{\parallel 0} = C_{s0} N_{i0} e$$

$$\varpi = N_{i0} \nabla_{\perp}^2 \phi$$

$$P = T_{e0} N_i$$

This is what happens with toroidal plasma without rotational transform!

In numerical simulations, blobs show two types of behavior: mushroom breakup or interchange breakup

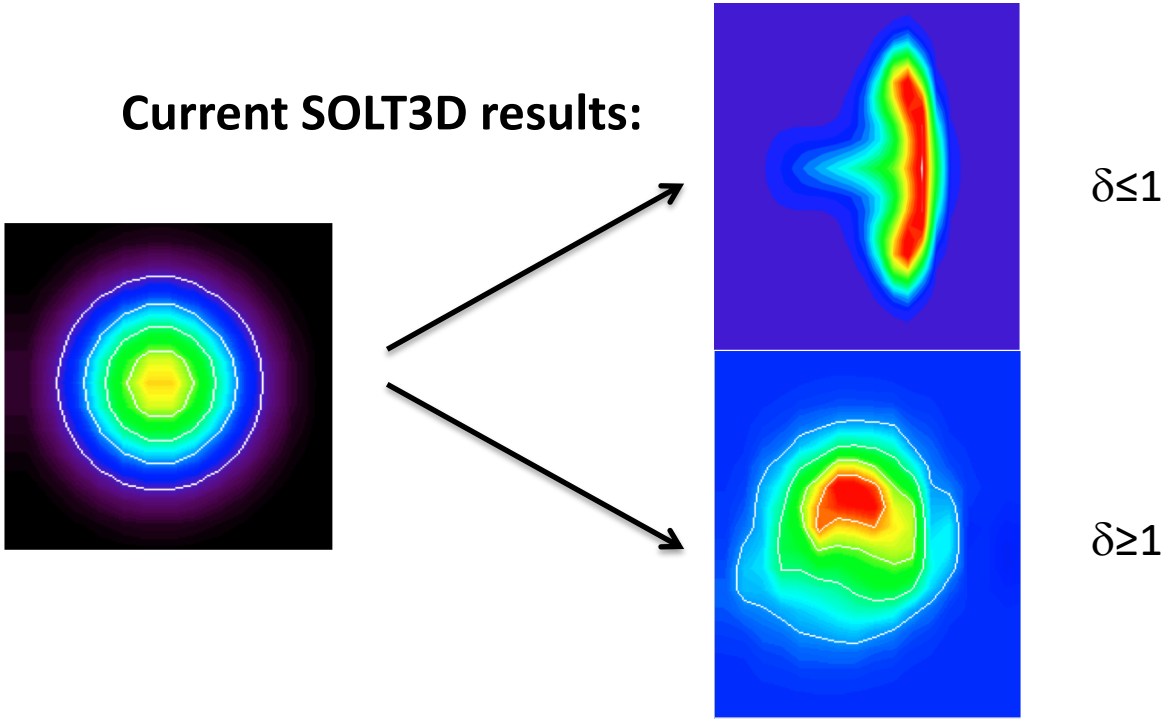
Normalized blob size

$$\delta = \frac{\delta_b}{\delta_*}$$

$$\delta_* = \rho_s \left(\frac{L_{\parallel}^2}{\rho_s R} \right)^{1/5}$$

- For small blobs, $\delta \ll 1$ KH mushroom
- For large blobs, $\delta \gg 1$ interchange breakup[1]

Current SOLT3D results:



[1]Krasheninnikov, D'Ippolito, and Myra,
J. Plas. Phys. (2008), vol. 74, part 5, pp. 679–717

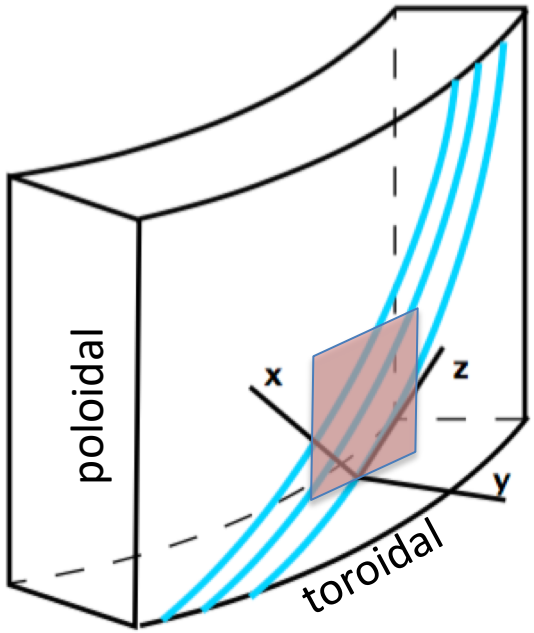
Coupling with RF physics: added terms in plasma equations represent sources of energy and momentum

Particle and heat flux driven by dissipative wave momentum and heat flux absorption

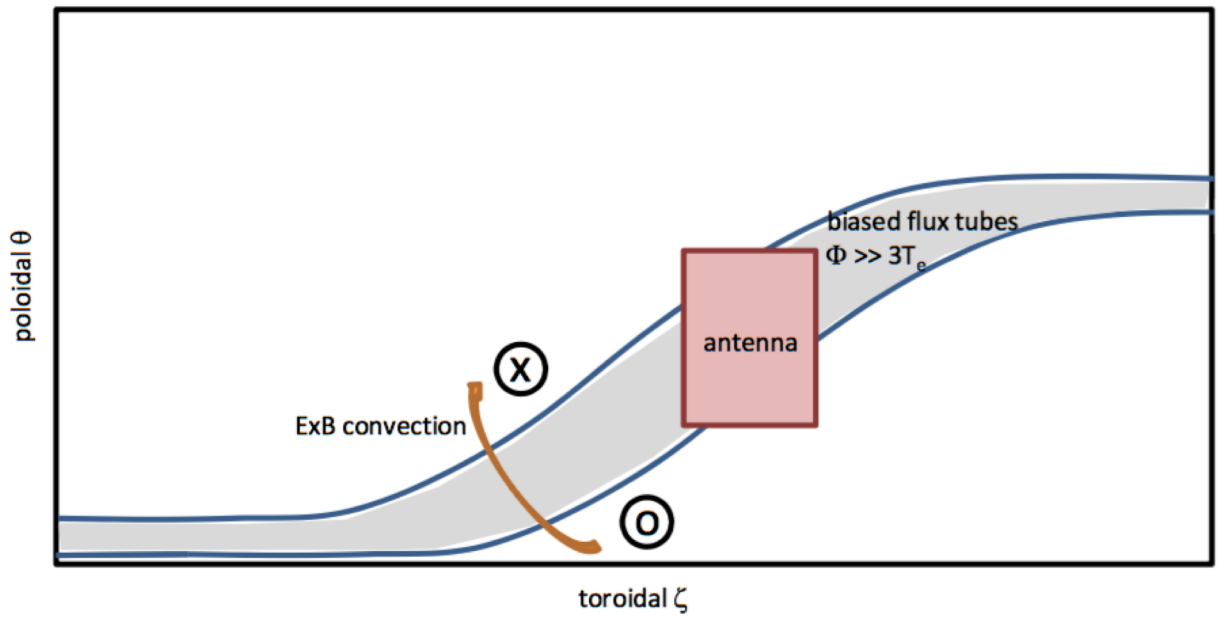
- **Forces:**
  - **dissipative wave momentum absorption, perpendicular wave particle energy exchange by dissipation**
  - **reactive (pressure-like) components of wave wave pressure and perpendicular wave-particle energy**
- **Can also drive convective structures that interact with turbulence**

Myra et al., Phys. Plasmas 11, 1786 (2004)

Coupling with RF physics: boundary conditions representing rectified sheath couple to turbulence



- **Initial antenna model: biased region on plasma surface**
- **Surface bias extends inwards via Laplacian in vorticity**
- **Drives convection**
- **Potentially causes transport and stabilization or destabilization of instabilities (and turbulence)**



Myra et al., Nucl. Fusion **46,** 455 (2006);
Myra and D'Ippolito, Phys. Plasmas **22**, 062507 (2015)

Summary

- Basic edge turbulence model SOLT3D implemented in BOUT++
- Similar to Lodestar's model SOLT but extended to 3D
- Simplified geometry initially, adding geometric details later
- Supports main plasma physics relevant to SOL and divertor:
  - Drift-resistive-ballooning mode (DRBM)
  - Conducting-wall mode (CWM)
  - Blobs (propagating coherent filamentary structures)
- Verified against analytic theory and previous simulations
- Implementation of terms & BC relevant to RF antenna ongoing
- Nonlinear simulations with turbulence ongoing