BOUT++ Physics development

Presented to 2015 BOUT++ mini-Workshop

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and BOUT++ team

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The author would like to acknowledge significant contributions to this presentation from the following collaborators:

Tokamak edge region encompasses boundary layer between hot core plasma and material walls

- Complex geometry
- Rich physics (plasma, atomic, material)
- Sets key engineering constraints for fusion reactor
- Sets global energy confinement

BOUT (BOUndary Turbulence) was originally developed at LLNL in late 1990s for modeling tokamak edge turbulence
BOUT++ is a successor to BOUT, developed in collaboration with Univ. York

Original BOUT, tokamak applications on boundary turbulence and ELMs with encouraging results

BOUT-06: code refactoring using differential operator approach, high order FD, verification

BOUT++: OOP, 2D parallelization, applications to tokamak ELMs and linear plasmas

2000  2005  2015

- Xu, Umansky, Dudson & Snyder, CiCP, V. 4, 949-979 (2008).
- Xu, Dudson, Snyder et al., PRL 105, 175005 (2010).
Principal BOUT++ Activities

- A suite of two-fluid multiple-field models has been implemented in BOUT++ for all ELM regimes and fluid turbulence
  
  T. Y. Xia, 3rd talk this afternoon

- A suite of 3+1 Gyro-Landau-Fluid models is also developed for pedestal kinetic turbulence and transport
  
  C.H. Ma, 2nd talk this afternoon

- Fluid neutral models are developed for SMBI, GAS puffing, Recycling

- A PIC module for impurity generation and transport

- Coupling BOUT++ and SOLPS for divertor heat flux contr.

- BOUT++ has been applied to a range of problems, including simulation of ELMs, plasma blobs, turbulence, & magnetic reconnection
Benchmarks
(module elm_pb)
BOUT++ 3-field reduce MHD model
(module elm_pb)

- 3-field reduced MHD equations evolve pressure $P$, vorticity $\omega$ and perturbed magnetic vector potential $A_\parallel$:

\[
\frac{\partial \tilde{\omega}}{\partial t} + v_E \cdot \nabla \tilde{\omega} = B_0 V_\parallel \tilde{J}_\parallel + 2b_0 \times k_0 \cdot \nabla \tilde{P}
\]

\[
\frac{\partial P}{\partial t} + v_E \cdot \nabla P = 0
\]

\[
\frac{\partial \tilde{A}_\parallel}{\partial t} = -V_\parallel \Phi + \frac{\eta}{\mu_0} V^2 \tilde{A}_\parallel
\]

- The variables in the equations are defined as:

\[
\tilde{\omega} = \frac{n_0 M_i}{B_0} \left( V^2 \tilde{\phi} + \frac{1}{n_0 Z_i e} V^2 \tilde{P} \right), \Phi = \tilde{\phi} + \Phi_0, k_0 = b_0 \cdot \nabla b_0
\]

\[
J_\parallel = J_{\parallel 0} - \frac{1}{\mu_0} V^2 \tilde{A}_\parallel, \quad v_E = \frac{1}{B_0} (b_0 \times \nabla \Phi)
\]

- In the equations, for any variable $F$, $\tilde{F}$ is the perturbed component,

$V_\parallel F = B \partial_\parallel (F/B), \partial_\parallel = \partial^0_\parallel + b \cdot \nabla, b = B/B = V_\parallel A_\parallel \times b_0/B, \partial^0_\parallel = b_0 \cdot \nabla$
JET-like Equilibrium model

- Based on the cbm18 equilibrium sequence
- Circular plasma, with a “vacuum” region
- BOUT++ computation region is $\psi$ [0.2, 1.4]
A set of JET-like equilibria with different edge current

- Based on the cbm18_dens8 equilibrium, using the CORSICA code, a sequence of equilibrium with different edge current are created
- Keep total current and pressure profile fixed
Edge current has stabilizing effects on the ballooning modes

- As the edge current increases, the medium $n$ ballooning modes are stabilized, the dominant mode is changed from ballooning modes to low-$n$ kink modes.
- The ballooning stabilization effect is due to the increase of local shear at the outer mid-plane.

![Linear growth rate v.s. toroidal mode number](chart1.png)

![RMS of perturbed pressure, the height is normalized with linear growth rate](chart2.png)
Good agreement between BOUT++, ELITE and GATO for both peeling and ballooning modes

- As edge current increases, the difference between BOUT++ and GATO/ELITE results becomes large.
- This difference is due to the vacuum treatment.

For the real “vacuum” model, the effect of resistivity should be included as BOUT++

Benchmark results for elongated plasma for both peeling and ballooning modes

- **Elongated dbm18 equilibrium**
  - $\text{Elong}=1.32$, $R_0=3\text{m}$, $I_p=2.25\text{MA}$, $B_t0=2.0\text{T}$, $\beta_N=1.68$

- **BOUT++ agrees with GATO if they use same vacuum model**

- **Vacuum model has small effects on the stability, though it is the ballooning dominated mode**

![Graph showing Z(m) vs. R(m) and y/ω_A vs. n with different models and vacuum assumptions]
Nonlinear ELM simulations show three stages of an ELM event

- BOUT++ 3-field reduce MHD model evolves
  - pressure $P$
  - vorticity $\omega$
  - magnetic vector potential $A_{\parallel}$

- Based on a set of JET-like magnetic equilibria
  - Circular plasma, with a "SOL" region
  - BOUT++ computation region in $\psi$ [0.1, 1.4]
The ELMs are quasiperiodic relaxations of the pedestal, resulting in a series of hot plasma eruptions.

✓ Potentially damage the ITER divertor plates and first walls

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Principal Results

- Demonstrated the linear and nonlinear characteristics of ELMs at different collisionality & Er via a density scan.

- By increasing collisionality, nonlinear simulations show that:
  - Power spectrum becomes broad, the dominant mode increases from n=6 to n=35.
  - Bispectrum analysis shows that nonlinear mode coupling becomes stronger, resulting in the lack of dominant filamentary structures and reduced ELM energy loss.

- The impact of radial electric field Er on peeling and ballooning modes is different.
  - The increase Er significantly enhances the linear growth rate of low-n peeling modes, but only weakly impacts on nonlinear ELM energy loss.
  - The increase Er leads to large suppression of nonlinear ballooning amplitudes, but only weakly impacts on their linear growth rates.
Eight cases: $n_e(0)=1, 3, 5, 7, 9, 12, 15, 20 \times 10^{19} \text{ m}^{-3}$.

$n_e = n_e(0) \times \left( \frac{P_0}{P_0(0)} \right)^{0.3}, T_e = \frac{P_0}{2n_e}$.

$V_{\perp 0} = \frac{1}{Z \epsilon n_i 0} \frac{b \times \nabla P_{i 0}}{B} + \frac{b \times \nabla \Phi_{\text{dia} 0}}{B}$.
Nonlinear ELM simulations show three stages of an ELM event

1) a linear growth phase
2) a fast crash phase
3) a slow inward spreading phase
   - In 3-field 2-fluid model, total energy loss (P) shows a similar spreading
   - In 3+1 GLF model, Electron perturbation provides the spreading, eventually dominates the total energy loss with a large conductive energy loss

The ballooning term dominates the high n modes. Because ion diamagnetic drift is inversely proportional to the density for fixed pressure, when density increases, the ion diamagnetic stabilization decreases and growth rate increases.

The kink term dominates the low n modes. Therefore, as the density increases, the edge current decreases and growth rate decreases.

As the edge density (collisionality) increases for fixed $E_r$, the growth rate of the P-B mode increases for high $n$ but decreases for low $n$ ($1<n<5$).
The growth time of linear drive is determined by nonlinear process via phase evolution for large ELM crash.

**Phase coherence time (PCT, \( \tau_c \)):** the length of time duration of the relative phase for linear growth

- Linear theory/simulations: unchanged \( \delta \phi \Rightarrow \tau_c \rightarrow \infty \)
- The growth time is determined by nonlinear Phase Scattering

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**Figure (a):** ELM size vs nonlinearity \( C_R \)

- \( A_{n \neq 0} \)
- \( \gamma(t) \) \( \exp(\gamma t) \)
- Dominant
- \( 2b_0 \times \mathbf{k} \cdot \nabla \tilde{P} \)
- Relative phase

**Figure (b):** PCT vs nonlinearity \( C_R \)

- \( \nabla \cdot \nabla \times B \cdot \nabla \sigma = \text{RHS} \)

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Xi, Xu, Diamond, Phys. Plasmas 21, 056110 (2014)
Stronger nonlinear mode coupling at high $n^*$ leads to the lack of dominant filamentary structures and the reduced ELM energy loss.

2D bispectrum shows that nonlinear mode coupling becomes stronger at high collisionality $\nu^*$
BOUT++ simulations show collisionality scaling of ELM energy losses consistent with ITPA multi-tokamak database.

As the edge collisionality decreases, both linear and nonlinear physics set ELM energy loss

- Linearly, the dominant P-B mode shifts to lower n and the spectrum width of the linear growth rate decreases
- Nonlinearly, Narrow mode spectrum → Weak nonlinear Phase Scattering → Long PCT → Large ELMs

Two factors determine if a single mode amplitude can grow to a large magnitude to trigger an ELM

- Linear growth rate
- Nonlinear growth time

\[ \exp(\gamma t) \begin{cases} 
\gamma < - & \text{linear} \\
 t < - & \text{nonlinear}
\end{cases} \]
Equilibrium electric field model

\[ V_{\perp 0} = \frac{1}{Z_{\text{en}i_0}} \frac{b \times \nabla P_{i0}}{B} + \frac{b \times \nabla \Phi_{\text{dia}0}}{B} \]

\[ \hat{\Phi}_{\text{dia}0} = -\frac{\bar{B}}{1.4 \mu_0 Z e V_A \bar{L} N \hat{n}_{i0}} \frac{\hat{P}_{00}^{0.3} \hat{P}_0^{0.7}}{\hat{B}_0} \]
Evolution of electric field during ELM crashes

At the beginning, the electric field was increased by a factor of about 3 times.
low density, strong current -> peeling dominant, sharp and narrow $\gamma(n)$
high density, weak current -> ballooning dominant, flat and wide $\gamma(n)$

Growth rate spectrums vs density & toroidal mode number

linear growth rate spectrum after turning off peeling drive
The increase Er only weakly impacts on the amplitude of peeling fluctuations; while the increase Er leads to large suppression of nonlinear ballooning amplitudes.
Increasing $E_r$ by a factor of 3, ELM size increases significantly at low collisionality ($n_0=5\times10^{19}/m^3$ and $n_0=9\times10^{19}/m^3$).

Increasing $E_r$ leads to the suppression of ELM size at high collisionality ($n_0=20\times10^{19}/m^3$).
By increasing collisionality, nonlinear simulations show that:
- amplitude spectrum becomes broad
- the dominant mode changes from $n=6$ to $n=35$
- nonlinear interactions are strongly enhanced.

The increase $E_r$ can also enhance the nonlinear coupling between modes.
ELM suppression by intermittent small scale turbulence induced by SMBI

Mitigations of ELM by SMBI have been observed on EAST, KSTAR and HL-2A;

After SMBI

Experimental results on EAST: Impact of collisionality (with SMBI) on ELM size

After SMBI (collisionality increased):

1. ELM size is mitigated and particle transport is mainly contributed by high frequency turbulence;
2. Bispectrum analysis indicate that the nonlinear interactions are greatly enhanced at high collisionality.

2. Bispectrum analysis results are provided by Adi Liu (USTC)
Preliminary experimental results on EAST:
Impact of toroidal rotation (Er?) on ELM size

ELM study with co NBI and con NBI:

1) ELM size is modulated by injecting direction of NBI with smaller ELM size at con NBI.
2) Stored energy and density profiles remain almost the same (collisionality remains the same?)
3) Toroidal rotation (Er) playing an important role in modifying the ELM?

Provided by C.B. Huang (ASIPP)
Impact of co-NBI and con-NBI on nonlinear coupling on EAST

- **Left Figure:** the electric field \( E_r \propto f_{\text{Doppler}} \) with con-NBI is larger than that with co-NBI;
- **Right Figure:** nonlinear interactions among different modes become stronger in con-NBI (or larger \( E_r \)) case comparing to the co-NBI case.

Those observations on EAST are consistent with the simulation results by BOUT++ mentioned before.
Principal Results

• Demonstrated the linear and nonlinear characteristics of ELMs at different collisionality & Er via a density scan

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  ✓ Bispectrum analysis shows that nonlinear mode coupling becomes stronger, resulting in the lack of dominant filamentary structures and reduced ELM energy loss.

• The impact of radial electric field Er on peeling and ballooning modes is different.
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• New ELM control tool: via radial electric field Er
Integrated multi-scale simulations for ELM crashes and recovery
Principal results for full ELM cycles with ELM dynamics

- Separate the dynamical equations into equations for the fluctuating and axisymmetric components to improve computational efficiency
- The axisymmetric components of the pressure and vorticity are on the slow transport timescale while the fluctuations are on a rapid timescale to changes in the profiles
- The axisymmetric projection of nonlinear fluxes that are bilinear in fluctuating quantities, such as, the energy flux, act as drive terms for the axisymmetric quantities that determine the profiles

- The equations are a set of coupled convection-diffusion equations, including
  - ELMs
  - Micro-turbulence & neoclassical transport
  - flux-limited parallel transport
  - sources and sinks

Simulation tracks five ELM cycles for 10000 Alfven times

- The pedestal pressure profile collapses and recovers to a steep gradient
- Strong poloidal non-uniformity, even with the flux-limited Spitzer-Harm parallel heat diffusivity
Simulated ELM cycles are similar to DIII-D expts.

- The pedestal pressure profile collapses and recovers to a steep gradient.
- Simulations are for small ELMs, DIII-D data for large ELMs.
In addition to ELMs,

Fluctuations are simulated for comparison between ELMs in H-mode &
During ELM-suppressed I-Modes

(module 6f_divertor_imp)

- **BOUT++ 3D 6-field 2-fluid electromagnetic model evolves**
  - density $n_i$,
  - ion temperature $T_i$
  - parallel velocity $v_{\|i}$
  - electron temperature $T_e$
  - vorticity $\omega$
  - magnetic vector potential $A_{\|}$
- **Based on a set of C-Mod and DIII-D experiments**
  - realistic X-point magnetic and plasma profiles
  - BOUT++ computation region across the magnetic separatrix
There is experimental evidence that Quasi-coherent fluctuations (QCFs) lead to the saturation of the pedestal between ELMs on C-Mod, DIII-D, AUG, & JET.

- QCFs Observed as density & magnetic fluctuations
- Pedestal-Localized QCFs with onsets for $V_T_e$

Weakly Coherent Modes (WCMs) in I-mode:
- mid-range frequency, (~200-400 kHz)
- lead to large $D_e$, $D_e > \chi_e$

I-modes: has a temperature pedestal but no density pedestal, ELM-free

ELMy H-mode on C-Mod

- J. W. Hughes, Nucl. Fusion 53 (2013) 043016
- A Diallo, APS invited, 2014
- A Diallo, PRL v112 115001(2014)
- JET-Perez PPCF 2004
- AUG-Laginner EPS 2015

Various coherent fluctuation structures observed in Alcator C-Mod and other Tokamaks
C-Mod experimental plasma profiles are used in Kinetic EFIT MHD equilibria calculations and BOUT++ simulations

- Dedicated ELMy discharges were performed
  - $B_T = 5.4 \text{ T} \& I_p = 0.9 \text{ MA}$
- Difference between the I-mode and H-mode pedestals
  - Lack of a particle barrier & no ELMs in I-mode

- I-mode has lower pressure & current than H-mode
  - Weakly Coherent Modes (WCMs) in I-mode
  - Quasi-coherent fluctuations (QCFs) between ELMs in ELMy H-mode
Linear simulations indicate that

- WCMs are unstable for resistive ballooning mode and drift-Alfven wave
- QCFs are marginal unstable near Peeling-Ballooning threshold

C-Mod pedestal simulations show ELMy H-mode and I-mode exhibit different underlying instabilities
BOUT++ simulations show similar evolution of Quasi-Coherent Fluctuations as C-Mod magnetic probe measurements and good agreement.

- Pedestal-Localized QCFs
- $k_\theta \sim 0.7 \text{ cm}^{-1}$, toroidal mode number $n \sim 10$
- $f \sim 300-400\text{kHz}$

**Figure:**

- BOUT++ simulations and C-Mod experimental data showing similar evolution of Quasi-Coherent Fluctuations.

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Linear and nonlinear BOUT++ analyses show similar frequency evolution as Quasi-Coherent Fluctuations (QCFs) on DIII-D expts.

- Normal $B_T = 1.9$ T with $q_{95} = 3$ in Type I ELMy H-mode
- Linear analysis indicates:
  - edge is marginally unstable for ballooning modes
  - $n=40$ is the most unstable when diamagnetic effects are included
  - drift Alfvén mode appears to be dominant
- Nonlinear analysis indicates
  - The frequency of the dominant mode is approx. $80$ kHz which is near that of observed QCFs

Increasing the edge should increase the QCF intensity, while increasing $\nabla T_i$ doesn’t.

**DIII-D expt.**

Evolution of QCF amplitude and Temperature gradient

$\nabla T_e [\text{keV/m}]$  

**BOUT++ for C-Mod**

Fixed pressure  

Scale $\propto (\nabla T_{i,e})$

- QCF onsets for a given critical temperature gradient $\nabla T_e$
- Temperature gradient $\nabla T_e$ and QCF amplitude track each other

Weakly Coherent Modes (WCMs) in I-modes have been simulated for comparison with expts.
The dominant mode $n=20$ near the position of the reflectometer shows a similar frequency peak in I-Modes.

- The spectrum of the mode $n=20$ is similar to the experimental result from the reflectometer.
- The spectrum of total modes also has the peak around 300kHz.

Z. X. Liu et al, GO4.00005, Analysis of Weakly Coherent mode on C-Mod with the BOUT++ code, 10:18AM - 10:30AM, Tuesday, November 17, 2015, in session of C-Mod, ADX, Lockheed Martin CFR.
I-mode: **Particle diffusivity is larger than thermal** ($D >> \chi_e$)

H-mode: **Particle diffusivity is smaller than thermal** ($D << \chi_e$)

- Larger particle diffusivity is consistent with the key feature of I-mode, $D >> \chi_e$
- Predicted $\chi_e$ and $\chi_i$ (dashed curves) are close to experimental $\chi_{eff}$ (from power balance over $0.95 < \psi < 1$) for I-mode
The high-fidelity BOUT++ two-fluid suites have demonstrated significant recent progress toward integrated multi-scale simulations

- including ELM dynamics, evolution of ELM cycles, and continuous fluctuations, as expts.

- Nonlinear ELM simulations show three stages of an ELM event
  - Collisionality scaling of ELM energy losses consistent with ITPA multi-tokamak database

- Nonlinear integrated multi-scale ELM simulations:
  - Simulation tracks five ELM cycles for 10000 Alfven times for small ELMs

- To validate BOUT++ simulations & find better regimes, both quasi-coherent fluctuations (QCFs) in ELMy H-modes and Weakly Coherent Modes (WCMs) in I-modes have been simulated for comparison with expts.
  1) H-mode simulations predict that
     - the QCFs are near marginal instability for ideal peeling-ballooning modes
     - the predicted particle diffusivity is smaller than the heat diffusivity, $D \gg \chi_e$
  2) I-mode simulation results are that
     - a strong instability exists at $n \geq 20$, for resistive ballooning modes and drift-Alfven wave
     - the predicted particle diffusivity is larger than the heat diffusivity, $D \ll \chi_e$

- A successful cooperation between simulation and experiment teams