BOUT++ 3+1 GLF tutorial

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BOUT GLF Team Meeting

Nov. 21, 2014

LLNL-PRES-664718

Outline

- Introduction
- Normalization
- Implementation
- Code structure
- Future work
- Installation
- Hands-on exercise

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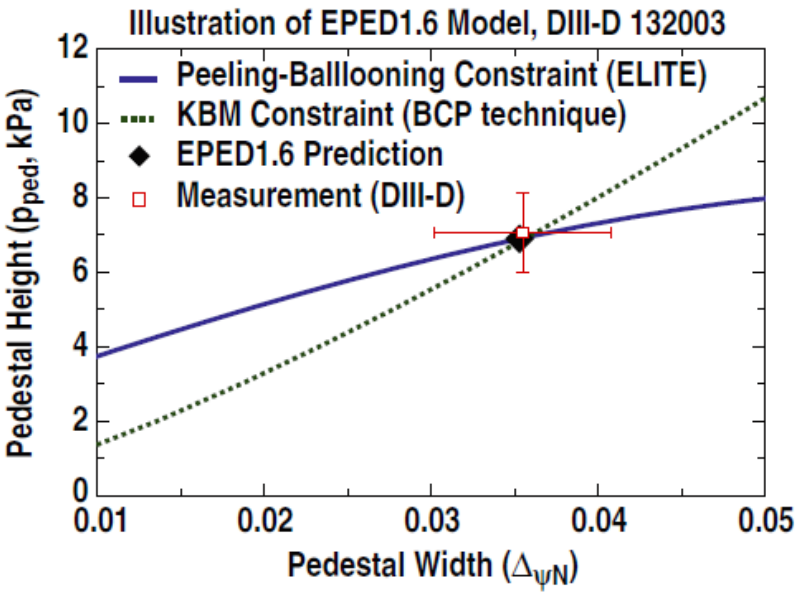
3+1 GLF simulations in BOUT++

- The linear KBM physics in EPED[1] successfully predicts the H-mode pedestal height and width;
- Turbulent transport is important near the ELM threshold;
- 3+1 gyro-Landau-fluid (GLF) model is implemented in BOUT++ to include the KBM turbulence effects in nonlinear ELM simulations.



1. P. B. Snyder , et al., NF(2011)

3+1 Gyro-Landau-fluid model

- We utilize the gyrofluid model[1] developed by P. Snyder and G. Hammett;
- 3+1 model: $(n_i, v_{\parallel i}, p_{\parallel i}, p_{\perp i}, \varpi, A_\parallel, p_{\parallel e}, p_{\perp e})$
  - Full FLR effects (Padé approximation): $k_\perp \rho_i \sim 1$;
  - Parallel Landau damping: non-local transport;
    - Collisional parallel closure is still developing
  - Toroidal resonance;
  - Non-isotropic response ($p_\parallel \neq p_\perp$)
- In long-wavelength limit ($k_\perp \rho_i \ll 1$) and isotropic assumption ($p_\parallel = p_\perp$), the set of equations is reduced to 6-field Landau-fluid model[2] with gyro-viscosity.

1. P. B. Snyder and G. W. Hammett, PoP (2001)
2. T. Y. Xia, X. Q. Xu and P. W. Xi, Nucl. Fusion (2013)

Full set of ion equations in 3+1 GLF model

$$\frac{\partial \tilde{n}_i}{\partial t} = -\frac{1}{B_0} b_0 \times \nabla\Phi \cdot \nabla n_i - \frac{1}{eB_0} b_0 \times \kappa \cdot \nabla (p_{\parallel i} + p_{\perp i})$$

$$-\frac{n_i}{B_0} b_0 \times \kappa \cdot \nabla \left(2 + \frac{1}{2}\hat{\nabla}_\perp^2\right)\Phi - n_0 B_0 \tilde{\nabla}_\parallel \frac{\tilde{u}_{\parallel i}}{B_0} - \frac{n_0}{2T_0 B_0} b_0 \times \nabla \left(\hat{\nabla}_\perp^2 \Phi\right) \cdot \nabla T_{\perp i}$$

$$n_0 \frac{\partial \tilde{u}_{\parallel i}}{\partial t} + n_0 v_\Phi \cdot \nabla \tilde{u}_{\parallel i} + n_0 v_{\Phi_0} \cdot \nabla \tilde{u}_{\parallel i} + \frac{B}{m_i} \tilde{\nabla}_\parallel \frac{\tilde{p}_\parallel}{B}$$

$$+ \left(\frac{\tilde{p}_{\perp i}}{m_i} + \frac{n_0 e}{2m_i} \hat{\nabla}_\perp^2 \Phi\right) \nabla_\parallel \ln B + \frac{1}{T_0} i\omega_d (\tilde{q}_{\parallel i} + \tilde{q}_{\parallel e}) + \frac{4}{T_0} i\omega_d p_{i0} \tilde{u}_{\parallel i} = 0$$

$$\frac{\partial \tilde{p}_{\parallel i}}{\partial t} = -\frac{1}{B_0} b_0 \times \nabla\Phi \cdot \nabla p_{\parallel i} - \frac{n_0}{2B_0} b_0 \times \nabla \left(\hat{\nabla}_\perp^2 \Phi\right) \cdot \nabla T_{\perp i}$$

$$-\frac{p_{\parallel i}}{B_0} b_0 \times \kappa \cdot \nabla \left(4 + \frac{1}{2}\hat{\nabla}_\perp^2\right)\Phi - 3B_0 \tilde{\nabla}_\parallel \frac{p_{i0} \tilde{u}_{\parallel i}}{B_0} - i\omega_d (\tilde{r}_{\parallel,\parallel} + \tilde{r}_{\parallel,\perp}) - B_0 \tilde{\nabla}_\parallel \frac{\tilde{q}_{\parallel i}}{B_0}$$

$$\frac{\partial \tilde{p}_{\perp i}}{\partial t} = -\frac{1}{B_0} b_0 \times \nabla\Phi_2 \cdot \nabla p_{\perp i} - \frac{n_0}{B_0} b_0 \times \left(\hat{\hat{\nabla}}_\perp^2 \Phi\right) \cdot \nabla T_{\perp i}$$

$$-\frac{p_{\perp i}}{B_0} b_0 \times \kappa \cdot \nabla \left(3 + \frac{3}{2}\hat{\nabla}_\perp^2 + \hat{\hat{\nabla}}_\perp^2\right)\Phi - B_0^2 \tilde{\nabla}_\parallel \frac{p_{\perp i} \tilde{u}_{\parallel i}}{B_0^2} - i\omega_d (\tilde{r}_{\parallel,\perp} + \tilde{r}_{\perp,\perp}) - B_0^2 \nabla_\parallel \frac{\tilde{q}_{\perp i}}{B_0^2}$$

FLR effect

Continuity

Compression

Landau damping

Toroidal closure

Full set of electron equations in 3+1 GLF model

$$\frac{\partial A_\parallel}{\partial t} = -\tilde{\nabla}_\parallel \phi + \frac{B_0}{n_0 e}\tilde{\nabla}_\parallel \frac{\tilde{p}_{\parallel e}}{B_0} + \frac{\eta}{\mu_0}\nabla_\perp^2 A_\parallel - \frac{\eta_H}{\mu_0}\nabla_\perp^4 A_\parallel$$

$$\frac{\partial \tilde{p}_{\parallel e}}{\partial t} = -\frac{1}{B_0} b_0 \times \nabla\phi \cdot \nabla p_{\parallel e} - \frac{4p_{\parallel e}}{B_0} b_0 \times \kappa \cdot \nabla\phi - 3B_0 \tilde{\nabla}_\parallel \frac{p_{\parallel e}\tilde{u}_{\parallel e}}{B_0} + \frac{T_{\parallel e}}{eB_0} b_0 \times \kappa \cdot \nabla(7\tilde{p}_{\parallel e} + \tilde{p}_{\perp e} - 4T_{e0}\tilde{n}_e) - B_0 \nabla_\parallel \frac{\tilde{q}_{\parallel e}}{B_0}$$

$$\frac{\partial \tilde{p}_{\perp e}}{\partial t} = -\frac{1}{B_0} b_0 \times \nabla\phi \cdot \nabla p_{\perp e} - \frac{3p_{\perp e}}{B_0} b_0 \times \kappa \cdot \nabla\phi - B_0^2 \tilde{\nabla}_\parallel \frac{p_{\perp e}\tilde{u}_{\parallel e}}{B_0^2} + \frac{T_{\perp e}}{eB_0} b_0 \times \kappa \cdot \nabla(\tilde{p}_{\parallel e} + 5\tilde{p}_{\perp e} - T_{e0}\tilde{n}_e) - B_0^2 \nabla_\parallel \frac{\tilde{q}_{\perp e}}{B_0^2}$$

$$\frac{\partial \tilde{\varpi}_G}{\partial t} = -\frac{1}{B_0} b_0 \times \nabla\phi \cdot \nabla \varpi_G + B_0^2 \tilde{\nabla}_\parallel \frac{\tilde{J}_\parallel}{B_0} + b_0 \times \kappa \cdot \nabla(\tilde{p}_{\parallel i} + \tilde{p}_{\perp i} + \tilde{p}_{\parallel e} + \tilde{p}_{\perp e})$$

$$+eB_0 b_0 \times \nabla\phi_f \cdot \nabla n_i + \frac{eB_0 n_0}{T_{i0}} b_0 \times \nabla\left(\hat{\nabla}_\perp^2 \Phi\right) \cdot \nabla T_{\perp i} + eB^2(\delta\bar{\boldsymbol{b}} - \delta\boldsymbol{b}) \cdot \nabla \frac{n_0 u_{\parallel i}}{B_0} + en_0 b_0 \times \kappa \cdot \nabla\left(2\phi_f + \hat{\nabla}_\perp^2 \Phi\right)$$

FLR effect

Continuity

Compression

Landau damping

Energy flux

Drift Alfven Wave

Poisson equation: $\varpi_G = eB\left[\bar{n}_i - \tilde{n}_i - n_0(1-\Gamma_0)\frac{e\phi}{T_0} + \frac{e\rho_i^2}{T_0}\nabla n_0 \cdot \nabla(\Gamma_0 - \Gamma_1)\phi\right]$

Carefully chosen closures are essential to match kinetic effects

Gyro averaging and Padé approximation:

$$\boldsymbol{v}_\Phi = \frac{1}{B}\boldsymbol{b}\times\nabla\Phi$$

$$\boldsymbol{v}_{\bar{A}_\parallel} = \frac{1}{B}\boldsymbol{b}\times\nabla\bar{A}_\parallel$$

$$(\Phi, \bar{A}_\parallel) = \Gamma_0^{\frac{1}{2}}(\phi, A_\parallel)$$

$$\begin{cases} \Gamma_0^{\frac{1}{2}}(b) \approx \frac{1}{1+b/2} \\ \Gamma_0(b) \approx \frac{1}{1+b} \\ \Gamma_0 - \Gamma_1 \approx 1 \end{cases} \qquad b = -\rho_i^2\nabla_\perp^2$$

Ion Landau closures:

$$\tilde{q}_{\parallel i} = -n_0\sqrt{\frac{8}{\pi}}v_{T_{th}}\frac{ik_\parallel\tilde{T}_{\parallel i}}{|k_\parallel|}$$

$$\tilde{q}_{\perp i} = -n_0\sqrt{\frac{2}{\pi}}v_{T_{th}}\frac{ik_\parallel}{|k_\parallel|}\left(\tilde{T}_{\perp i} + \frac{e}{2}\hat{\nabla}_\perp^2\Phi\right)$$

Electron Landau closures:

$$\tilde{q}_{\parallel e} = -n_0\sqrt{\frac{8}{\pi}}v_{T_{th}}\frac{ik_\parallel\tilde{T}_{\parallel e}}{|k_\parallel|}$$

$$\tilde{q}_{\perp e} = -n_0\sqrt{\frac{2}{\pi}}v_{T_{th}}\frac{ik_\parallel\tilde{T}_{\perp e}}{|k_\parallel|}$$

Toroidal closures:

$$i\omega_d\left(\tilde{r}_{\parallel,\parallel} + \tilde{r}_{\parallel,\perp}\right) = i\omega_d\left(7\tilde{p}_\parallel + \tilde{p}_\perp - 4T_0\tilde{n} - 2i\frac{|\omega_d|}{\omega_d}\left(\nu_1\tilde{T}_\parallel + \nu_2\tilde{T}_\perp\right)\right)$$

$$i\omega_d\left(\tilde{r}_{\parallel,\perp} + \tilde{r}_{\perp,\perp}\right) = i\omega_d\left(\tilde{p}_\parallel + 5\tilde{p}_\perp - 3T_0\tilde{n} - 2i\frac{|\omega_d|}{\omega_d}\left(\nu_3\tilde{T}_\parallel + \nu_4\tilde{T}_\perp\right)\right)$$

$$i\omega_d\left(\tilde{q}_{\parallel i} + \tilde{q}_{\perp i}\right) = 2n_0T_0\nu_5|\omega_d|\tilde{u}_{\parallel i}$$

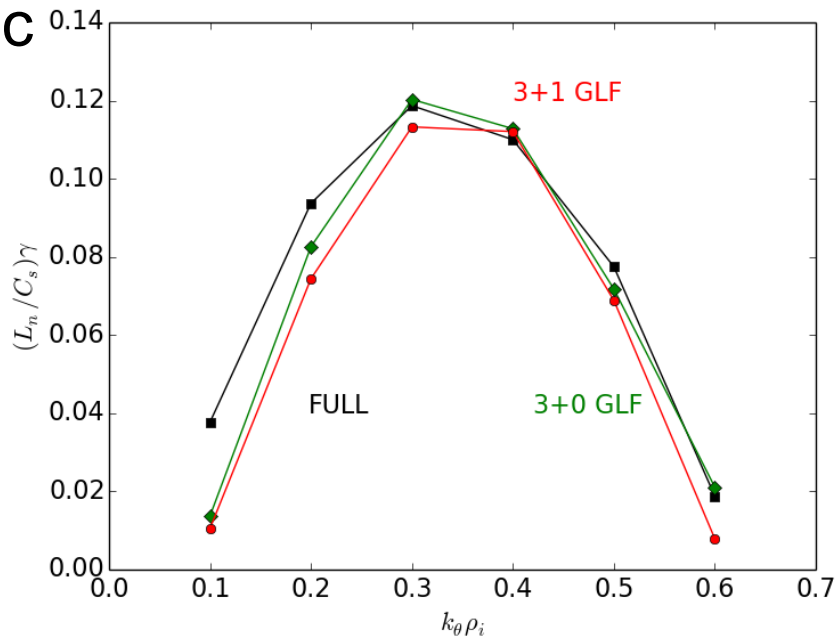
Energy flux

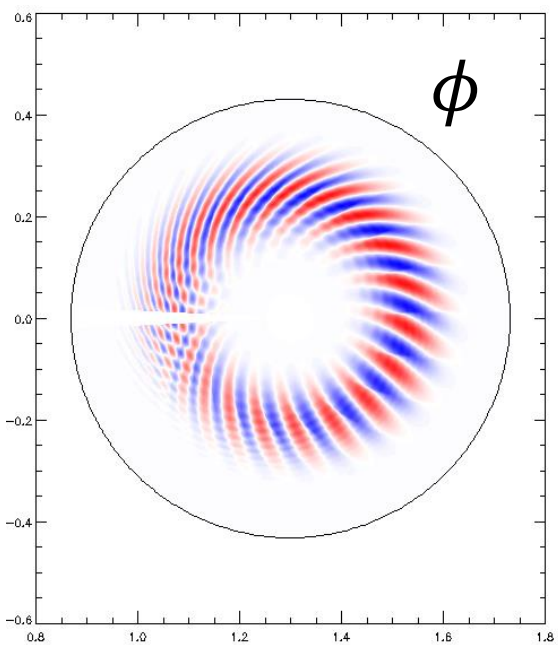
Toroidal closure 2 (Imaginary)

Toroidal closure 3 (Real)

Our 3+1 GLF code is benchmarked with other gyro-kinetic and gyro-fluid code in ES ITG simulations

- With adiabatic electron response, the 3+1 GLF results are in good agreement with other gyrofluid code (GLF 3+0) and gyorokinetic code (FULL).

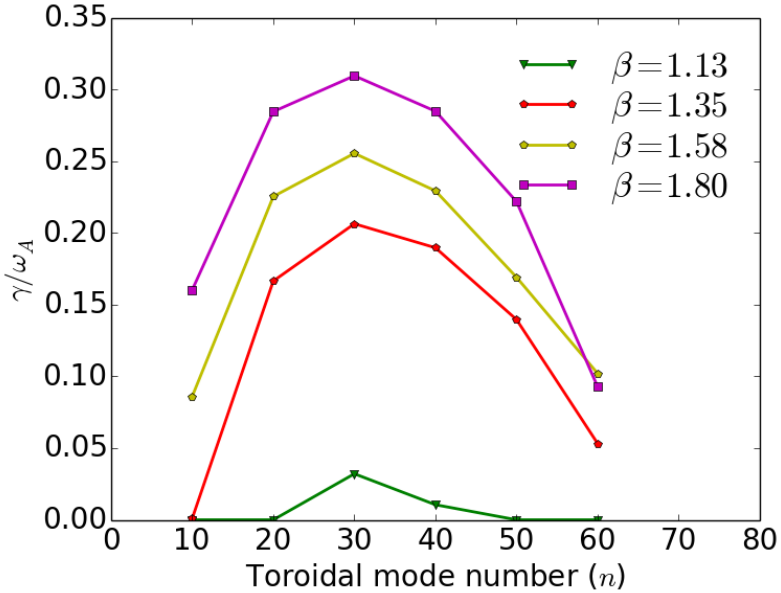


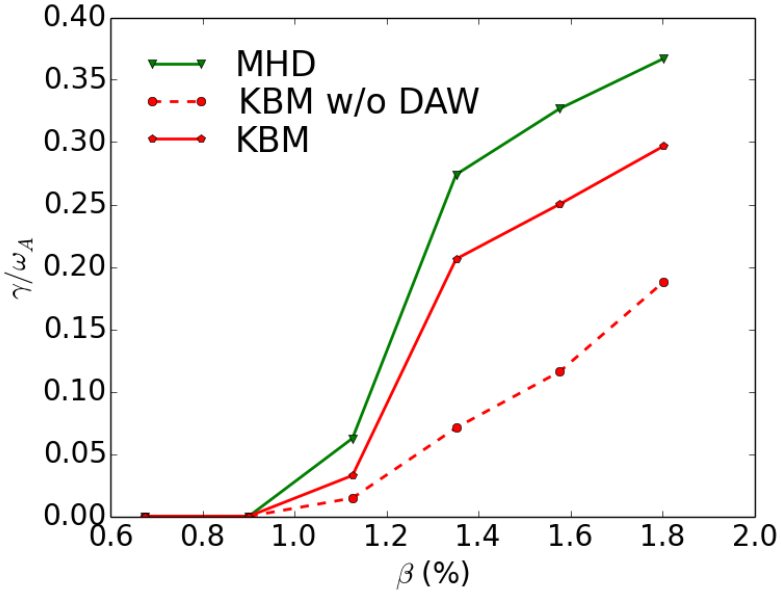


Adiabatic response:

$$\frac{n_0\tilde{\phi}}{T_{e0}} + \frac{n_0\tilde{\phi}}{T_{i0}}(1-\Gamma_0)\tilde{\phi} = \Gamma_0^{1/2}\tilde{n}_i + \frac{n_0}{T_{i0}} b \frac{\partial \Gamma_0^{1/2}}{\partial b}\tilde{T}_{i\perp}$$

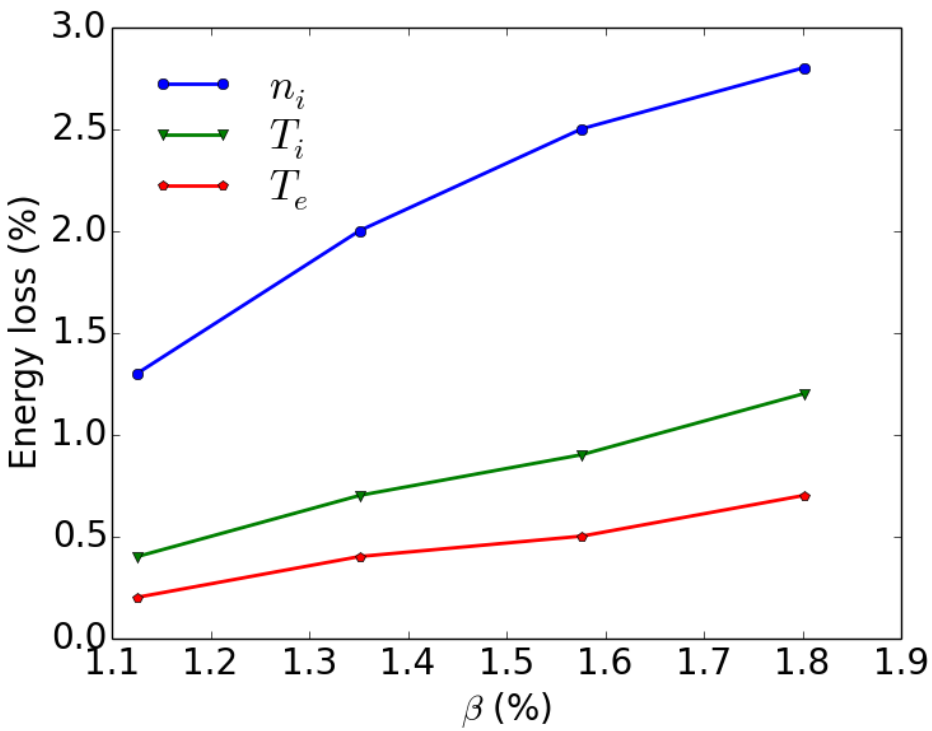
Kinetic physics has stabilizing effects on ballooning modes





- The linear growth rate of Kinetic ballooning mode increases when beta increases;
- Kinetic effects decrease the growth rate of ballooning mode;
- The drift Alfvén wave has the destabilizing effect on the mode;
- This is KBM because there is no instability without curvature drive;
- Since $\eta_i < 1$, threshold of KBM and IBM is about the same.

The relative energy loss increases with increasing beta



- When beta increases, the relative energy loss of initial crash from all channels increase;
- Convective energy loss is dominant because of the large density height;
- Energy loss from ion is larger than the loss from electron;
- Electron perturbation is damped by the Landau damping effect, which is larger than ions by a factor of $\sqrt{m_i/m_e}$.

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Basic normalized quantities

- Normalization parameters: $(\bar{L}, \bar{T}, \bar{N}, \bar{B})$

$$\bar{V} = \bar{L}/\bar{T}\,, \bar{V}^2 = V_A^2 = \bar{B}^2/\mu_0 m_i \bar{N}\,, \bar{\Omega} = e\bar{B}/m_i\,, C_{nor} = \bar{\Omega}\bar{T}$$

$$\hat{t} = \frac{t}{\bar{T}}, \quad \hat{B} = \frac{B}{\bar{B}}, \quad \hat{\nabla} = \bar{L}\nabla, \quad \hat{\kappa} = \bar{L}\kappa.$$

- Define

$$\psi = A_{\|}/B, \Psi = \bar{A}_{\|}/B, U = \tilde{\varpi}_G/m_i$$

- Evolving variables

$$\hat{n} = \frac{\tilde{n}}{\bar{N}},$$

$$\hat{u}_{\|} = \frac{\tilde{u}_{\|}}{\bar{V}},$$

$$\hat{p} = \frac{\tilde{p}}{m_i \bar{V}^2 \bar{N}},$$

$$\hat{U} = \frac{\bar{T}}{\bar{N}} U,$$

$$\hat{\psi} = \frac{\psi}{\bar{L}}.$$

other important variables

$$\hat{T} = \frac{\tilde{T}}{m_i \bar{V}^2},$$

$$\hat{\eta} = \frac{\bar{T}}{\bar{L}^2 \mu_0} \eta,$$

$$\hat{J}_{\|c} = \bar{L} J_{\|c},$$

$$\hat{\varpi}_G = \frac{\varpi_G}{e\bar{B}\bar{N}},$$

$$\hat{\phi} = \frac{\phi}{\bar{L}\bar{V}\bar{B}}.$$

Compare the normalization of 3-field, 6-field, and 3+1

- Normalization parameters: $(\bar{L}, \bar{T}, \bar{N}, \bar{B})$ same!

$$\bar{V} = \bar{L}/\bar{T}\,, \bar{V}^2 = V_A^2 = \bar{B}^2/\mu_0 m_i \bar{N}$$

- Difference:

	3-field	6-field Landau-fluid	3+1
$\hat{\phi}$	$\frac{\phi}{\bar{L}\bar{V}B_0}$	$\frac{\phi}{\bar{L}\bar{V}B_0}$	$\frac{\phi}{\bar{L}\bar{V}\bar{B}}$
$\hat{n}$	$\frac{n}{\bar{N}}$	$\frac{n}{\bar{N}}$	$\frac{n}{\bar{N}}$
$\hat{T}$	$\frac{2T_j}{m_i\bar{V}^2}$	$\frac{T_j}{\bar{T}_j}$	$\frac{T_j}{m_i\bar{V}^2}$
$\hat{P}$	$\frac{2P_j}{m_i\bar{V}^2\bar{N}}$	$\frac{P_j}{k_B\bar{N}\bar{T}_j}$	$\frac{P_j}{m_i\bar{V}^2\bar{N}}$

Normalized ion equations

$$\frac{\partial \hat{n}_i}{\partial \hat{t}} = -[\hat{\Phi}, \hat{n}_{G0}] - [\hat{\Phi}_0, \hat{n}_{G0}] - [\hat{\Phi}, \hat{n}_i] - \frac{1}{C_{nor}\hat{B}_0} b_0 \times \hat{\kappa} \cdot \hat{\nabla}(\hat{p}_{\parallel i} + \hat{p}_{\perp i})$$
$$- \frac{\hat{n}_0}{\hat{B}_0} b_0 \times \hat{\kappa} \cdot \hat{\nabla}\left(2 + \frac{1}{2}\hat{\nabla}_\perp^2\right)\hat{\Phi} - \frac{\hat{n}_i}{\hat{B}_0} b_0 \times \hat{\kappa} \cdot \hat{\nabla}\left(2 + \frac{1}{2}\hat{\nabla}_\perp^2\right)\hat{\Phi}$$
$$- \hat{n}_0 \hat{B}_0 \hat{\nabla}_\parallel \frac{\hat{u}_{\parallel i}}{\hat{B}_0} - \hat{n}_i \hat{B}_0 \hat{\nabla}_\parallel \frac{\hat{u}_{\parallel i}}{\hat{B}_0} - \frac{\hat{n}_0}{2\hat{T}_0}[\hat{\nabla}_\perp^2 \hat{\Phi}, \hat{T}_{i0}] - \frac{1}{2}\left[\hat{\nabla}_\perp^2 \hat{\Phi}, \frac{\hat{n}_0 \hat{T}_{\perp i}}{\hat{T}_{i0}}\right]$$

$$\frac{\partial \hat{u}_{\parallel i}}{\partial \hat{t}} = -[\hat{\Phi}, \hat{u}_{\parallel i}] - \frac{1}{\tilde{n}_0}\tilde{\nabla}_\parallel \hat{p}_\parallel + \frac{\hat{B}_0}{\hat{n}_0}[\hat{\psi}, \hat{P}_0]$$
$$- \frac{4}{C_{nor}\hat{B}_0} b_0 \times \hat{\kappa} \cdot \hat{\nabla}\hat{T}_{i0}\hat{u}_{\parallel i} - \frac{\hat{p}_{\perp i}}{\hat{n}_0 \hat{B}_0}\nabla_{\parallel 0} B_0 - \frac{C_{nor}}{B_0}\hat{\nabla}_\perp^2 \hat{\Phi}\hat{\nabla}_{\parallel 0} B - \frac{1}{\hat{p}_{i0}} i\omega_d(\hat{q}_{\parallel i} + \hat{q}_{\perp i})$$

$$\frac{\partial \hat{p}_{\parallel i}}{\partial \hat{t}} = -[\hat{\Phi}, \hat{p}_{i0}] - [\hat{\Phi}, \hat{p}_{\parallel i}] - \frac{\hat{n}_0}{2}[\hat{\nabla}_\perp^2 \Phi, \hat{T}_{i0}] - \frac{\hat{n}_0}{2}[\hat{\nabla}_\perp^2 \hat{\Phi}, \hat{T}_{\perp i}]$$
$$- \frac{\hat{p}_{i0}}{\hat{B}_0} b_0 \times \hat{\kappa} \cdot \hat{\nabla}\left(4 + \frac{1}{2}\hat{\nabla}_\perp^2\right)\hat{\Phi} - \frac{\hat{p}_{\parallel i}}{\hat{B}_0} b_0 \times \hat{\kappa} \cdot \hat{\nabla}\left(4 + \frac{1}{2}\hat{\nabla}_\perp^2\right)\hat{\Phi}$$
$$- 3\hat{B}_0 \hat{\nabla}_\parallel \frac{\hat{p}_{i0}\hat{u}_{\parallel i}}{\hat{B}_0} - 3\hat{B}_0 \hat{\nabla}_{\parallel 0} \frac{\hat{p}_{\parallel i}\hat{u}_{\parallel i}}{\hat{B}_0} - i\omega_d(\hat{r}_{\parallel,\parallel} + \hat{r}_{\parallel,\perp}) - \hat{B}_0 \hat{\nabla}_\parallel \frac{\hat{q}_{\parallel i}}{\hat{B}_0}$$

$$\frac{\partial \hat{p}_{\perp i}}{\partial \hat{t}} = -[\hat{\Phi}_2, \hat{p}_{i0}] - [\hat{\Phi}_2, \hat{p}_{\parallel i}] - \hat{n}_0[\hat{\hat{\nabla}}_\perp^2 \hat{\Phi}, \hat{T}_{i0}] - \hat{n}_0[\hat{\hat{\nabla}}_\perp^2 \hat{\Phi}, \hat{T}_{\perp i}]$$
$$- \frac{\hat{p}_{i0}}{\hat{B}_0} b_0 \times \hat{\kappa} \cdot \hat{\nabla}\left(3 + \frac{3}{2}\hat{\nabla}_\perp^2 + \hat{\hat{\nabla}}_\perp^2\right)\hat{\Phi} - \frac{\hat{p}_{\perp i}}{\hat{B}_0} b_0 \times \hat{\kappa} \cdot \hat{\nabla}\left(3 + \frac{3}{2}\hat{\nabla}_\perp^2 + \hat{\hat{\nabla}}_\perp^2\right)\hat{\Phi}$$
$$- \hat{B}_0^2 \hat{\nabla}_\parallel \frac{\hat{p}_{i0}\hat{u}_{\parallel i}}{\hat{B}_0^2} - \hat{B}_0^2 \hat{\nabla}_{\parallel 0} \frac{\hat{p}_{\perp i}\hat{u}_{\parallel i}}{B_0^2} - i\omega_d(\hat{r}_{\parallel,\perp} + \hat{r}_{\perp,\perp}) - \hat{B}_0^2 \hat{\nabla}_{\parallel 0} \frac{\hat{q}_{\perp i}}{\hat{B}_0^2} \quad (66$$

Normalized electron equations

$$\frac{\partial \hat{\psi}}{\partial \hat{t}} = -\frac{1}{\hat{B}_0}\hat{\nabla}_{\parallel}\hat{\phi} + \frac{1}{C_{nor}\hat{n}_0\hat{B}_0}\hat{\nabla}_{\parallel}\hat{p}_{\parallel e} + \frac{1}{C_{nor}\hat{n}_0}[\hat{\psi}, \hat{P}_{e0}] + \hat{\eta}\hat{J}_{\parallel c} - \hat{\eta}_H\hat{\nabla}_{\perp}^2\hat{J}_{\parallel c}$$

$$\frac{\partial \hat{p}_{\parallel e}}{\partial \hat{t}} = -[\hat{\phi}, \hat{p}_{e0}] - [\hat{\phi}, \hat{p}_{\parallel e}] - \frac{4\hat{p}_{e0}}{\hat{B}_0}b_0 \times \hat{\kappa}\cdot\hat{\nabla}\hat{\phi} - \frac{4\hat{p}_{\parallel e}}{\hat{B}_0}b_0 \times \hat{\kappa}\cdot\hat{\nabla}\hat{\phi}$$
$$- 3\hat{B}_0\hat{\nabla}_{\parallel}\frac{\hat{p}_{e0}\hat{u}_{\parallel e}}{B_0} - 3\hat{B}_0\hat{\nabla}_{\parallel 0}\frac{\hat{p}_{\parallel e}\hat{u}_{\parallel e}}{B_0}$$
$$+\frac{\hat{T}_{e0}}{C_{nor}\hat{B}_0}b_0\times\hat{\kappa}\cdot\hat{\nabla}(7\hat{p}_{\parallel e}+\hat{p}_{\perp e}-4\hat{T}_{e0}\hat{n}_e)+\frac{\hat{T}_{\parallel e}}{C_{nor}\hat{B}_0}b_0\times\hat{\kappa}\cdot\hat{\nabla}(7\hat{p}_{\parallel e}+\hat{p}_{\perp e}-4\hat{T}_{e0}\hat{n}_e)-\hat{B}_0\hat{\nabla}_{\parallel}\frac{\hat{q}_{\parallel e}}{\hat{B}_0}$$

$$\frac{\partial \hat{p}_{\perp e}}{\partial \hat{t}} = -[\hat{\phi}, \hat{p}_{e0}] - [\hat{\phi}, \hat{p}_{\perp e}] - \frac{3\hat{p}_{e0}}{\hat{B}_0}b_0 \times \hat{\kappa}\cdot\hat{\nabla}\hat{\phi} - \frac{3\hat{p}_{\perp e}}{\hat{B}_0}b_0 \times \hat{\kappa}\cdot\hat{\nabla}\hat{\phi}$$
$$- \hat{B}_0^2\hat{\nabla}_{\parallel}\frac{\hat{p}_{e0}\hat{u}_{\parallel e}}{\hat{B}_0^2} - \hat{B}_0^2\hat{\nabla}_{\parallel 0}\frac{\hat{p}_{\perp e}\hat{u}_{\parallel e}}{\hat{B}_0^2}$$
$$+\frac{\hat{T}_{e0}}{C_{nor}\hat{B}_0}b_0\times\hat{\kappa}\cdot\hat{\nabla}(\hat{p}_{\parallel e}+5\hat{p}_{\perp e}-\hat{T}_{e0}\hat{n}_e)+\frac{\hat{T}_{\perp e}}{C_{nor}\hat{B}_0}b_0\times\hat{\kappa}\cdot\hat{\nabla}(\hat{p}_{\parallel e}+5\hat{p}_{\perp e}-\hat{T}_{e0}\hat{n}_e)-\hat{B}_0^2\hat{\nabla}_{\parallel}\frac{\hat{q}_{\perp e}}{\hat{B}_0^2}$$

Normalized vorticity equations

$$\frac{\partial \hat{U}}{\partial \hat{t}} = -[\hat{\phi}, \hat{U}_0] - [\hat{\phi}, \hat{U}] - \frac{V_A^2}{\bar{V}^2}\hat{B}_0^2 \hat{\nabla}_{\parallel} \hat{J}_{\parallel c} - \hat{B}_0^3 [\hat{\psi}, \hat{J}_0] + b_0 \times \hat{\kappa} \cdot \hat{\nabla}(\hat{p}_{\parallel i} + \hat{p}_{\perp i} + \hat{p}_{\parallel e} + \hat{p}_{\perp e})$$

$$+ C_{nor}\hat{B}_0[\hat{\phi}_f, \hat{n}_0] + C_{nor}\hat{B}_0[\hat{\phi}_f, \hat{n}_i] + \frac{C_{nor}\hat{B}_0\hat{n}_0}{\hat{T}_{i0}}[\hat{\nabla}_{\perp}^2\hat{\Phi}, \hat{T}_{i0}] + \frac{C_{nor}\hat{B}_0\hat{n}_0}{\hat{T}_{i0}}[\hat{\nabla}_{\perp}^2\hat{\Phi}, \hat{T}_{\perp i}]$$

$$- C_{nor}\hat{B}_0^3 \left[\hat{\Psi} - \hat{\psi}, \frac{\hat{n}_0 \hat{u}_{\parallel i}}{\hat{B}_0}\right] + C_{nor}\hat{n}_0 b_0 \times \hat{\kappa} \cdot \hat{\nabla}\left(2\hat{\phi}_f + \hat{\nabla}_{\perp}^2\hat{\Phi}\right) \quad (81)$$

Where:

$$U = \frac{\varpi_G}{m_i} \qquad \hat{\varpi}_G = \hat{B}(\hat{n}_e - \hat{n}_i), \qquad \hat{U} = C_{nor}\hat{\varpi}_G.$$

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Poisson equation

- The gyro-kinetic Poisson equation and substituted to U

$$U = \frac{eB}{m_i}\left[\bar{n}_i - \tilde{n}_i - n_0\left(1-\Gamma_0\right)\frac{e\phi}{T_0} + \frac{e\rho_i^2}{T_0}\nabla n_0 \cdot \nabla(\Gamma_0 - \Gamma_1)\phi\right]$$

- Define

$$\hat{n}_{mid} = \frac{\hat{T}_0}{C_{nor}\hat{n}_0}\left(\hat{\bar{n}}_i - \hat{n}_i - \frac{\hat{U}}{C_{nor}\hat{B}_0}\right)$$

- Poisson equation becomes

$$\left(1-\Gamma_0\right)\hat{\phi} - \frac{\hat{\rho}_i^2}{\hat{n}_0}\hat{\nabla}\hat{n}_0 \cdot \hat{\nabla}(\Gamma_0 - \Gamma_1)\hat{\phi} = \hat{n}_{mid}.$$

- With Pade approximation

$$\hat{\nabla}_\perp^2\hat{\phi} + \frac{1}{\hat{n}_0}\hat{\nabla}\hat{n}_0 \cdot \hat{\nabla}\hat{\phi} = -\frac{1}{\hat{\rho}_i^2}\hat{n}_{mid} + \hat{\nabla}_\perp^2\hat{n}_{mid}.$$

Gyro-average operators

- Gyro-average operator with Pade approximation

$$\Phi = \Gamma_0^{\frac{1}{2}}\phi = \frac{1}{1+\frac{b}{2}}\phi,$$

- becomes

$$-\frac{1}{2}\rho_i^2\nabla_\perp^2\Phi + \Phi = \phi.$$

Modified Laplacian operators

- Modified Laplacian operators can be expressed as the subtract of gyro-average operators:

$$
\begin{aligned}
\hat{\nabla}_\perp^2 \Phi =& 2b \frac{\partial \Gamma_0^{\frac{1}{2}}}{\partial b} \phi \\
=& -\frac{b}{\left(1+\frac{b}{2}\right)^2} \phi \\
=& 2\left[\frac{1}{\left(1+\frac{b}{2}\right)^2} - \frac{1}{1+\frac{b}{2}}\right] \phi \\
=& 2\left[\Phi_2 - \Phi\right],
\end{aligned}
$$

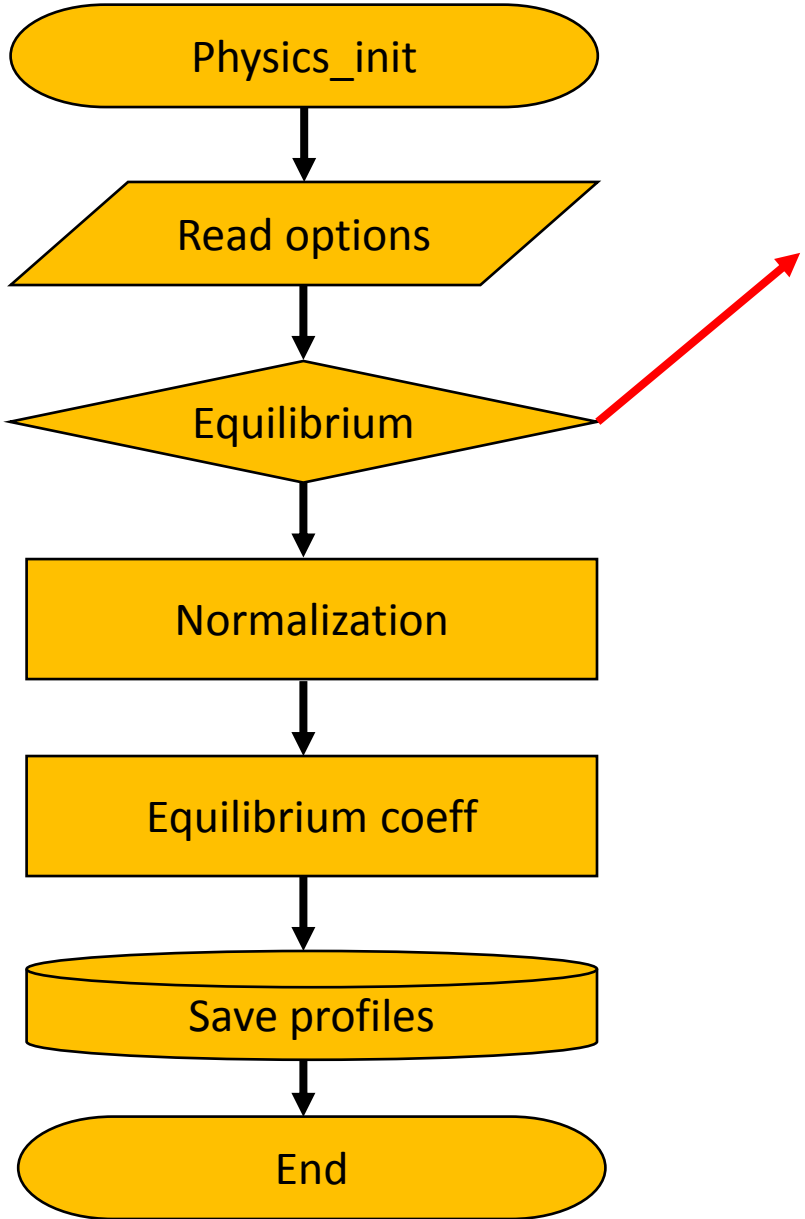
$$
\begin{aligned}
\hat{\hat{\nabla}}_\perp^2 \Phi =& b \frac{\partial^2}{\partial b^2} \left(b \Gamma_0^{\frac{1}{2}}\right) \phi \\
=& -\frac{b}{\left(1+\frac{b}{2}\right)^3} \phi \\
=& 2\left[\frac{1}{\left(1+\frac{b}{2}\right)^3} - \frac{1}{\left(1+\frac{b}{2}\right)^2}\right] \phi \\
=& 2\left[\Phi_3 - \Phi_2\right],
\end{aligned}
$$

where $\Phi_2 = \Gamma_0^{\frac{1}{2}} \Phi$ and $\Phi_3 = \Gamma_0^{\frac{1}{2}} \Phi_2$.

Outline

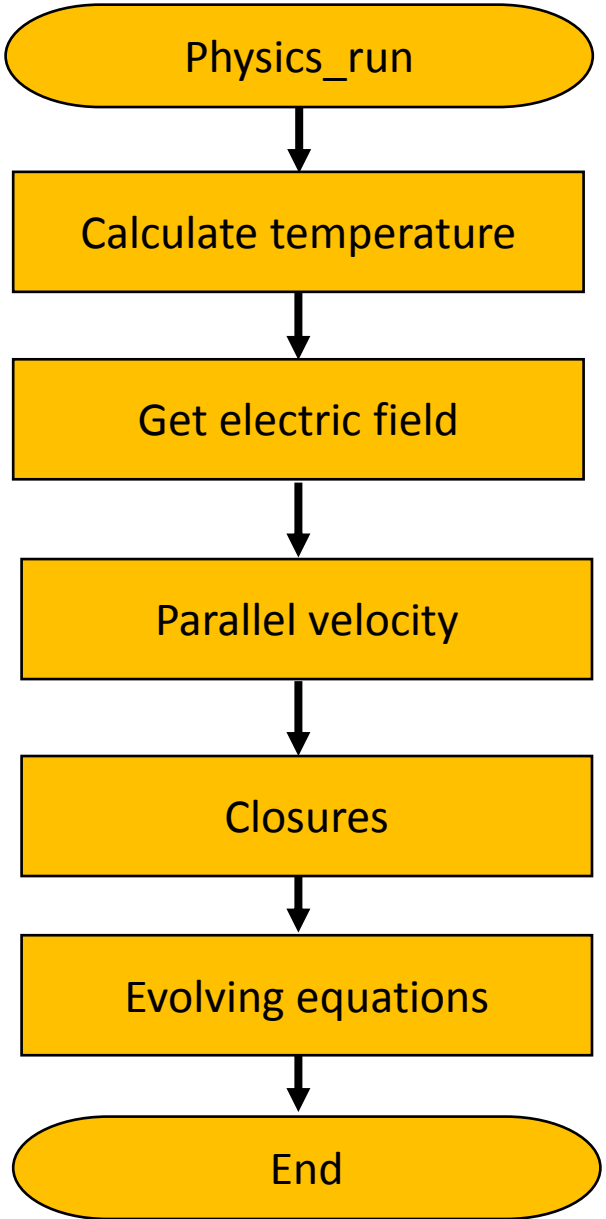
- Introduction
- Normalization
- Implementation
- Code structure
- Future work
- Hands-on exercise

Initialization



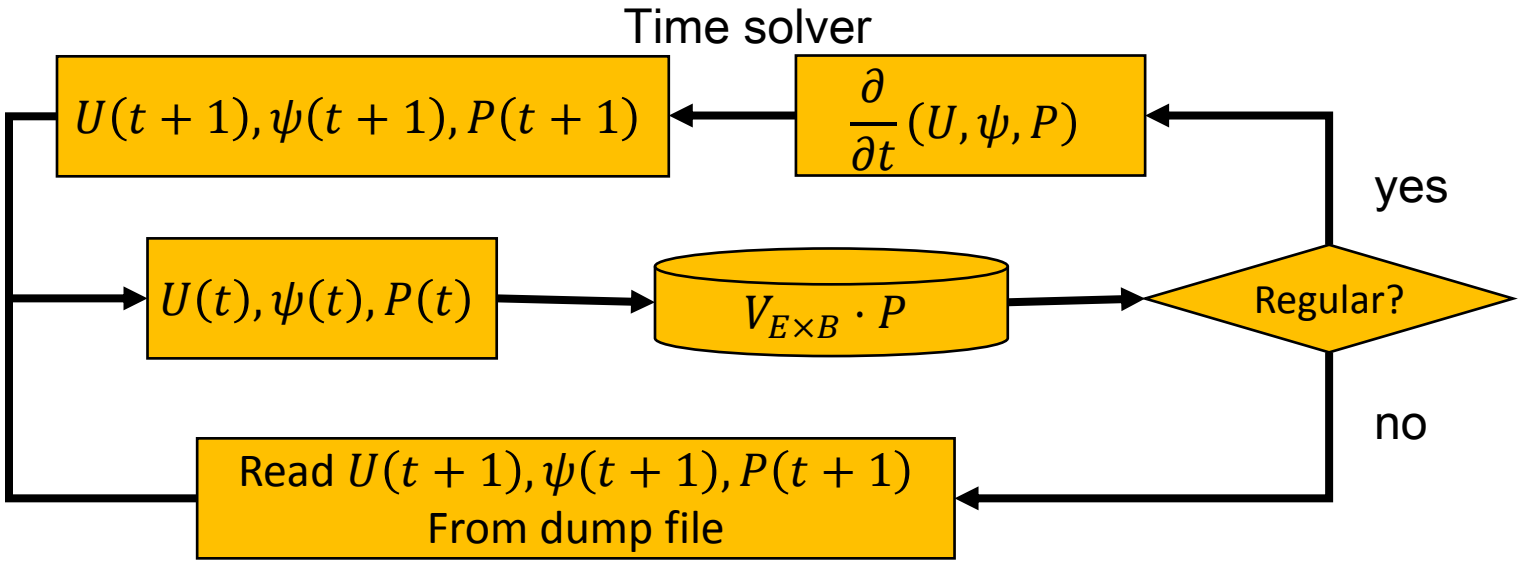
- Equilibrium cases (P0, N0, T0):
  - 1: η_i scan profiles;
  - 2: cyclone case;
  - 4: tanh function profiles;
  - 5: Self-consistent bootstrap current grid;
  - 6: Real geometry with experimental profiles.

Time evolution



Rerun feature

- Fakerun feature:
  - Calculate more intermediate results or interesting parameters (i.e. $V_{E\times B}\cdot P$);
  - Generate results from the dump file of previous run (faster than rerun).



- To use
  - Set fakerun=true
  - Set path="../glfkbm3-1/data" # The path to the data folder

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Future work

- Code development
  - Collisional Landau closure
  - Toroidal closure in X-point geometry
  - GLF thermal force
  - Isotropic pressure response
  - Sheath boundary condition
- Applications
  - KBM turbulent transport near marginal instability
  - Pedestal rebuild between two ELMs

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Install the 3+1 module

- Get the `bout_glf` git repository

```
git clone ssh://user@portal-auth.nersc.gov/project/
projectdirs/bout_glf/www/git/bout_glf.git
```

- Switch to the `modomegad` branch

```
git checkout bout_modomegad
```

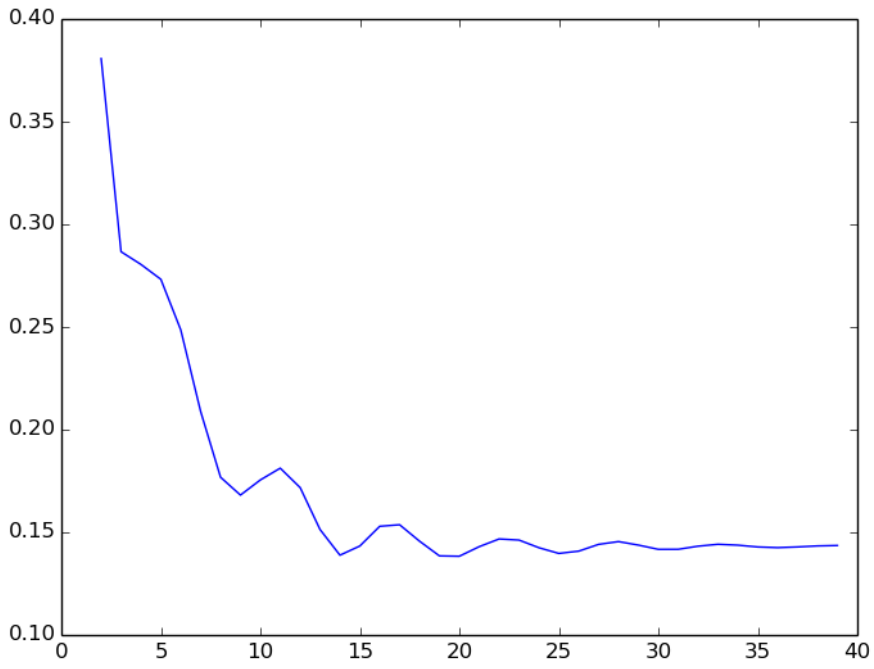
- Compile the code

```
./configure
make
cd examples/glfkbm3-1
make
```

Running example

- Launch the linear run job

```
qsub bout_hopper_debug.cmd
```

- Get linear growth rate

```
python growthrate.py
```