# ELM and KBM simulations in BOUT++

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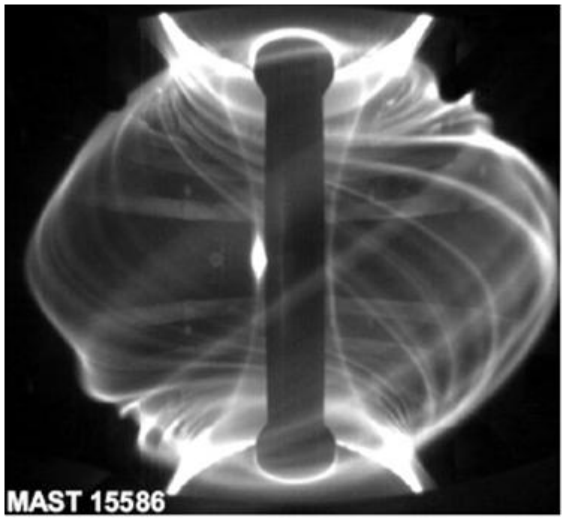
# OUTLINE

1. Introduction
2. P-B turbulence and ELM crashes
3. 3+1 gyrofluid model for KBM simulations
4. Summary

# Background : Peeling-ballooning model for ELMs

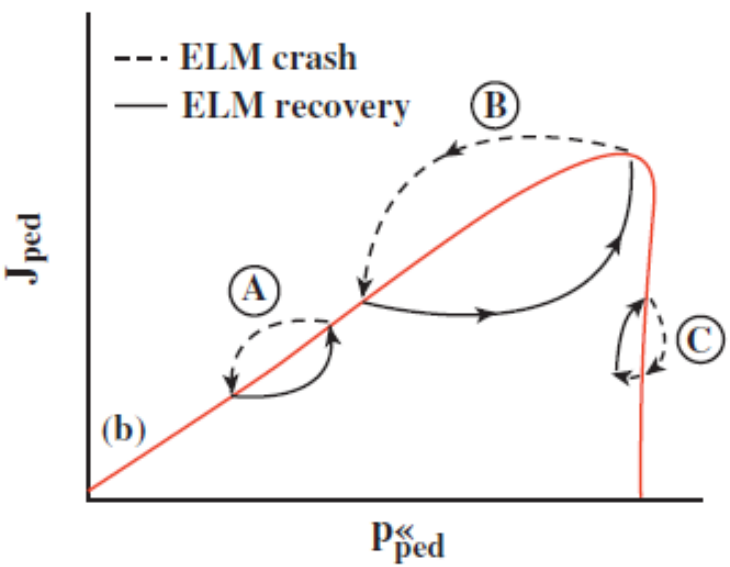



A. Kirk, PRL **96**, 185001 (2006)

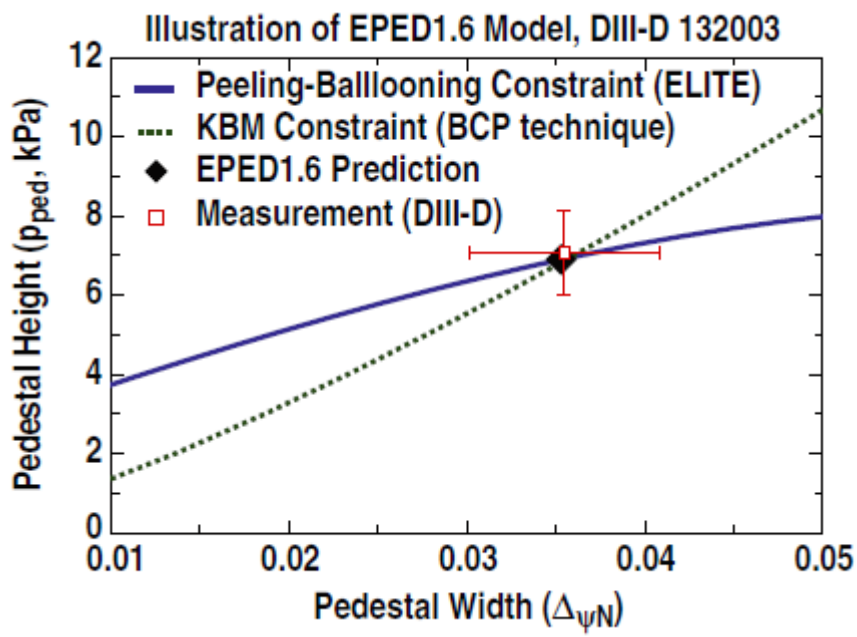
- **Peeling-ballooning model for ELMs**
  - ✓ ELM crash is triggered by linear peeling-ballooning instability;
  - ✓ Criterion for ELM crash:

$$\gamma_{PB} > 0$$

- **KBM at pedestal (EPED)**
  - ✓ KBM turbulence before ELM crash




P.B. Snyder, et.al *Nucl. Fusion* **47** (2007) 961




P.B. Snyder, et.al *Nucl. Fusion* **51** (2011) 103016

# What is the impact of KBM turbulence on ELM crashes?

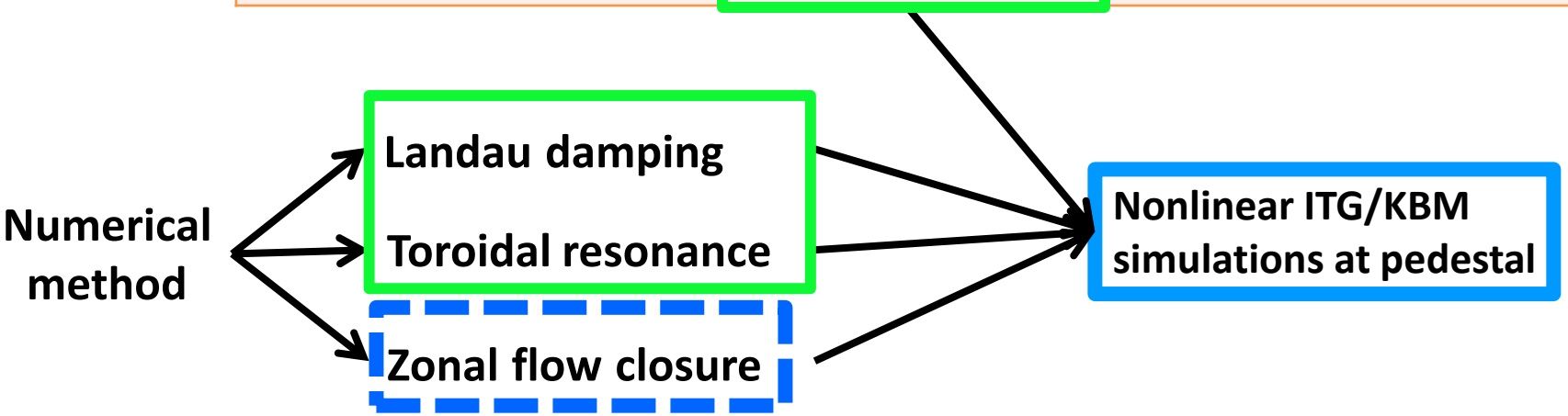
→The role of pedestal turbulence in ELM crash

→The characteristics of KBM turbulence at the pedestal

# BOUT++: A framework for nonlinear twofluid and gyrofluid simulations

- **Different twofluid and gyrofluid models are developed under BOUT++ framework for ELM simulations**

| Twofluid | Gyrofluid (FLR effect) | Physics |
|---|---|---|
| **3-field** $(\varpi, P, A_\parallel)$ | **1+0** $(n_{iG}, n_e, A_\parallel)$ | **Peeling-ballooning mode** |
| **4-field** $(\varpi, P, A_\parallel, V_\parallel)$ | **2+0** $(n_{iG}, n_e, A_\parallel, V_\parallel)$ | **+ Acoustic wave** |
| **6-field** $(\varpi, n_i, A_\parallel, V_\parallel, T_i, T_e)$ | **3+1** $(n_{iG}, n_e, A_\parallel, V_\parallel, T_{i\perp}, T_{i\parallel}, T_e)$ | **+ Thermal transport** |



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# Simulation model and equilibrium

- **3-field model for nonlinear ELM simulations**
  - ✓ **Including essential physics for the onset of ELMs**

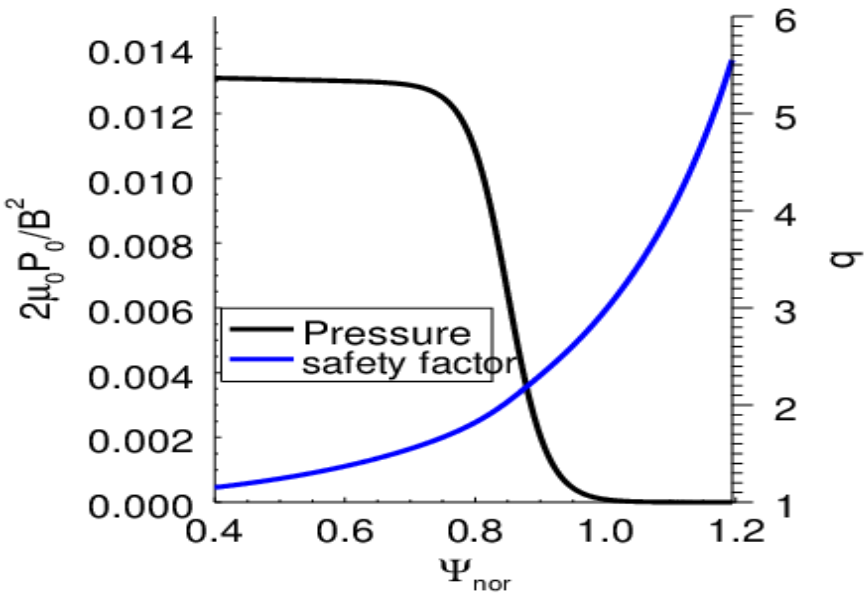
$$\frac{d\varpi}{dt} = B\nabla_{\parallel} J_{\parallel} + 2\mathbf{b}_0 \times \boldsymbol{\kappa} \cdot \nabla \tilde{P} + \mu_{i,\parallel} \partial_{\parallel}^2 \varpi$$

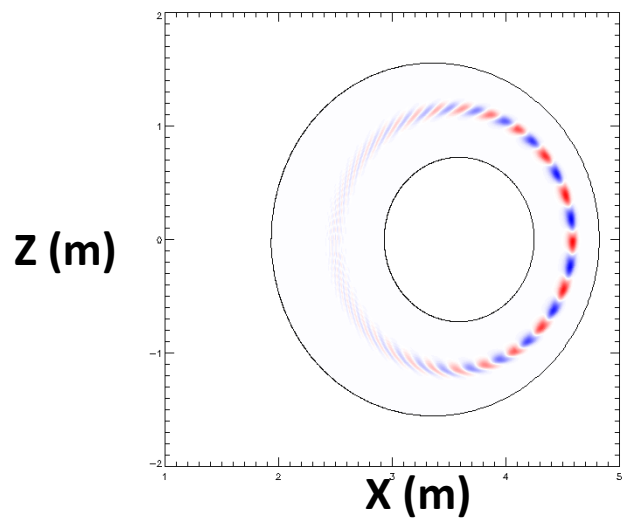
$$\frac{d\tilde{P}}{dt} + \mathbf{V}_E \cdot \nabla P_0 = 0$$

$$\frac{\partial A_{\parallel}}{\partial t} + \partial_{\parallel} \phi_T = \frac{\eta}{\mu_0} \nabla_{\perp}^2 A_{\parallel} - \frac{\eta_H}{\mu_0} \nabla_{\perp}^4 A_{\parallel}$$

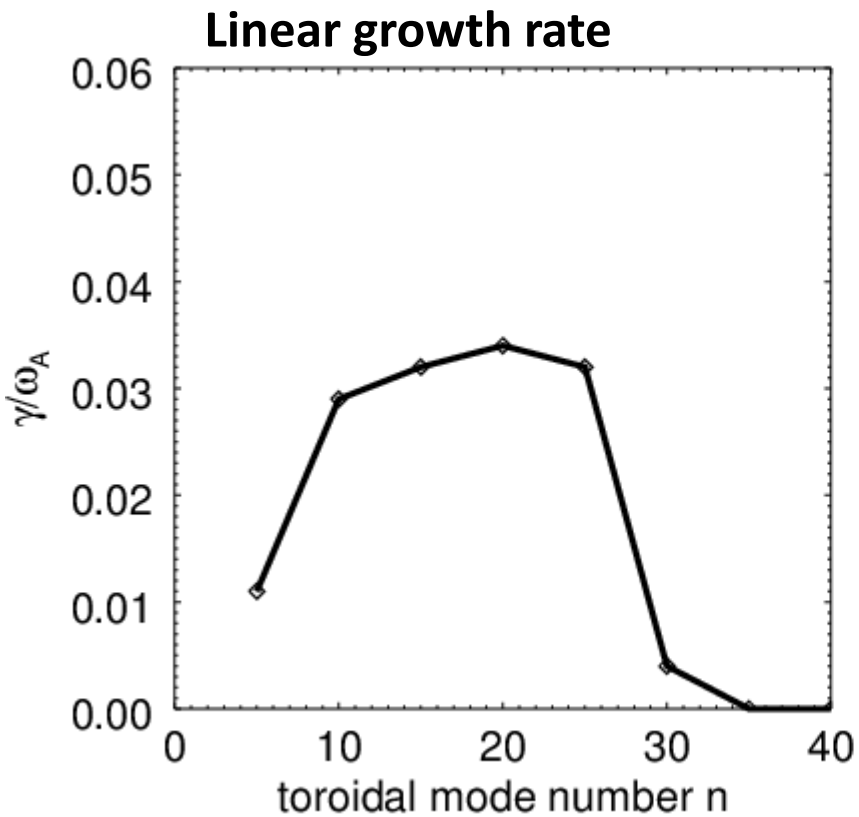
$$\varpi = \frac{m_i n_0}{B} \left( \nabla_{\perp}^2 \phi + \frac{1}{e n_0} \nabla_{\perp}^2 \tilde{P}_i \right)$$

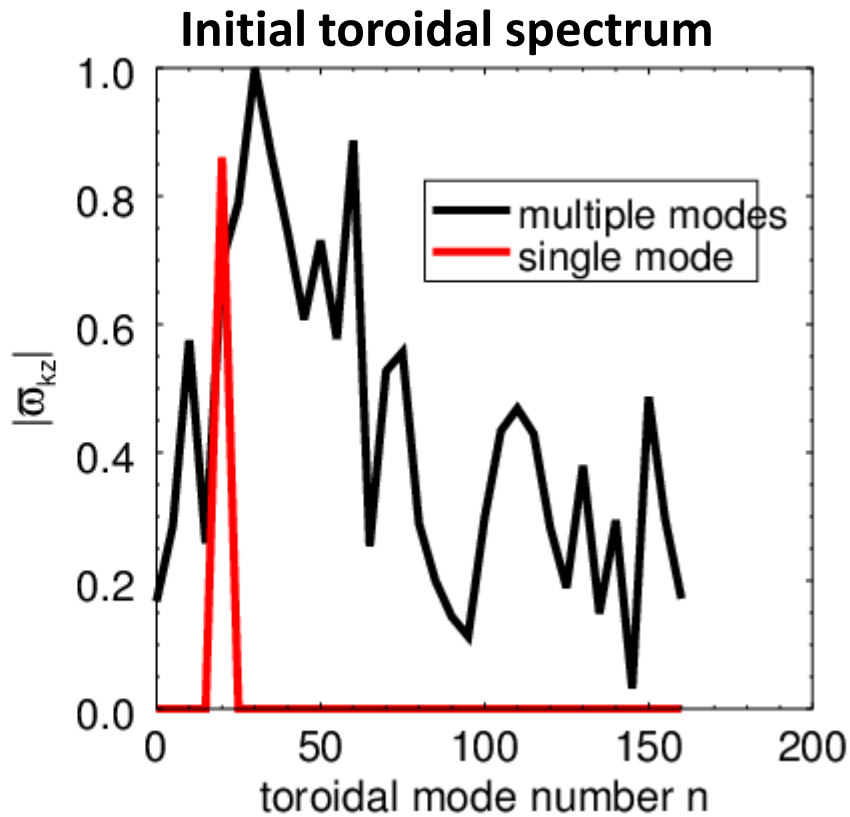
$$d/dt = \partial/\partial t + \mathbf{V}_{ET} \cdot \nabla,\ \mathbf{V}_{ET} = \frac{1}{B} \mathbf{b}_0 \times \nabla \phi_T,\ \phi_T = \phi_0 + \phi,\ \nabla_{\parallel} f = B \partial_{\parallel} \frac{f}{B},\ \partial_{\parallel} = \partial_{\parallel}^0 + \delta\mathbf{b} \cdot \nabla,\ \delta\mathbf{b} = \frac{1}{B} \nabla A_{\parallel} \times \mathbf{b}_0,\ J_{\parallel} = J_{\parallel 0} + \tilde{J}_{\parallel},\ \tilde{J}_{\parallel} = -\nabla_{\perp}^2 A_{\parallel} / \mu_0$$



# Initial perturbation: single mode and multiple modes




- **Peeling-ballooning unstable**
  - **ELM crash according to P-B model**

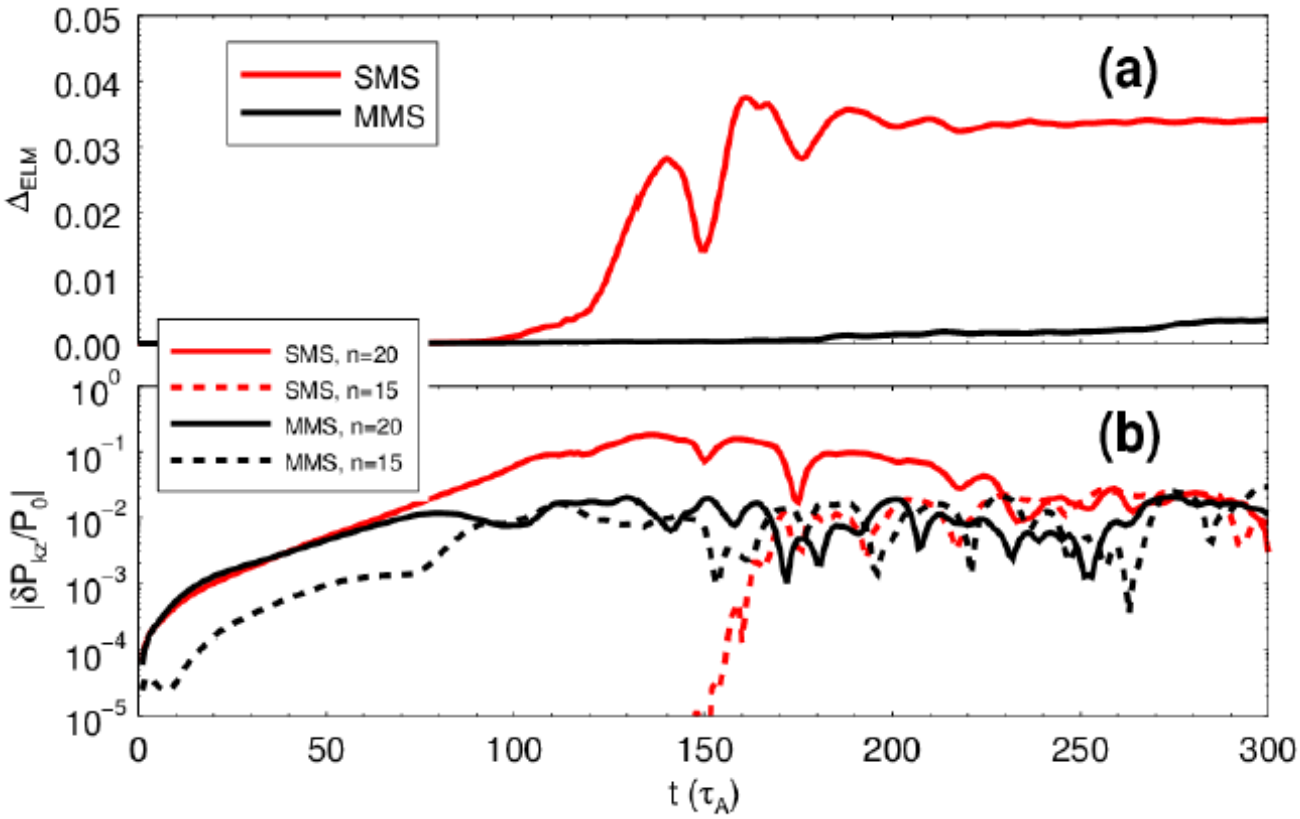
**Single mode** $\tilde{p}_s = A(x,y)e^{inz}$

**Multiple modes** $\tilde{p}_m = \sum_n A_n(x,y)e^{inz}$

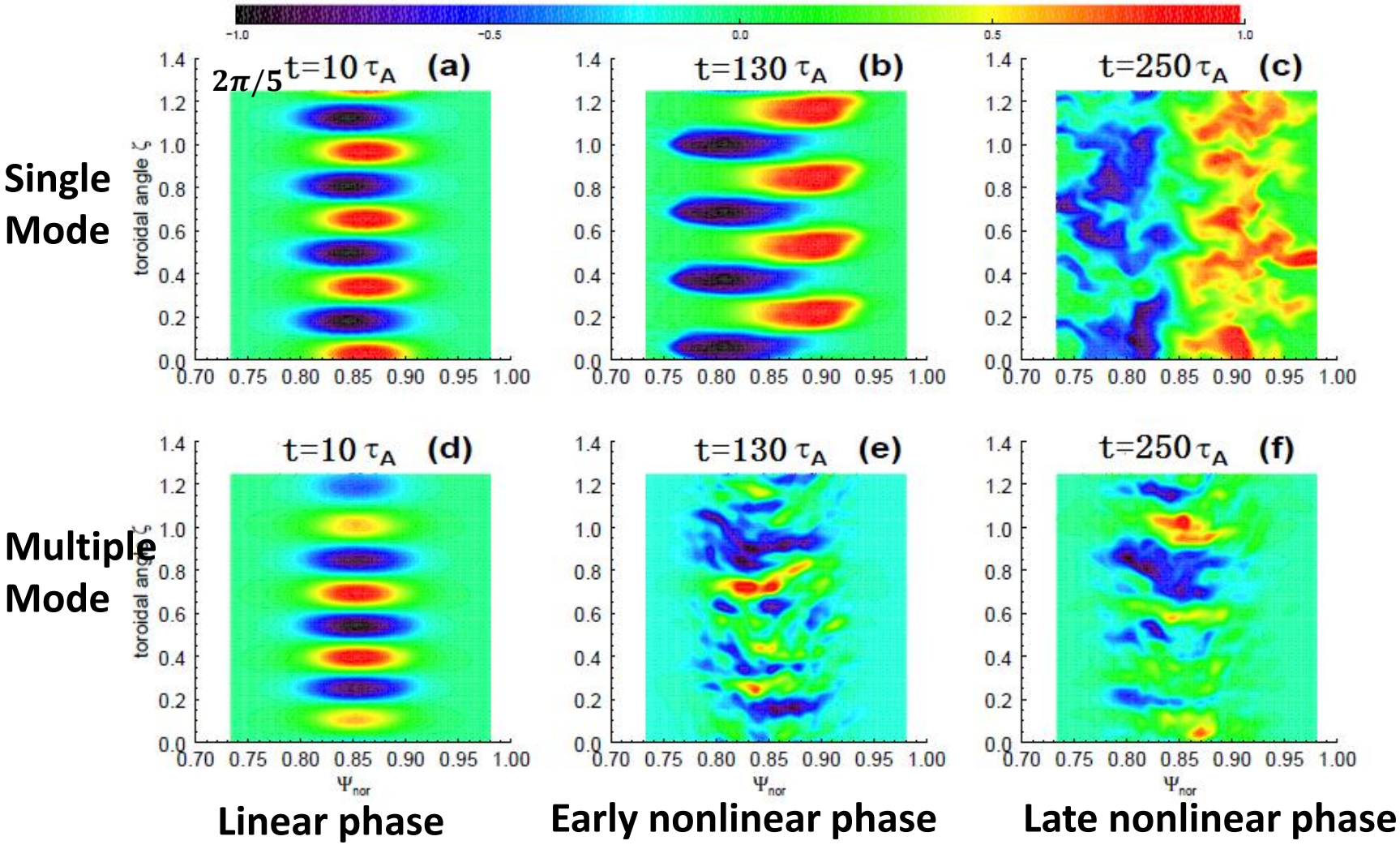
# Single mode: ELM crash || Multiple modes: P-B turbulence

- ELM size

$$\Delta_{ELM} = \frac{\Delta W_{ped}}{W_{ped}} = \frac{\int dx^3 (P_0 - \langle P \rangle_\zeta)}{\int dx^3 P_0}$$



# Different perturbation patterns (1/5 of the torus)




- **Single mode: Filamentary structure is generated by linear instability;**
- **Multiple modes: Linear mode structure is interrupted by nonlinear mode interaction and no filamentary structure appears**

# The growth time

- **The growth time of a instability**

$$\tilde{A} \propto \exp(\gamma t)$$

  - **How long can a mode grow?**
  - **Linear theory presumes $t \to \infty$ or long enough**
  - **But, the growth time is determined nonlinearly**

- **To extract the free energy from the background pressure gradient, pressure and potential perturbation need to stay in favorable phase**

**Relative phase** $\delta\varphi(n,\psi,\theta,t) = \arg\left(\frac{\hat{P}_n(\psi,\theta,t)}{\hat{\phi}_n(\psi,\theta,t)}\right)$

$$\frac{\partial \tilde{V}^2_{E\times B,n}}{\partial t} \propto -2\,\mathrm{Re}\left(in\hat{\phi}_n^* \hat{P}_n b_0 \times \kappa \cdot \nabla \xi\right) \propto \sin\delta\varphi \Rightarrow \begin{cases} 0 < \delta\varphi < \pi & \textbf{Growing phase} \\ -\pi < \delta\varphi < 0 & \textbf{Damping phase} \end{cases}$$

**Phase coherence time (PCT, $\tau_c$):** the length of time duration for the growing phase

# The phase coherence time is limited by nonlinear mode interaction

- $\tau_c$ is determined by nonlinear mode interaction and is a random quantity;
- Linear simulation/theory: no mechanism to perturb $\delta\varphi$

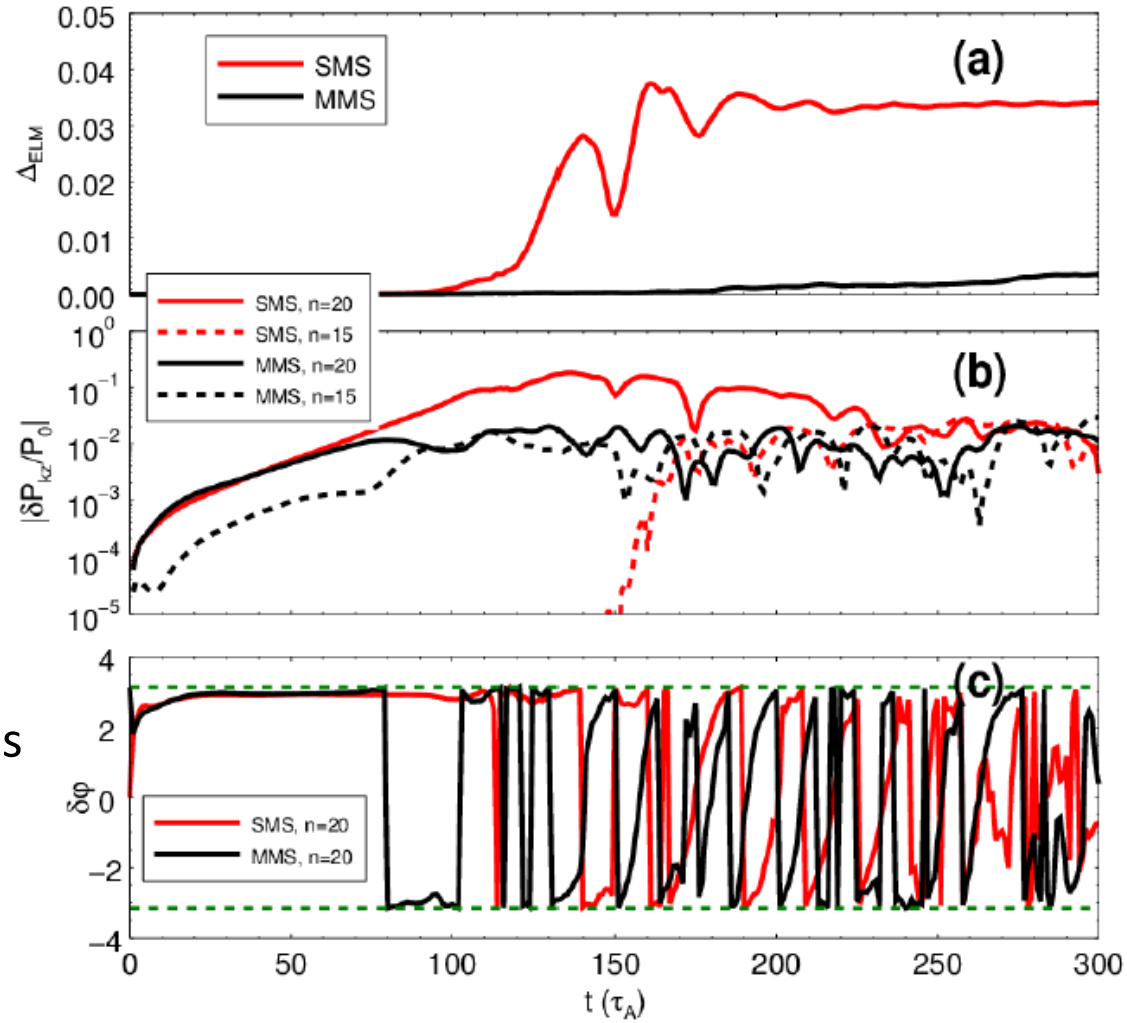
$$\tau_c \to \infty$$

- Nonlinear simulation: $\tau_c$ depends on wave-wave interaction

- Criterion for the onset of ELMs

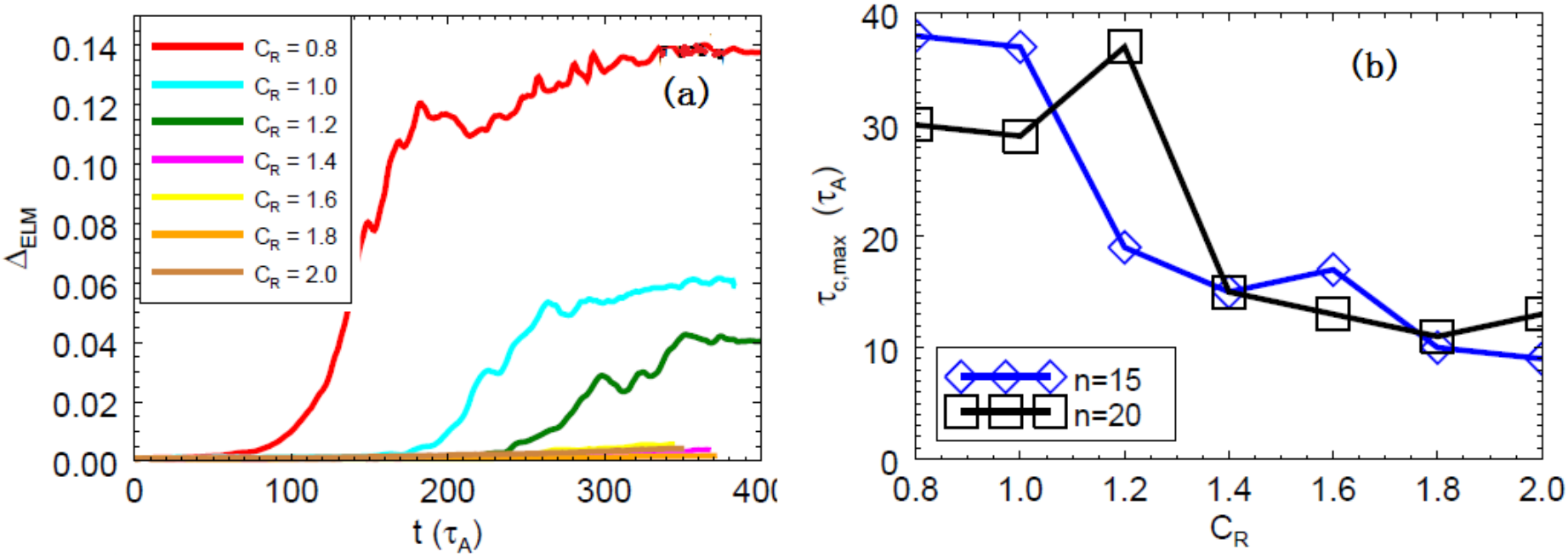
$$\gamma\tau_c > \ln 10 \Rightarrow \gamma > \frac{\ln 10}{\tau_c}$$

$$\lim_{\tau_c \to \infty} \Rightarrow \gamma > 0$$



# ELMs can be controlled by reducing phase coherence time alone

$$\frac{\partial \varpi}{\partial t} + C_R \frac{\mathbf{b} \times \nabla \phi}{B} \cdot \nabla \varpi = RHS$$




- ELMs are determined by the product of $\gamma(n)\tau_c(n)$;
- Reducing the phase coherence time can limit the growth of instability;
- Experimental methods: Shear flow; Collisionality; Plasma shape;

- **Different turbulence lead to different phase coherence time and, thus different ELMs → KBM turbulence**

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# Ion equations*

$$\frac{d\tilde{n}_i}{dt}+B\tilde{\nabla}_{\parallel}\frac{n_0\tilde{u}_{\parallel i}}{B}+\frac{n_0}{2T_0}\left[\hat{\nabla}_{\perp}^2\mathbf{V}_{\Phi}\right]\cdot\nabla\tilde{T}_{\perp i}-n_0(1+\frac{1}{2}\eta_i\hat{\nabla}_{\perp}^2)i\omega_*\frac{e\Phi}{T_0}+n_0(2+\frac{1}{2}\hat{\nabla}_{\perp}^2)i\omega_d\frac{e\Phi}{T_0}+\frac{1}{T_0}i\omega_d(\tilde{p}_{\parallel i}+\tilde{p}_{\perp i})=0$$

$$n_0\frac{d\tilde{u}_{\parallel i}}{dt}+\frac{n_0e}{m_i}\tilde{\nabla}_{\parallel}\Phi+\frac{B}{m_i}\tilde{\nabla}_{\parallel}\frac{\tilde{p}_{\parallel i}}{B}-\frac{n_0}{2m_i}\left[\hat{\nabla}_{\perp}^2\mathbf{V}_{\bar{A}}\right]\cdot\nabla\tilde{T}_{\perp i}+\frac{n_0e}{m_i}\frac{\partial\bar{A}_{\parallel}}{\partial t}+\frac{n_0T_0}{m_i}\left(1+\eta_i+\frac{\eta_i}{2}\hat{\nabla}_{\perp}^2\right)i\omega_*\frac{e\bar{A}_{\parallel}}{T_0}$$

$$+\left(\frac{\tilde{p}_{\perp i}}{m_i}+\frac{n_0e}{2m_i}\hat{\nabla}_{\perp}^2\Phi\right)\nabla_{\parallel}\ln B+\frac{1}{T_0}i\omega_d4P_{i0}\tilde{u}_{\parallel i}+\boxed{2\frac{1}{T_0}|\omega_d|\nu_5P_{i0}\tilde{u}_{\parallel i}}=0$$

$$\frac{d\tilde{p}_{\parallel i}}{dt}+\boxed{B\tilde{\nabla}_{\parallel}\frac{\tilde{q}_{\parallel i}}{B}}+B\tilde{\nabla}_{\parallel}\frac{3P_{i0}\tilde{u}_{\parallel i}}{B}+\frac{n_0}{2}\left[\hat{\nabla}_{\perp}^2\mathbf{V}_{\Phi}\right]\cdot\nabla\tilde{T}_{\perp i}+2(\tilde{q}_{\perp i}+P_{i0}\tilde{u}_{\parallel i})\nabla_{\parallel}\ln B$$

$$-n_0T_0(1+\eta_i+\frac{\eta_i}{2}\hat{\nabla}_{\perp}^2)i\omega_*\frac{e\Phi}{T_0}+n_0T_0(4+\frac{1}{2}\hat{\nabla}_{\perp}^2)i\omega_d\frac{e\Phi}{T_0}+\boxed{i\omega_d(\tilde{r}_{\parallel,\parallel}+\tilde{r}_{\parallel,\perp})}=0$$

$$\frac{d\tilde{p}_{\perp i}}{dt}+\boxed{B^2\tilde{\nabla}_{\parallel}\frac{\tilde{q}_{\perp i}}{B^2}}+B^2\tilde{\nabla}_{\parallel}\frac{P_{i0}\tilde{u}_{\parallel i}}{B^2}+\frac{1}{2}\left[\hat{\nabla}_{\perp}^2\mathbf{V}_{\Phi}\right]\cdot\nabla\tilde{p}_{\perp i}-\frac{P_{i0}}{2}\left[\hat{\nabla}_{\perp}^2\mathbf{V}_{\bar{A}}\right]\cdot\nabla\tilde{u}_{\parallel i}$$

$$-n_0T_0\left[1+\frac{1}{2}\hat{\nabla}_{\perp}^2+\eta_i\left(1+\frac{1}{2}\hat{\nabla}_{\perp}^2+\hat{\hat{\nabla}}_{\perp}^2\right)\right]i\omega_*\frac{e\Phi}{T_0}+n_0T_0(3+\frac{3}{2}\hat{\nabla}_{\perp}^2+\hat{\hat{\nabla}}_{\perp}^2)i\omega_d\frac{e\Phi}{T_0}+\boxed{i\omega_d(\tilde{r}_{\perp,\parallel}+\tilde{r}_{\perp,\perp})}=0$$

## Definitions:

$$\mathbf{V}_{\Phi}=\frac{1}{B}\mathbf{b}\times\nabla\Phi,\ \mathbf{V}_{\bar{A}_{\parallel}}=\frac{1}{B}\mathbf{b}\times\nabla\bar{A}_{\parallel}$$

$$\boxed{(\Phi,\bar{A}_{\parallel})=\Gamma_0^{1/2}(\phi,A_{\parallel})},\ \tilde{\nabla}_{\parallel}=\nabla_{\parallel}-\mathbf{V}_{\bar{A}_{\parallel}}\cdot\nabla$$

$$\nabla_{\parallel}=\mathbf{b}\cdot\nabla,\ \eta_i=\frac{L_{ni}}{L_{Ti}}$$

$$i\omega_*=\frac{T_0}{eBn_0}\mathbf{b}\times\nabla n_0\cdot\nabla$$

$$i\omega_d=\frac{1}{2}\frac{T_0}{eB}\left(\frac{\mathbf{b}\times\nabla B}{B}\cdot\nabla+\mathbf{b}\times\boldsymbol{\kappa}\cdot\nabla\right)$$

## Ion Landau damping

$$\boxed{\tilde{q}_{\parallel i}=-n_0\sqrt{\frac{8}{\pi}}v_{th}\frac{ik_{\parallel}\tilde{T}_{\parallel i}}{|k_{\parallel}|},\quad \tilde{q}_{\perp i}=-n_0\sqrt{\frac{2}{\pi}}v_{th}\frac{ik_{\parallel}}{|k_{\parallel}|}\left(\tilde{T}_{\perp i}+\frac{e}{2}\hat{\nabla}_{\perp}^2\Phi\right)}$$

## Toroidal closure

$$\boxed{\begin{aligned}i\omega_d(\tilde{r}_{\parallel,\parallel}+\tilde{r}_{\parallel,\perp})&=i\omega_d(7\tilde{p}_{\parallel}+\tilde{p}_{\perp}-4T_0\tilde{n}_i-2i\frac{|\omega_d|}{\omega_d}\nu_1\tilde{T}_{\parallel}+\nu_2\tilde{T}_{\perp})\\ i\omega_d(\tilde{r}_{\perp,\parallel}+\tilde{r}_{\perp,\perp})&=i\omega_d(\tilde{p}_{\parallel}+5\tilde{p}_{\perp}-3T_0\tilde{n}_i-2i\frac{|\omega_d|}{\omega_d}\nu_3\tilde{T}_{\parallel}+\nu_4\tilde{T}_{\perp})\end{aligned}}$$

## FLR effect: Padé approximation

$$\begin{cases}\Gamma_0^{1/2}\approx\dfrac{1}{1+b/2}\\ \Gamma_0\approx\dfrac{1}{1+b}\\ \Gamma_0-\Gamma_1\approx1\end{cases},\quad b=-\rho_i^2\nabla_{\perp}^2$$

*P. Snyder thesis

# Electron equations

$$\frac{d\varpi_G}{dt} \boxed{-eB(\mathbf{V}_\Phi - \mathbf{V}_E)\cdot\nabla\tilde{n}_{iG}} = B^2\tilde{\nabla}_\parallel \frac{J_\parallel}{B} + \frac{eB}{T_0} i\omega_d(\tilde{p}_{\parallel e} + \tilde{p}_{\perp e} + \tilde{p}_{\parallel i} + \tilde{p}_{\perp i})$$

$$\boxed{\begin{aligned} &-eB^2\tilde{\nabla}_\parallel \frac{n_0(\bar{u}_{\parallel i} - \tilde{u}_{\parallel i})}{B} + eB^2\left(\delta\bar{\mathbf{b}} - \delta\mathbf{b}\right)\cdot\nabla\frac{n_0\tilde{u}_{\parallel i}}{B} - eBn_0 i\omega_* \frac{e\phi_f}{T_0} + 2eBn_0 i\omega_d \frac{e\phi_f}{T_0} \\ &\frac{en_0}{2T_0}\mathbf{b}\times\nabla\Phi^a\cdot\nabla\tilde{T}_{\perp i} - eB\frac{\eta_i}{2}n_0 i\omega_* \frac{e\Phi^a}{T_0} + \frac{eB}{2}n_0 i\omega_d \frac{e\Phi^a}{T_0} \end{aligned}}$$

$$\frac{\partial A_\parallel}{\partial t} + \tilde{\nabla}_\parallel\phi - \frac{B}{n_0 e}\tilde{\nabla}_\parallel \frac{\tilde{p}_{\parallel e}}{B} - i\omega_* A_\parallel = \eta\tilde{J}_\parallel$$

$$\frac{d\tilde{p}_{\parallel e}}{dt} + \boxed{B\tilde{\nabla}_\parallel \frac{\tilde{q}_{\parallel e}}{B}} + B\tilde{\nabla}_\parallel \frac{3P_{e0}\tilde{u}_{\parallel e}}{B} - en_0 i\omega_*\phi + 4en_0 i\omega_d\phi = 0$$

$$\frac{d\tilde{p}_{\perp e}}{dt} + \boxed{B^2\tilde{\nabla}_\parallel \frac{\tilde{q}_{\perp e}}{B^2}} + B^2\tilde{\nabla}_\parallel \frac{P_{e0}\tilde{u}_{\parallel e}}{B^2} - en_0 i\omega_*\phi + 3en_0 i\omega_d\phi = 0$$

**Gyrokinetic vorticity:**

$$\varpi_G = eB\left(\tilde{n}_e - \tilde{n}_{iG}\right)$$

**Electron Landau damping**

$$\tilde{q}_{\parallel e} = -n_0\sqrt{\frac{8}{\pi}}\sqrt{\frac{m_i}{m_e}}v_{th}\frac{ik_\parallel\tilde{T}_{\parallel e}}{|k_\parallel|}$$

$$\tilde{q}_{\perp e} = -n_0\sqrt{\frac{2}{\pi}}\sqrt{\frac{m_i}{m_e}}v_{th}\frac{ik_\parallel\tilde{T}_{\perp e}}{|k_\parallel|}$$

**Quasi-neutral condition**

$$\bar{n}_i - n_0(1-\Gamma_0)\frac{e\phi}{T_0} + \boxed{\frac{e\rho_i^2}{T_0}\nabla n_0\cdot\nabla(\Gamma_0 - \Gamma_1)\phi} = \tilde{n}_e$$

$$\bar{n}_i = \Gamma_0^{1/2}\tilde{n}_i - \frac{n_0}{T_0}\frac{2b}{(2+b)^2}\tilde{T}_{\perp i}$$

**Large density gradient is taken into account for H-mode pedestal**

**Ampere's Law**

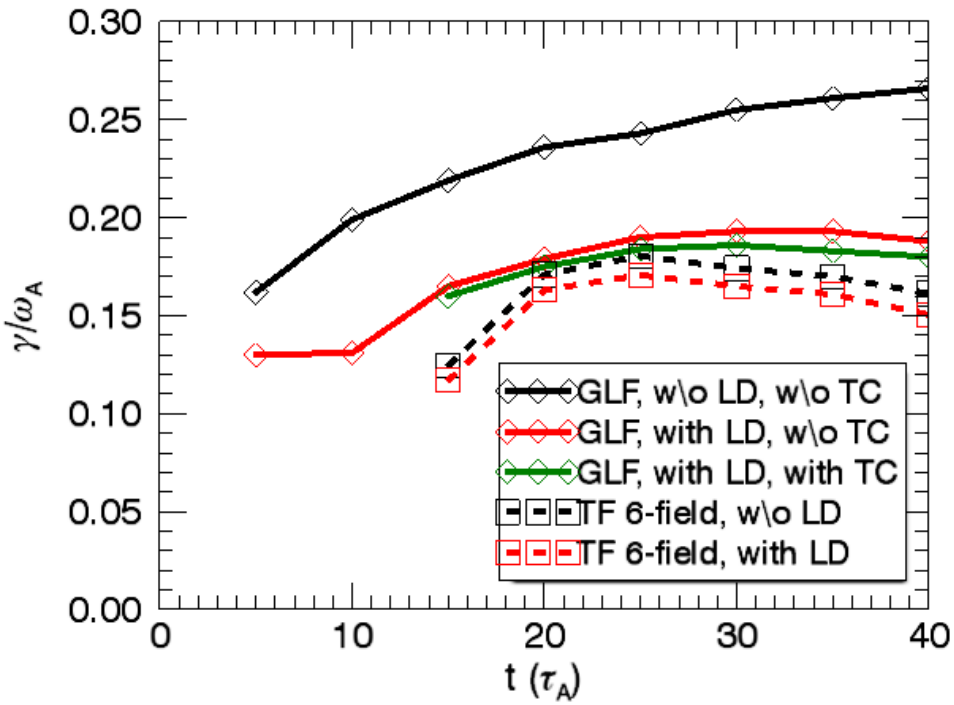
$$\nabla_\perp^2 A_\parallel = -\mu_0 en_0\left(\bar{u}_{\parallel i} - \tilde{u}_{\parallel e}\right) = -\mu_0\tilde{J}_\parallel$$

$$\bar{u}_{\parallel i} = \Gamma_0^{1/2}\tilde{u}_{\parallel i} - \frac{2b}{(2+b)^2}\tilde{q}_{\perp i}$$

**Improvement compared with G-S model**

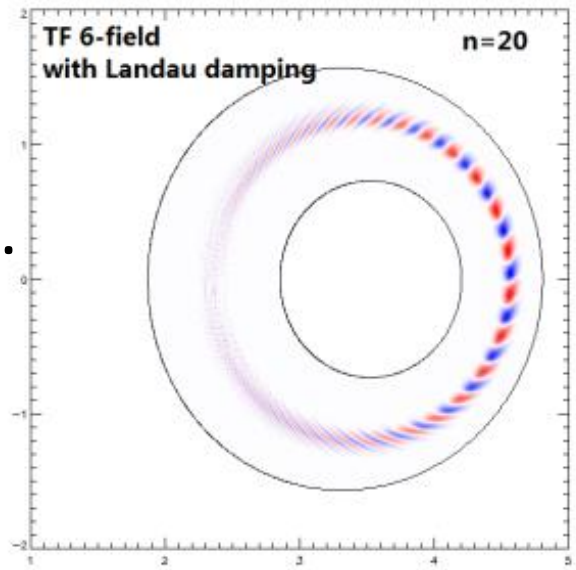
- ✓ Gyrokinetic vorticity formula is used, which is good for numerical calculation
- ✓ Electron pressure equations
- ✓ Density gradient in quasi-neutral condition

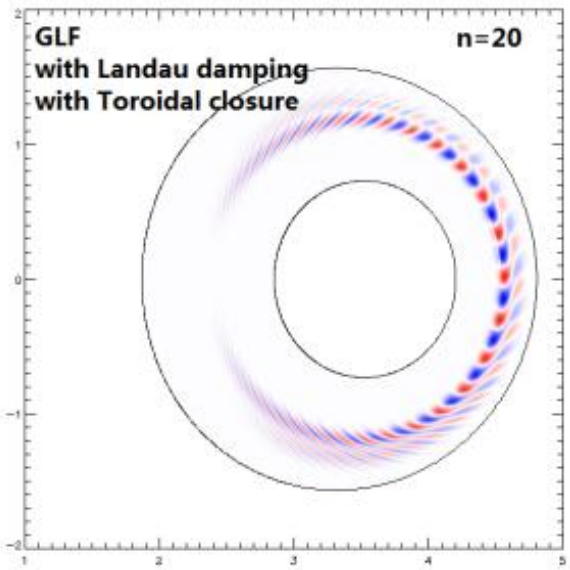
# Basic comparison with 6-field for constant temperature



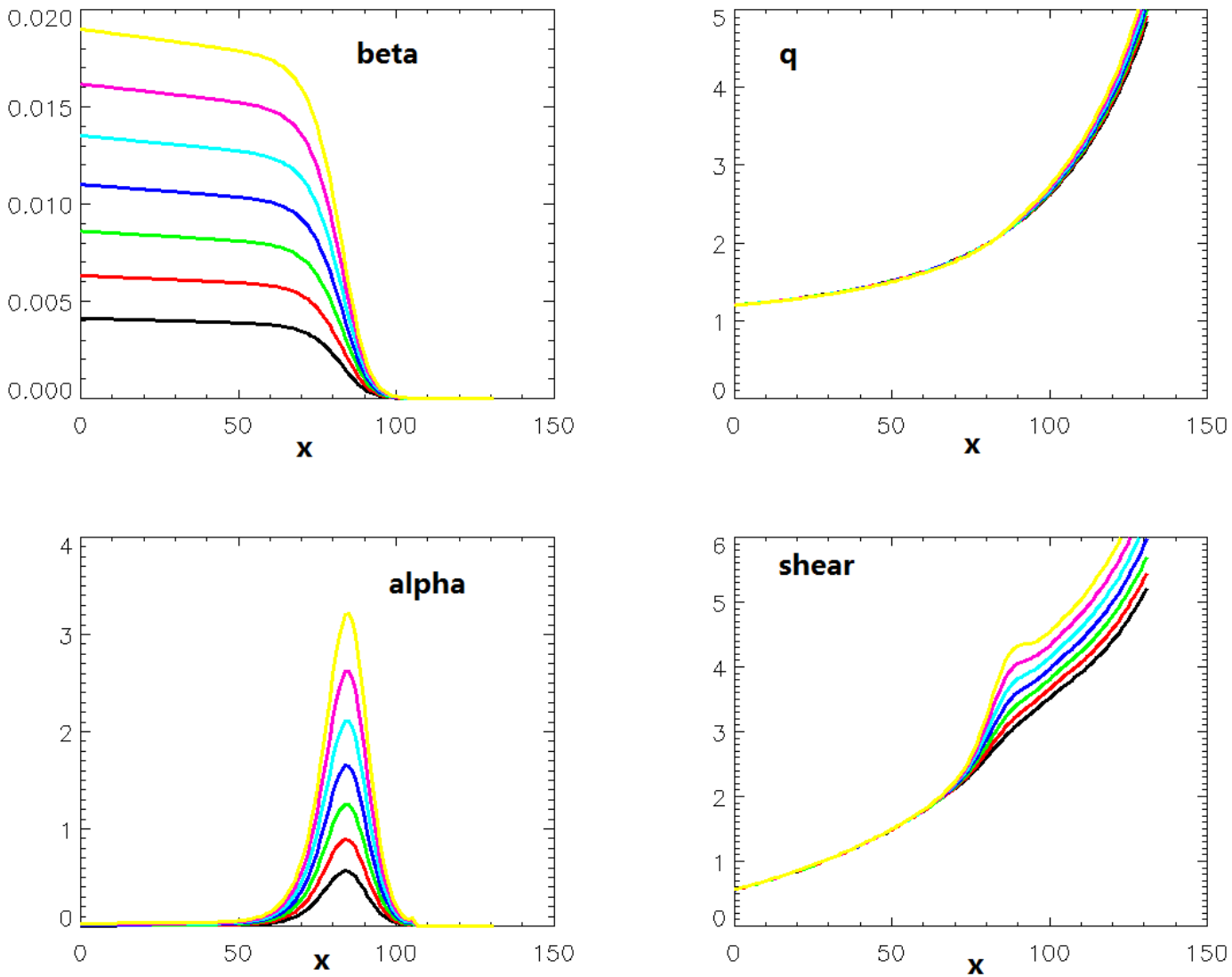

- Landau damping has stronger impact in GLF-KBM code, while toroidal closure has much weaker effect. This is different with GLF-ITG simulation.
- Comparable linear growth rate between GLF and TF in peeling-ballooning mode simulation
- Similar ballooning structure

**Benchmark with global gyrokinetic code is needed.**

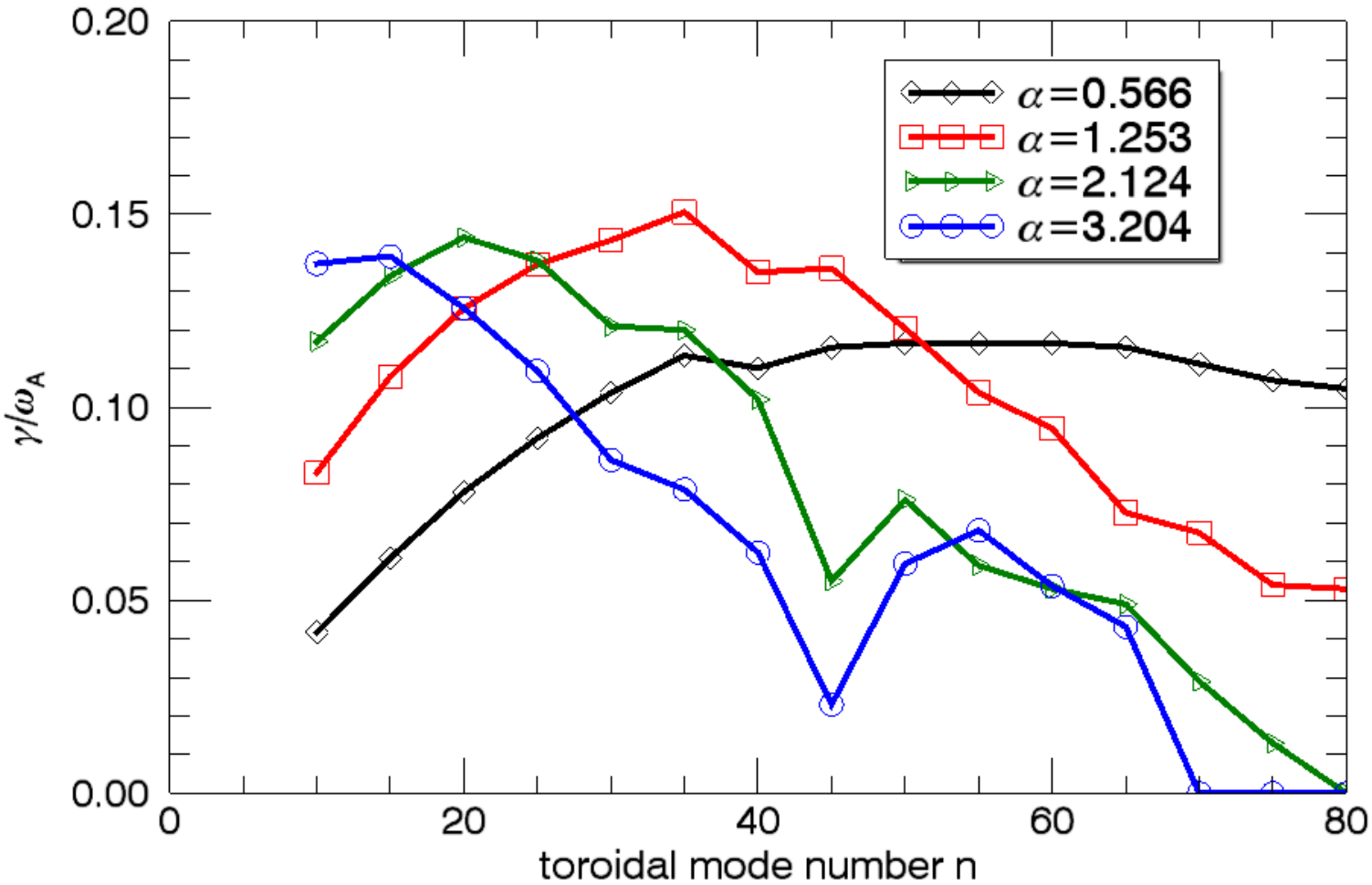


# Setup for $\alpha$ scan



# Linear results



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# Summary

- **Once pedestal becomes linearly unstable, the self-generated turbulence appears at first;**
- **ELM is triggered when the fast growing mode becomes dominant for enough time period;**
- **Filamentary structure can be different from the most unstable mode due to nonlinear mode interaction;**
- **ELM crash is determined by the nonlinear threshold $\gamma > \gamma_c$;**
- **A 3+1 gyrofluid model for KBM simulations is developed under BOUT++ framework;**