Introduction to Eigenvalue Solver for ITG and Gyrofluid Simulation using BOUT++ GLF code in Core Region

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Introduction to

Eigenvalue Solver for ITG

Governing equations for ITG

(Ottaviani et. al., PoP '99)

- Vorticity equation

$$\frac{\partial \Omega_i}{\partial t} + \mathbf{V}_E \cdot \nabla \Omega_i = -n \nabla_\parallel V_\parallel + n\left(\mathbf{V}_E + \mathbf{V}_{p_i}\right) \cdot \left(\boldsymbol{\kappa} + \nabla \ln B\right) + n \mathbf{V}_{p_i} \cdot \nabla \left(\frac{n_1 - \Omega_i}{n} \right) - \mathbf{V}_E \cdot \nabla n_{eq} + D_c \Omega_i$$

where $\Omega_i = n_1 - n_{pol}$: generalized vorticity, $n_{pol} = \frac{ne}{m_i \omega_{ci}^2} \nabla_\perp^2 \varphi$, $n_1 = n_{eq} \frac{e\varphi_1}{T_e}$: adiabatic response

$$\mathbf{V}_E = \frac{c}{B} \mathbf{b} \times \nabla \varphi, \quad \mathbf{V}_{p_i} = \frac{c}{neB} \mathbf{b} \times \nabla p_i$$

- Ion temperature equation

$$\frac{\partial T_i}{\partial t} + \mathbf{V}_E \cdot \nabla T_i = -\frac{2}{3} T_i \nabla_\parallel V_\parallel + D_c T_i + D_{glf} T_i$$

- Ion parallel velocity equation

$$\frac{\partial V_\parallel}{\partial t} + \mathbf{V}_E \cdot \nabla V_\parallel = -\frac{e}{m_i} \nabla_\parallel \varphi - \frac{1}{m_i n} \nabla_\parallel p_i + D_c V_\parallel$$

where $D_c F = \mu_1 \nabla_\perp^2 F - \mu_2 \nabla_\perp^4 F$: viscous damping, $D_{glf} T_i = -\sqrt{\frac{8 T_{i,eq}}{\pi m_i}} \left| \nabla_\parallel \right| T_{i1}$: Landau damping.

Linearized equations

$$\frac{\partial \tilde{\Omega}_i}{\partial \bar{t}} = -\hat{n}_i \hat{\nabla}_{\parallel} \tilde{V}_{i\parallel} + i\left(\hat{n}_i \omega_{di} \tilde{\varphi} + \omega_{di} \tilde{p}_i\right) + \left[\frac{\hat{p}_i}{\rho_*}, \tilde{\nabla}_{\perp}^2 \tilde{\varphi}\right] - \left[\tilde{\Psi}, \frac{\hat{n}_i}{\rho_*}\right] + \overline{D}_c \tilde{\Omega}_i$$

$$\frac{\partial \tilde{T}_i}{\partial \bar{t}} + \left[\tilde{\Psi}, \frac{\hat{T}_i}{\rho_*}\right] = -\frac{2}{3} \hat{T}_i \nabla_{\parallel} \tilde{V}_{i\parallel} + \overline{D}_c \tilde{T}_i + \overline{D}_{glf} \tilde{T}_i$$

$$\frac{\partial \tilde{V}_{i\parallel}}{\partial \bar{t}} = -\hat{\nabla}_{\parallel} \tilde{\varphi} - \frac{1}{\hat{n}_i} \hat{\nabla}_{\parallel} \tilde{p}_i + \overline{D}_c \tilde{V}_{i\parallel}$$

where

$$\tilde{\Omega}_i = \tilde{n}_i - \hat{n}_i \tilde{\nabla}_{\perp}^2 \tilde{\varphi}, \quad \tilde{n}_i = \frac{\hat{n}_i}{\hat{T}_e} \tilde{\varphi},$$

$$\tilde{\Psi} = \Gamma_0^{1/2} \tilde{\varphi} = \left(1 - \frac{1}{2} \tilde{\nabla}_{\perp}^2\right)^{-1} \tilde{\varphi}$$

is gyroaveraged potential.

following normalizations are used,

$$F = F_{eq} + F_1, \quad F = \left(\Omega, V_{\parallel}, n, T, \varphi\right), \quad F_0 = \left(n_0, c_{s0}, n_0, T_0, \frac{T_0}{e}\right)$$

$$\hat{F} = \frac{F_{eq}}{F_0}, \quad \tilde{F} = \frac{F_1}{\rho_* F_0}, \quad \tilde{\nabla}_{\perp} = \rho_* a \nabla_{\perp}, \quad \hat{\nabla}_{\parallel} = a \nabla_{\parallel}, \quad \rho = \frac{r}{a}, \quad \bar{t} = \frac{t}{t_0}$$

$$\rho_* = \frac{\rho_{s0}}{a}, \quad \rho_{s0} = \frac{c_{s0}}{\omega_{c0}}, \quad c_{s0} = \sqrt{\frac{T_0}{m_i}}, \quad \omega_{c0} = \frac{e_i B_0}{m_i c}, \quad t_0 = \frac{a}{c_{s0}}$$

$$\left[f, g\right] \equiv \rho_{s0}^2 \mathbf{b} \times \nabla f \cdot \nabla g = \frac{\rho_*^2}{\rho} \left(\frac{\partial f}{\partial \rho} \frac{\partial g}{\partial \theta} - \frac{\partial g}{\partial \rho} \frac{\partial f}{\partial \theta}\right)$$

$$i \omega_{di} \tilde{F} \equiv 2 \rho_{s0} a \mathbf{b} \times \nabla \tilde{F} \cdot \nabla \ln B = 2 \rho_* \frac{a}{R} \left(\frac{1}{\rho} \cos\theta \frac{\partial \tilde{F}}{\partial \theta} + \sin\theta \frac{\partial \tilde{F}}{\partial \rho}\right)$$

Note: F_0 is constant (not the value on magnetic axis)

$$\frac{\partial \tilde{\Omega}_i}{\partial \bar{t}} = -\hat{n}_i \hat{\nabla}_{\parallel} \tilde{V}_{i\parallel} + i\left(\hat{n}_i \omega_{di} \tilde{\varphi} + \omega_{di} \tilde{p}_i\right) + \left[\frac{\hat{p}_i}{\rho_*}, \tilde{\nabla}_{\perp}^2 \tilde{\varphi}\right] + \left[\frac{\hat{n}_i}{\rho_*}, \tilde{\Psi}\right] + \bar{D}_c \tilde{\Omega}_i$$

$$\Rightarrow \quad \omega M \tilde{\varphi} = \hat{n}_i k_{\parallel} \tilde{V}_{i\parallel} - a_W \tilde{\varphi} - b_W \tilde{T}_i + G_{\hat{p}_i} L \tilde{\varphi} + G_{\hat{n}_i} J_0 \tilde{\varphi} + i\bar{D}_c M \tilde{\varphi}$$

$$\frac{\partial \tilde{T}_i}{\partial \bar{t}} = \left[\frac{\hat{T}_i}{\rho_*}, \tilde{\Psi}\right] - \frac{2}{3} \hat{T}_i \nabla_{\parallel} \tilde{V}_{i\parallel} + \bar{D}_c \tilde{T}_i + \bar{D}_{glf} \tilde{T}_i$$

$$\Rightarrow \quad \omega \tilde{T}_i = G_{\hat{T}_i} J_0 \tilde{\varphi}_i + \frac{2}{3} \hat{T}_i k_{\parallel} \tilde{V}_{i\parallel} + i\bar{D}_c \tilde{T}_i + i\bar{D}_{glf} \tilde{T}_i$$

$$\frac{\partial \tilde{V}_{i\parallel}}{\partial \bar{t}} = -\hat{\nabla}_{\parallel} \tilde{\varphi} - \frac{1}{\hat{n}_i} \hat{\nabla}_{\parallel} \tilde{p}_i + \bar{D}_c \tilde{V}_{i\parallel}$$

$$\Rightarrow \quad \omega \tilde{V}_{i\parallel} = \left(1 + \frac{\hat{T}_i}{\hat{T}_e}\right) k_{\parallel} \tilde{\varphi} + k_{\parallel} \tilde{T}_i + i\bar{D}_c \tilde{V}_{i\parallel}$$

where we introduce

$$\frac{\partial}{\partial \bar{t}} \equiv -i\omega, \qquad \tilde{\nabla}_{\perp}^2 \tilde{\varphi} \equiv L\tilde{\varphi}, \qquad \hat{\nabla}_{\parallel} \equiv -ik_{\parallel}$$

$$\tilde{\Omega}_i = \left(\frac{\hat{n}_i}{\hat{T}_e} - \hat{n}_i \tilde{\nabla}_{\perp}^2\right) \tilde{\varphi} \equiv M\tilde{\varphi}$$

$$\tilde{\Psi} = \left(1 - \frac{1}{2} \tilde{\nabla}_{\perp}^2\right)^{-1} \tilde{\varphi} \equiv J_0 \tilde{\varphi}$$

$$\left[\frac{\hat{F}}{\rho_*}, \quad\right] = \rho_{s0}^2 \mathbf{b} \times \nabla \frac{\hat{F}}{\rho_*} \cdot \nabla \equiv -iG_{\hat{F}}$$

$$i\left(\hat{n}_i \omega_{di} \tilde{\varphi} + \omega_{di} \tilde{p}_i\right) \equiv i\left(a_W \tilde{\varphi} + b_W \tilde{T}_i\right)$$

and use

$$\tilde{p}_i = \frac{\hat{p}_i}{\hat{T}_e} \tilde{\varphi}_i + \hat{n}_i \tilde{T}_i.$$

$$\frac{\partial \tilde{\Omega}_i}{\partial \bar{t}} = -\hat{n}_i \hat{\nabla}_{\parallel} \tilde{V}_{i\parallel} + i\left(\hat{n}_i \omega_{di} \tilde{\varphi} + \omega_{di} \tilde{p}_i\right) + \left[\frac{\hat{p}_i}{\rho_*}, \tilde{\nabla}_{\perp}^2 \tilde{\varphi}\right] + \left[\frac{\hat{n}_i}{\rho_*}, \tilde{\Psi}\right] + \bar{D}_c \tilde{\Omega}_i$$

$$\Rightarrow \quad \omega M \tilde{\varphi} = \hat{n}_i k_{\parallel} \tilde{V}_{i\parallel} - a_W \tilde{\varphi} - b_W \tilde{T}_i + G_{\hat{p}_i} L \tilde{\varphi} + G_{\hat{n}_i} J_0 \tilde{\varphi} + i\bar{D}_c M \tilde{\varphi}$$

$$\Rightarrow \quad \boxed{\omega \tilde{\varphi} = M^{-1}\left(-a_W + G_{\hat{p}_i} L + G_{\hat{n}_i} J_0 + i\bar{D}_c M\right)\tilde{\varphi} - M^{-1} b_W \tilde{T}_i + M^{-1} \hat{n}_i k_{\parallel} \tilde{V}_{i\parallel}}$$

$$\frac{\partial \tilde{T}_i}{\partial \bar{t}} = \left[\frac{\hat{T}_i}{\rho_*}, \tilde{\Psi}\right] - \frac{2}{3} \hat{T}_i \nabla_{\parallel} \tilde{V}_{i\parallel} + \bar{D}_c \tilde{T}_i + \bar{D}_{glf} \tilde{T}_i$$

$$\Rightarrow \quad \omega \tilde{T}_i = G_{\hat{T}_i} J_0 \tilde{\varphi}_i + \frac{2}{3} \hat{T}_i k_{\parallel} \tilde{V}_{i\parallel} + i\bar{D}_c \tilde{T}_i + i\bar{D}_{glf} \tilde{T}_i$$

$$\Rightarrow \quad \boxed{\omega \tilde{T}_i = G_{\hat{T}_i} J_0 \tilde{\varphi}_i + \left(i\bar{D}_c + i\bar{D}_{glf}\right)\tilde{T}_i + \frac{2}{3} \hat{T}_i k_{\parallel} \tilde{V}_{i\parallel}}$$

$$\frac{\partial \tilde{V}_{i\parallel}}{\partial \bar{t}} = -\hat{\nabla}_{\parallel} \tilde{\varphi} - \frac{1}{\hat{n}_i} \hat{\nabla}_{\parallel} \tilde{p}_i + \bar{D}_c \tilde{V}_{i\parallel}$$

$$\Rightarrow \quad \boxed{\omega \tilde{V}_{i\parallel} = \left(1 + \frac{\hat{T}_i}{\hat{T}_e}\right) k_{\parallel} \tilde{\varphi} + k_{\parallel} \tilde{T}_i + i\bar{D}_c \tilde{V}_{i\parallel}}$$

where we introduce

$$\frac{\partial}{\partial \bar{t}} \equiv -i\omega, \qquad \tilde{\nabla}_{\perp}^2 \tilde{\varphi} \equiv L\tilde{\varphi}, \qquad \hat{\nabla}_{\parallel} \equiv -ik_{\parallel}$$

$$\tilde{\Omega}_i = \left(\frac{\hat{n}_i}{\hat{T}_e} - \hat{n}_i \tilde{\nabla}_{\perp}^2\right)\tilde{\varphi} \equiv M\tilde{\varphi}$$

$$\tilde{\Psi} = \left(1 - \frac{1}{2}\tilde{\nabla}_{\perp}^2\right)^{-1} \tilde{\varphi} \equiv J_0 \tilde{\varphi}$$

$$\left[\frac{\hat{F}}{\rho_*}, \quad\right] = \rho_{s0}^2 \mathbf{b} \times \nabla \frac{\hat{F}}{\rho_*} \cdot \nabla \equiv -iG_{\hat{F}}$$

$$i\left(\hat{n}_i \omega_{di} \tilde{\varphi} + \omega_{di} \tilde{p}_i\right) \equiv i\left(a_W \tilde{\varphi} + b_W \tilde{T}_i\right)$$

and use

$$\tilde{p}_i = \frac{\hat{p}_i}{\hat{T}_e} \tilde{\varphi}_i + \hat{n}_i \tilde{T}_i.$$

$$\frac{\partial \tilde{\Omega}_i}{\partial \bar{t}} = -\hat{n}_i \hat{\nabla}_{\parallel} \tilde{V}_{i\parallel} + i\left(\hat{n}_i \omega_{di} \tilde{\varphi} + \omega_{di} \tilde{p}_i\right) + \left[\frac{\hat{p}_i}{\rho_*}, \tilde{\nabla}_{\perp}^2 \tilde{\varphi}\right] + \left[\frac{\hat{n}_i}{\rho_*}, \tilde{\Psi}\right] + \bar{D}_c \tilde{\Omega}_i$$

$$\Rightarrow \quad \omega M \tilde{\varphi} = \hat{n}_i k_{\parallel} \tilde{V}_{i\parallel} - a_W \tilde{\varphi} - b_W \tilde{T}_i + G_{\hat{p}_i} L \tilde{\varphi} + G_{\hat{n}_i} J_0 \tilde{\varphi} + i\bar{D}_c M \tilde{\varphi}$$

$$\Rightarrow \quad \boxed{\omega \tilde{\varphi} = M^{-1}\left(-a_W + G_{\hat{p}_i} L + G_{\hat{n}_i} J_0 + i\bar{D}_c M\right)\tilde{\varphi} - M^{-1} b_W \tilde{T}_i + M^{-1} \hat{n}_i k_{\parallel} \tilde{V}_{i\parallel}}$$

Those operators can be represented as matrices in a functional space.

$$\frac{\partial \tilde{T}_i}{\partial \bar{t}} = \left[\frac{\hat{T}_i}{\rho_*}, \tilde{\Psi}\right] - \frac{2}{3} \hat{T}_i \nabla_{\parallel} \tilde{V}_{i\parallel} + \bar{D}_c \tilde{T}_i + \bar{D}_{glf} \tilde{T}_i$$

$$\Rightarrow \quad \omega \tilde{T}_i = G_{\hat{T}_i} J_0 \tilde{\varphi}_i + \frac{2}{3} \hat{T}_i k_{\parallel} \tilde{V}_{i\parallel} + i\bar{D}_c \tilde{T}_i + i\bar{D}_{glf} \tilde{T}_i$$

$$\Rightarrow \quad \boxed{\omega \tilde{T}_i = G_{\hat{T}_i} J_0 \tilde{\varphi}_i + \left(i\bar{D}_c + i\bar{D}_{glf}\right)\tilde{T}_i + \frac{2}{3} \hat{T}_i k_{\parallel} \tilde{V}_{i\parallel}}$$

$$\frac{\partial \tilde{V}_{i\parallel}}{\partial \bar{t}} = -\hat{\nabla}_{\parallel} \tilde{\varphi} - \frac{1}{\hat{n}_i} \hat{\nabla}_{\parallel} \tilde{p}_i + \bar{D}_c \tilde{V}_{i\parallel}$$

$$\Rightarrow \quad \boxed{\omega \tilde{V}_{i\parallel} = \left(1 + \frac{\hat{T}_i}{\hat{T}_e}\right) k_{\parallel} \tilde{\varphi} + k_{\parallel} \tilde{T}_i + i\bar{D}_c \tilde{V}_{i\parallel}}$$

where we introduce

$$\frac{\partial}{\partial \bar{t}} \equiv -i\omega, \qquad \tilde{\nabla}_{\perp}^2 \tilde{\varphi} \equiv L\tilde{\varphi}, \qquad \hat{\nabla}_{\parallel} \equiv -ik_{\parallel}$$

$$\tilde{\Omega}_i = \left(\frac{\hat{n}_i}{\hat{T}_e} - \hat{n}_i \tilde{\nabla}_{\perp}^2\right)\tilde{\varphi} \equiv M\tilde{\varphi}$$

$$\tilde{\Psi} = \left(1 - \frac{1}{2}\tilde{\nabla}_{\perp}^2\right)^{-1} \tilde{\varphi} \equiv J_0 \tilde{\varphi}$$

$$\left[\frac{\hat{F}}{\rho_*}, \quad\right] = \rho_{s0}^2 \mathbf{b} \times \nabla \frac{\hat{F}}{\rho_*} \cdot \nabla \equiv -iG_{\hat{F}}$$

$$i\left(\hat{n}_i \omega_{di} \tilde{\varphi} + \omega_{di} \tilde{p}_i\right) \equiv i\left(a_W \tilde{\varphi} + b_W \tilde{T}_i\right)$$

and use

$$\tilde{p}_i = \frac{\hat{p}_i}{\hat{T}_e} \tilde{\varphi}_i + \hat{n}_i \tilde{T}_i.$$

Final form of eigenvalue equation

$$\omega\tilde{\varphi} = A_{11}\tilde{\varphi} + A_{12}\tilde{T}_i + A_{13}\tilde{V}_{i\|}$$

$$\omega\tilde{T}_i = A_{21}\tilde{\varphi} + A_{22}\tilde{T}_i + A_{23}\tilde{V}_{i\|}$$

$$\omega\tilde{V}_{i\|} = A_{31}\tilde{\varphi} + A_{32}\tilde{T}_i + A_{33}\tilde{V}_{i\|}$$

$$\boxed{\omega\mathbf{F} = \mathbf{A}\cdot\mathbf{F}}$$

$$[\mathbf{F},\omega] = \mathrm{eig}(\mathbf{A});$$

in *matlab*

$$\mathbf{F} = \begin{pmatrix} \tilde{\boldsymbol{\varphi}} \\ \tilde{\mathbf{T}}_{\mathbf{i}} \\ \tilde{\mathbf{V}}_{\mathbf{i}\|} \end{pmatrix}, \quad \mathbf{A} = \begin{pmatrix} \mathbf{A}_{11} & \mathbf{A}_{12} & \mathbf{A}_{13} \\ \mathbf{A}_{21} & \mathbf{A}_{22} & \mathbf{A}_{23} \\ \mathbf{A}_{31} & \mathbf{A}_{32} & \mathbf{A}_{33} \end{pmatrix}$$

$$\tilde{\boldsymbol{\varphi}} = \sum_k \tilde{\varphi}_k |k\rangle = \begin{pmatrix} \tilde{\varphi}_1 \\ \tilde{\varphi}_2 \\ \tilde{\varphi}_3 \\ \vdots \end{pmatrix}, \quad \ldots$$

$|k\rangle$ is a basis function

where

$$A_{11} = M^{-1}\left(-a_W + G_{\hat{p}_i}L + G_{\hat{n}_i}J_0 + i\overline{D}_c M\right), \qquad A_{12} = -M^{-1}b_w, \qquad A_{13} = M^{-1}\hat{n}_i k_\|$$

$$A_{21} = G_{\hat{T}_i}J_0, \qquad A_{22} = i\overline{D}_c + i\overline{D}_{glf}, \qquad A_{23} = \frac{2}{3}\hat{T}_i k_\|$$

$$A_{31} = \left(1 + \frac{\hat{T}_i}{\hat{T}_e}\right)k_\|, \qquad A_{32} = k_\|, \qquad A_{33} = i\overline{D}_c$$

Basis functions

Field F can represented as $F(\rho,\theta,\zeta,t)=e^{-i\omega t}\sum_{k=mnp}F_k W_k(\rho)e^{i(m\theta-n\zeta)}$ in concentric - circular geometry :

$$\mathbf{F}=|F\rangle=e^{-i\omega t}\sum_k F_k|k\rangle,\quad \boxed{|k\rangle=|mnp\rangle=W_{mnp}(\rho)e^{i(m\theta-n\zeta)}},$$

inner product:

$$\langle k|F\rangle\equiv\frac{1}{4\pi^2}\int_0^1\rho d\rho\int_0^{2\pi}d\theta\int_0^{2\pi}d\zeta W_{mnp}(\rho)e^{-i(m\theta-n\zeta)}F,\qquad \langle k|k'\rangle=\delta_{kk'}$$

- Radial basis function

For Hermite basis ,

$$W_{mnp}(\rho)=\frac{1}{\sqrt{2\rho w_{mn}}\nu_p}H_p(x)e^{-x^2/2}$$

where $x=\dfrac{\rho-\rho_{mn}}{w_{mn}}$ $\left(\rho_{mn}\text{ is radial position of rational surface satisfying } q=\dfrac{m}{n}\right)$, $\nu_p=2^{p/2}\Gamma(p+1)^{1/2}\pi^{1/4}$

For Bessel basis,

$$W_{mnp}(\rho)=\frac{\sqrt{2}}{J_{m+1}(\alpha_{mp})}J_m(\alpha_{mp}\rho)\quad\text{where } J_m(\alpha_{mp})=0$$

Matrix (M, J0, Dc) involving Laplacian L

For Bessel basis,

$$L_{kk'} = \langle k | \tilde{\nabla}_\perp^2 | k' \rangle = \rho_*^2 \langle k | \left(\frac{\partial^2}{\partial \rho^2} + \frac{1}{\rho}\frac{\partial}{\partial \rho} - \frac{m'^2}{\rho^2} \right) | k' \rangle = -k_\perp^2 \delta_{kk'}$$

where $k_\perp = \rho_* \alpha_{mp}$

$$M_{kk'} \approx \left. \frac{\hat{n}_i}{\hat{T}_e} \right|_{\rho=\rho_{mn}} \delta_{kk'} - \hat{n}_i \big|_{\rho=\rho_{mn}} L_{kk'}$$

$$J_{0,kk'} = \left(1 - \frac{1}{2} L_{kk'} \right)^{-1}$$

$$\overline{D}_{c,kk'} = \overline{\mu}_1 L_{kk'} - \overline{\mu}_2 L_{kk'}^2$$

$$L\tilde{\varphi} = \tilde{\nabla}_\perp^2 \tilde{\varphi}$$

$$M\tilde{\varphi} = \left(\frac{\hat{n}_i}{\hat{T}_e} - \hat{n}_i \tilde{\nabla}_\perp^2 \right) \tilde{\varphi} = \tilde{\Omega}_i$$

$$J_0 \tilde{\varphi} = \left(1 - \frac{1}{2} \tilde{\nabla}_\perp^2 \right)^{-1} \tilde{\varphi} = \tilde{\Psi}$$

$$\overline{D}_c F = \overline{\mu}_1 \tilde{\nabla}_\perp^2 F - \overline{\mu}_2 \tilde{\nabla}_\perp^4 F$$

Matrix ($k_{\|}$, D_{glf}) involving parallel wavenumber

$$\hat{F}k_{\|,kk'} = i\langle k|\hat{F}\hat{\nabla}_{\|}|k'\rangle = \delta_{mm'}\delta_{nn'}\frac{a}{R}\int_0^1 \hat{F}(\rho)\left(\frac{m}{q(\rho)} - n\right)W_{mnp}(\rho)W_{mnp'}(\rho)\rho d\rho$$

$$\overline{D}_{glf,kk'} = -\langle k|\sqrt{\frac{8\hat{T}_i}{\pi}}\left|\hat{\nabla}_{\|}\right||k'\rangle = -\delta_{mm'}\delta_{nn'}\sqrt{\frac{8}{\pi}}\frac{a}{R}\int_0^1 \sqrt{\hat{T}_i(\rho)}\left|\frac{m}{q(\rho)} - n\right|W_{mnp}(\rho)W_{mnp'}(\rho)\rho d\rho$$

Matrix (G) for ExB and diamagnetic drifts

$$G_{\hat{F}}\tilde{\varphi} = i\left[\frac{\hat{F}}{\rho_*}, \tilde{\varphi}\right]:$$

$$G_{\hat{F},kk'} = i\langle k|\left[\frac{\hat{F}}{\rho_*}, \quad\right]|k'\rangle = i\rho_*\langle k|\frac{1}{\rho}\frac{\partial\hat{F}}{\partial\rho}\frac{\partial}{\partial\theta}|k'\rangle = -\delta_{mm'}\delta_{nn'}\rho_* m\int_0^1 \frac{\partial\hat{F}}{\partial\rho}W_{mnp}(\rho)W_{mnp'}(\rho)d\rho$$

Matrix (a_W, b_W) involving curvature

$$\hat{\beta}\omega_{di}\hat{\alpha}\left|k'\right\rangle = -2i\rho_* \frac{a}{R}\hat{\beta}\left(\frac{1}{\rho}\cos\theta\frac{\partial}{\partial\theta} + \sin\theta\frac{\partial}{\partial\rho}\right)\hat{\alpha}\left|k'\right\rangle$$

$$= \rho_* \frac{a}{R}\left[\left(\frac{m'}{\rho}\hat{\beta}\hat{\alpha}W_{k'} + \hat{\beta}\frac{\partial\hat{\alpha}W_{k'}}{\partial\rho}\right)e^{i(m'-1)\theta} + \left(\frac{m'}{\rho}\hat{\beta}\hat{\alpha}W_{k'} - \hat{\beta}\frac{\partial\hat{\alpha}W_{k'}}{\partial\rho}\right)e^{i(m'+1)\theta}\right]e^{-in'\zeta}$$

$$\Rightarrow \quad \left\langle k\left|\hat{\beta}\omega_{di}\hat{\alpha}\right|k'\right\rangle = \rho_* \frac{a}{R}\left[\begin{array}{l}\left\{(m+1)\int_0^1 \hat{\beta}\hat{\alpha}W_{mnp}W_{m+1,n,p'}d\rho + \int_0^1 \hat{\beta}W_{mnp}\frac{\partial\hat{\alpha}W_{m+1,n,p'}}{\partial\rho}\rho d\rho\right\}\delta_{m+1,m'}\delta_{nn'} \\ +\left\{(m-1)\int_0^1 \hat{\beta}\hat{\alpha}W_{mnp}W_{m-1,n,p'}d\rho - \int_0^1 \hat{\beta}W_{mnp}\frac{\partial\hat{\alpha}W_{m-1,n,p'}}{\partial\rho}\rho d\rho\right\}\delta_{m-1,m'}\delta_{nn'}\end{array}\right]$$

$$\Downarrow$$

$$a_{W,kk'} = \left\langle k\left|\left(\hat{n}_i\omega_{di} + \omega_{di}\frac{\hat{p}_i}{\hat{T}_e}\right)\right|k'\right\rangle = a_W^+(k,p')\delta_{m+1,m'}\delta_{nn'} + a_W^-(k,p')\delta_{m-1,m'}\delta_{nn'}$$

$$b_{W,kk'} = \left\langle k\left|\omega_{di}\hat{n}_i\right|k'\right\rangle = b_W^+(k,p')\delta_{m+1,m'}\delta_{nn'} + b_W^-(k,p')\delta_{m-1,m'}\delta_{nn'}$$

$$a_W\tilde{\varphi} = \hat{n}_i\omega_{di}\tilde{\varphi} + \omega_{di}\frac{\hat{p}_i}{\hat{T}_e}\tilde{\varphi}$$

$$b_W\tilde{T}_i = \omega_{di}\hat{n}_i\tilde{T}_i$$

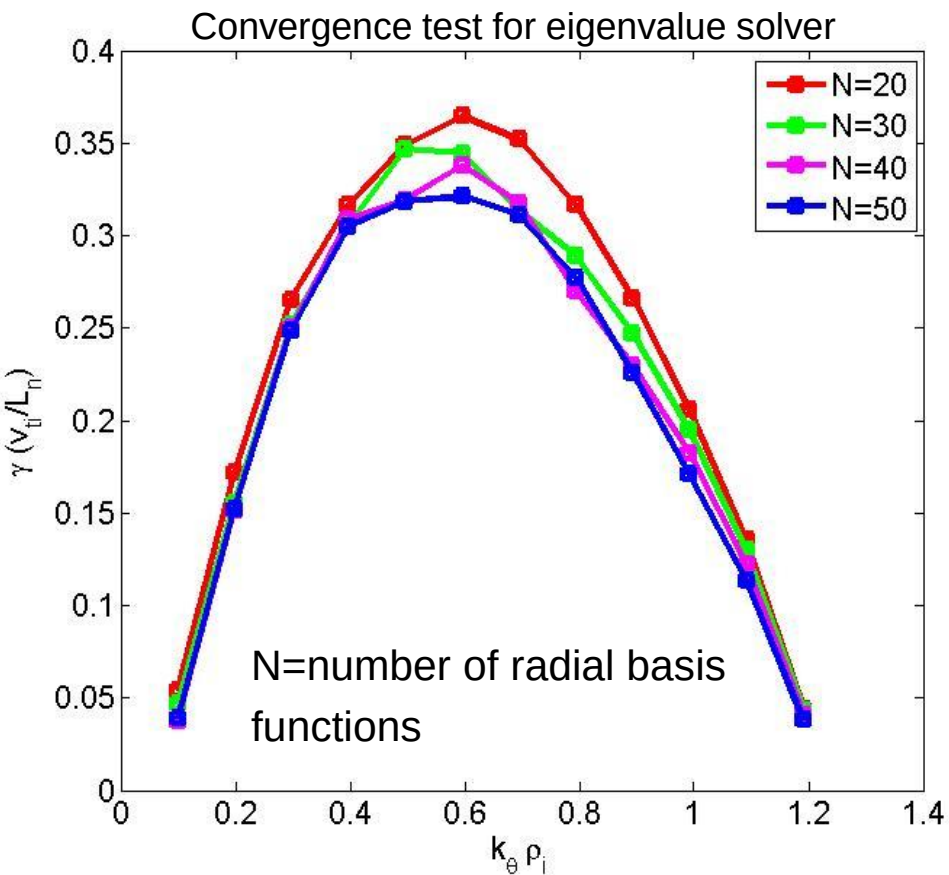
where

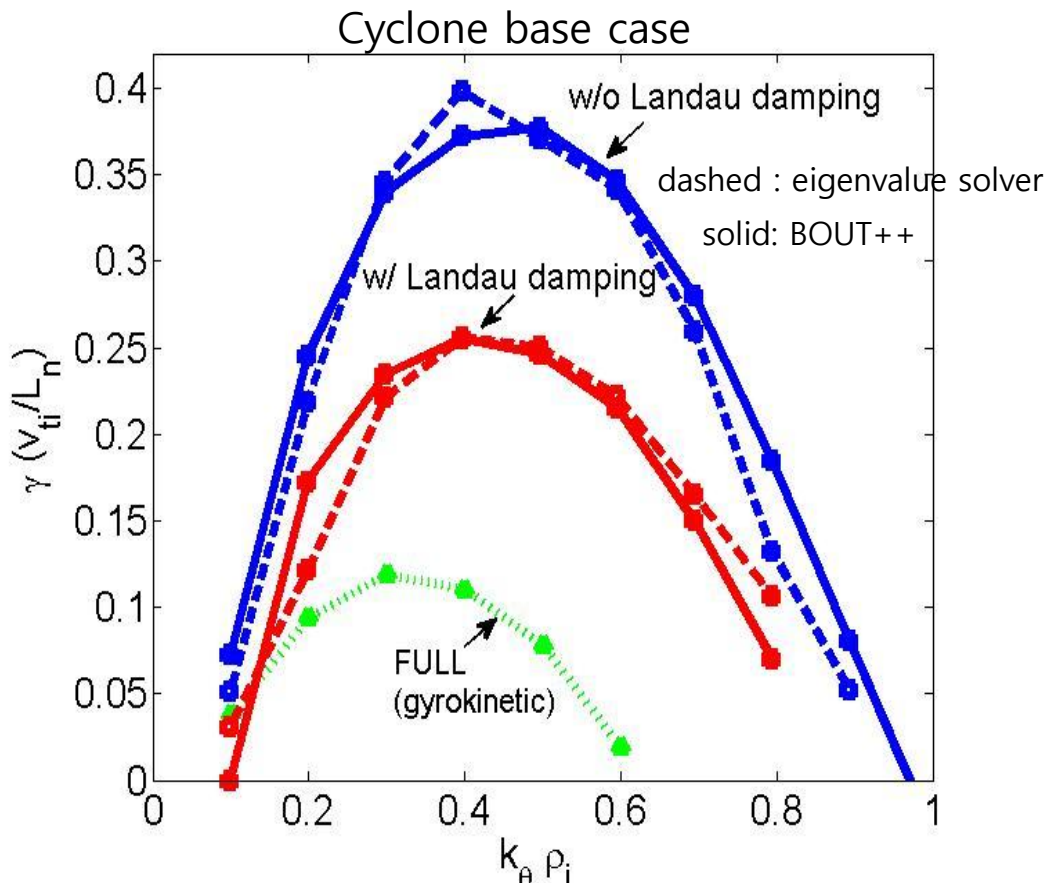
$$a_W^\pm(k,p') = \rho_* \frac{a}{R}\left[\int_0^1 \hat{n}_i\left\{(m\pm1)(1+\tau) \pm \left(\frac{\partial\ln\hat{n}_i}{\partial\rho} + \frac{\partial\ln\tau}{\partial\rho}\right)\tau\right\}W_{mnp}W_{m\pm1,n,p'}d\rho \pm \int_0^1 \hat{n}_i(1+\tau)W_{mnp}\frac{\partial W_{m\pm1,n,p'}}{\partial\rho}\rho d\rho\right]$$

$$b_W^\pm(k,p') = \rho_* \frac{a}{R}\left[\int_0^1\left(m\pm1+\frac{\partial\ln\hat{n}_i}{\partial\rho}\right)\hat{n}_i W_{mnp}W_{m\pm1,n,p'}d\rho \pm \int_0^1 \hat{n}_i W_{mnp}\frac{\partial W_{m\pm1,n,p'}}{\partial\rho}\rho d\rho\right], \quad \tau = \frac{\hat{T}_i}{\hat{T}_e}$$

Comparison of BOUT++ with eigenvalue solver

- In eigenvalue solver, $W_{mnp} = \frac{\sqrt{2}}{J_{m+1}(\alpha_{mp})} J_m\left(\frac{\alpha_{mp}}{a} r\right) e^{i(m\theta - n\zeta)}$ $(p = 1 \sim N)$ is used as basis function.
- The fluid equations are then projected on to the set of basis functions.
- Eigenvalues are obtained by using matlab.
- BOUT++ results agree well with eigenvalues, which means BOUT++ correctly solves the given equations.





Gyrofluid Simulation using BOUT++ GLF Code in Core Region

Gyrofluid equation for ITG

(M. A. Beer and G. W. Hammett, PoP '96)

- **Guiding center density equation**

$$\frac{\partial \tilde{n}_i}{\partial t}+\mathbf{V}_{\tilde{\Phi}}\cdot\nabla\tilde{n}_i+\frac{n_0}{2T_{i0}}\left[\hat{\nabla}_\perp^2\mathbf{V}_{\tilde{\Phi}}\right]\cdot\nabla\tilde{T}_{i\perp}+B\nabla_\parallel\frac{n_0\tilde{V}_{i\parallel}}{B}-\frac{en_0}{T_{i0}}\left(1+\frac{1}{2}\eta_i\hat{\nabla}_\perp^2\right)i\omega_*\tilde{\Phi}+\frac{en_0}{T_{i0}}\left(2+\frac{1}{2}\hat{\nabla}_\perp^2\right)i\omega_d\tilde{\Phi}+\frac{1}{T_{i0}}i\omega_d\left(\tilde{p}_{i\parallel}+\tilde{p}_{i\perp}\right)=0$$

- **Guiding center parallel velocity equation**

$$\frac{\partial \tilde{V}_{i\parallel}}{\partial t}+\mathbf{V}_{\tilde{\Phi}}\cdot\nabla\tilde{V}_{i\parallel}+\frac{e}{m_i}\nabla_\parallel\tilde{\Phi}+\frac{B}{m_in_0}\nabla_\parallel\frac{\tilde{p}_{i\parallel}}{B}+\left(\frac{\tilde{T}_{i\perp}}{m_i}+\frac{e}{2m_i}\hat{\nabla}_\perp^2\tilde{\Phi}\right)\nabla_\parallel\ln B+\frac{1}{p_{i0}}i\omega_d\left(\tilde{q}_{i\parallel}+\tilde{q}_{i\perp}\right)+4i\omega_d\tilde{V}_{i\parallel}=0$$

- **Guiding center parallel & perpendicular pressure equation**

$$\frac{\partial \tilde{p}_{i\parallel}}{\partial t}+\mathbf{V}_{\tilde{\Phi}}\cdot\nabla\tilde{p}_{i\parallel}+\frac{n_0}{2}\left[\hat{\nabla}_\perp^2\mathbf{V}_{\tilde{\Phi}}\right]\cdot\nabla\tilde{T}_{i\perp}+B\nabla_\parallel\frac{1}{B}\left(\tilde{q}_{i\parallel}+p_{i0}\tilde{V}_{i\parallel}\right)+2p_{i0}B\nabla_\parallel\frac{\tilde{V}_{i\parallel}}{B}+2\tilde{q}_{i\perp}\nabla_\parallel\ln B$$
$$-en_0\left(1+\eta_i+\frac{1}{2}\eta_i\hat{\nabla}_\perp^2\right)i\omega_*\tilde{\Phi}+en_0\left(4+\frac{1}{2}\hat{\nabla}_\perp^2\right)i\omega_d\tilde{\Phi}+4i\omega_d\tilde{p}_{i\parallel}+n_0\,i\omega_d\left(3\tilde{T}_{i\parallel}+\tilde{T}_{i\perp}\right)+2n_0\left|\omega_d\right|\left(\nu_1\tilde{T}_{i\parallel}+\nu_2\tilde{T}_{i\perp}\right)=0$$

$$\frac{\partial \tilde{p}_{i\perp}}{\partial t}+\mathbf{V}_{\tilde{\Phi}}\cdot\nabla\tilde{p}_{i\perp}+\frac{1}{2}\left[\hat{\nabla}_\perp^2\mathbf{V}_{\tilde{\Phi}}\right]\cdot\nabla\tilde{p}_{i\perp}+n_0\left[\hat{\hat{\nabla}}_\perp^2\mathbf{V}_{\tilde{\Phi}}\right]\cdot\nabla\tilde{T}_{i\perp}+B^2\nabla_\parallel\frac{1}{B^2}\left(\tilde{q}_{i\perp}+p_{i0}\tilde{V}_{i\parallel}\right)$$
$$-en_0\left[1+\frac{1}{2}\hat{\nabla}_\perp^2+\eta_i\left(1+\frac{1}{2}\hat{\nabla}_\perp^2+\hat{\hat{\nabla}}_\perp^2\right)\right]i\omega_*\tilde{\Phi}+en_0\left(3+\frac{3}{2}\hat{\nabla}_\perp^2+\hat{\hat{\nabla}}_\perp^2\right)i\omega_d\tilde{\Phi}+3i\omega_d\tilde{p}_{i\perp}+n_0i\omega_d\left(\tilde{T}_{i\parallel}+2\tilde{T}_{i\perp}\right)+2n_0\left|\omega_d\right|\left(\nu_3\tilde{T}_{i\parallel}+\nu_4\tilde{T}_{i\perp}\right)=0$$

- **Quasi-neutrality**

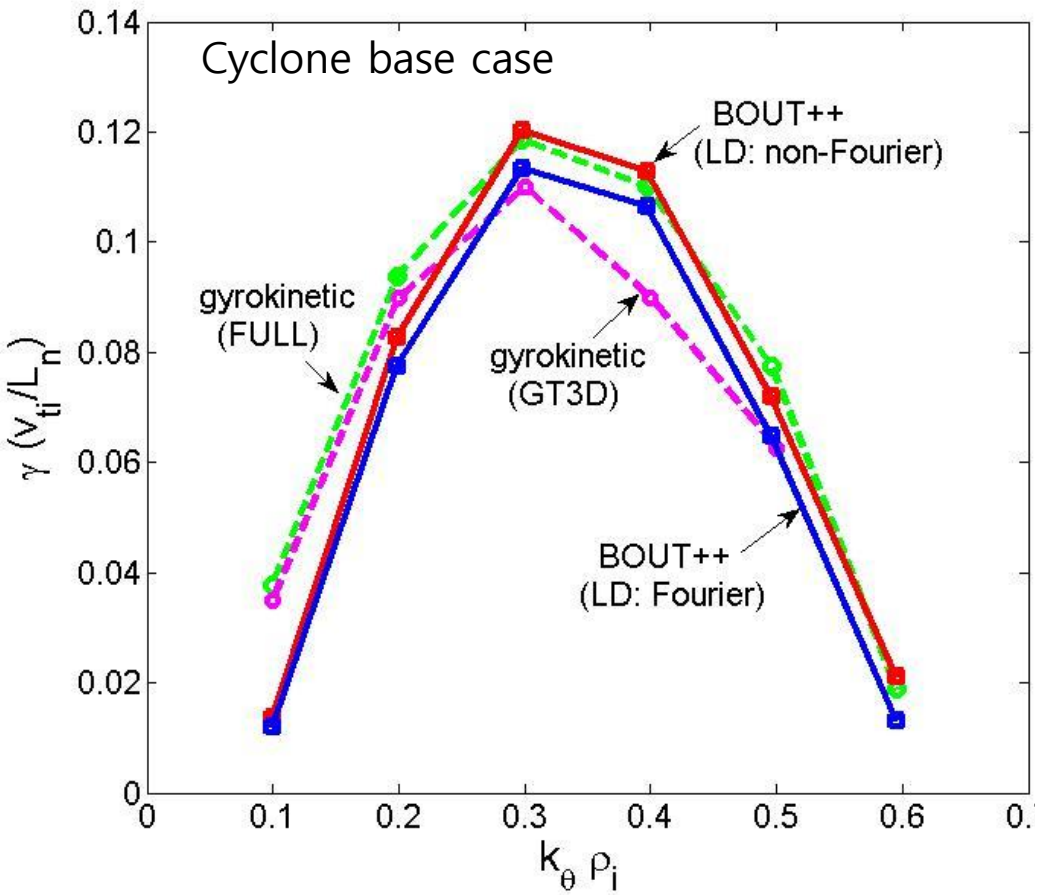
$$\frac{n_0}{T_{e0}}\tilde{\varphi}+\frac{n_0}{T_{i0}}\left(1-\Gamma_0\right)\tilde{\varphi}=\Gamma_0^{1/2}\tilde{n}_i+\frac{n_0}{T_{i0}}b\frac{\partial\Gamma_0^{1/2}}{\partial b}\tilde{T}_i$$

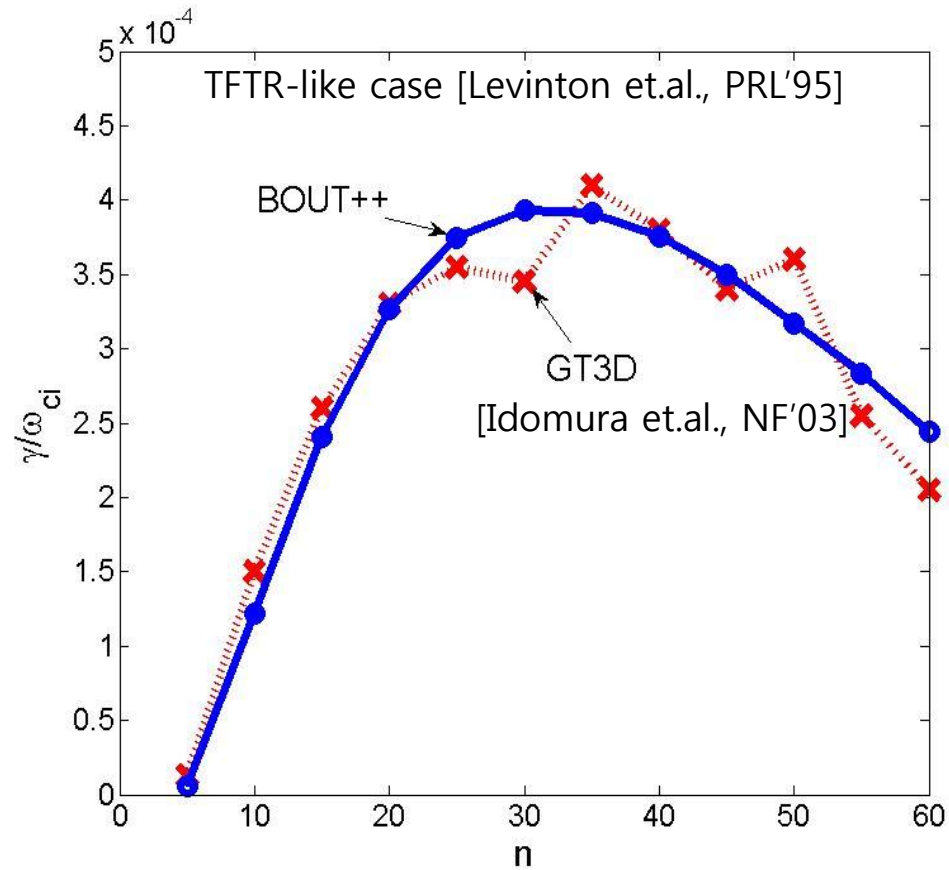
where $\tilde{\Phi}=\Gamma_0^{1/2}\tilde{\varphi}$, $\frac{1}{2}\hat{\nabla}_\perp^2\tilde{\Phi}=\left(b\frac{\partial\Gamma_0^{1/2}}{\partial b}\right)\tilde{\varphi}$, $\hat{\hat{\nabla}}_\perp^2\tilde{\Phi}=b\frac{\partial^2}{\partial b^2}\left(b\Gamma_0^{1/2}\right)\tilde{\varphi}$, $\Gamma_0^{1/2}\cong\frac{1}{1+b/2}$

$$i\omega_d=\left(v_t^2/\omega_c\right)\mathbf{b}\times\nabla\ln B\cdot\nabla,\quad b=-\rho_i^2\nabla_\perp^2$$

Linear benchmarks

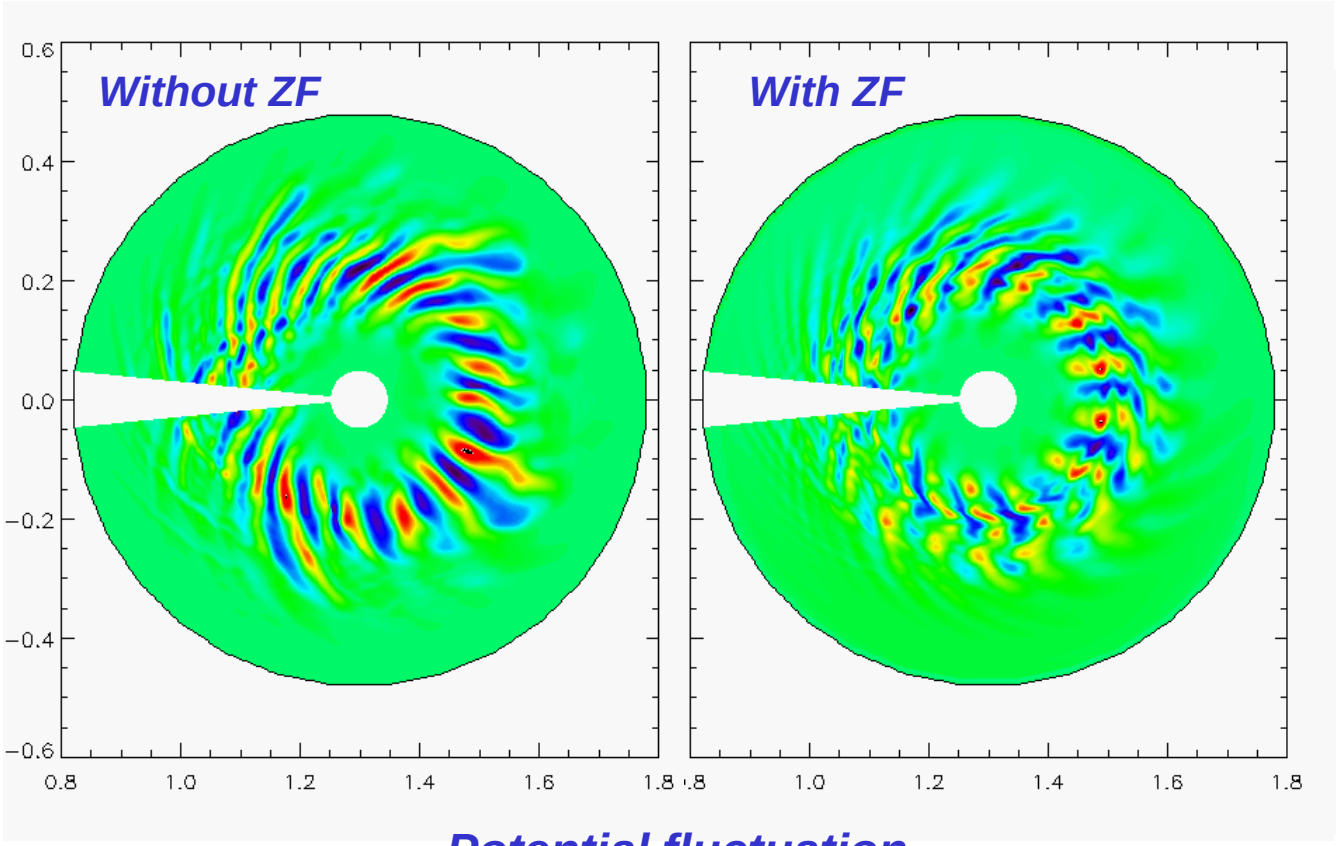
- BOUT++ using Beer's "3+1" field model agrees well with gyrokinetic codes.
- Non-Fourier method for Landau damping shows good agreement with Fourier method.



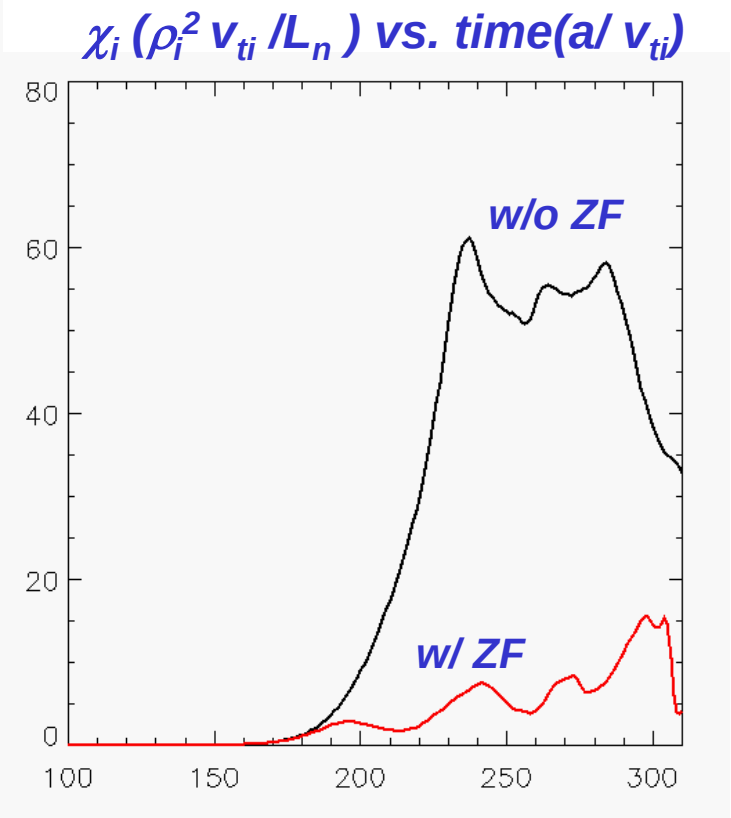


Nonlinear simulations of ITG

- Global nonlinear simulations using Beer model performed at fixed profile
- Turbulence suppression by zonal flow observed



Potential fluctuation



Hands-on Exercise for Eigenvalue Solver using MATLAB

0) Get eigenvalue solver and Run matlab :

1) Run eigensolver_init.m and eigensolver_ITG.m in matlab:

>> eigensolver_init : assign equilibrium profiles, calculate radial basis functions,
arrange mode index, evaluate matrix elements involving integrals

>> eigensolver_ITG : set up matrix and solve the eigenvalue equation for ITG

2) Run plot_eigenvalue.m and plot_eigenmode.m in matlab:

>> plot_eigenvalue : plot linear ITG growth rate and real frequency vs. k_theta*rho_i
>> plot_eigenmode : choose toroidal mode number and plot eigenmode structure

Get eigenvalue solver using git and run matlab :

```
> mkdir eigensolver-ex
> cd eigensolver-ex
> git clone https://github.com/boutproject/BOUT-2.0.git
> cd BOUT-2.0/tools/eigensolver/GLF_ITG
> module load matlab-nofonts
> matlab
```

File Edit Debug Parallel Desktop Window Help

Current Folder: /global/u2/t/train258/eigensolver-ex/BOUT-2.0/tools/eigensolver/GLF_ITG

Shortcuts How to Add What's New

Current Folder

« GLF_ITG

Name

- eigensolver_init.m
- eigensolver_ITG.m
- eigensolver_ITG.pdf
- plot_eigenmode.m
- plot_eigenvalue.m
- README

Details

Select a file to view details

Command Window

New to MATLAB? Watch this Video, see Demos, or read Getting Started.

```
>>
```

Workspace

Select dat...

Name	Value

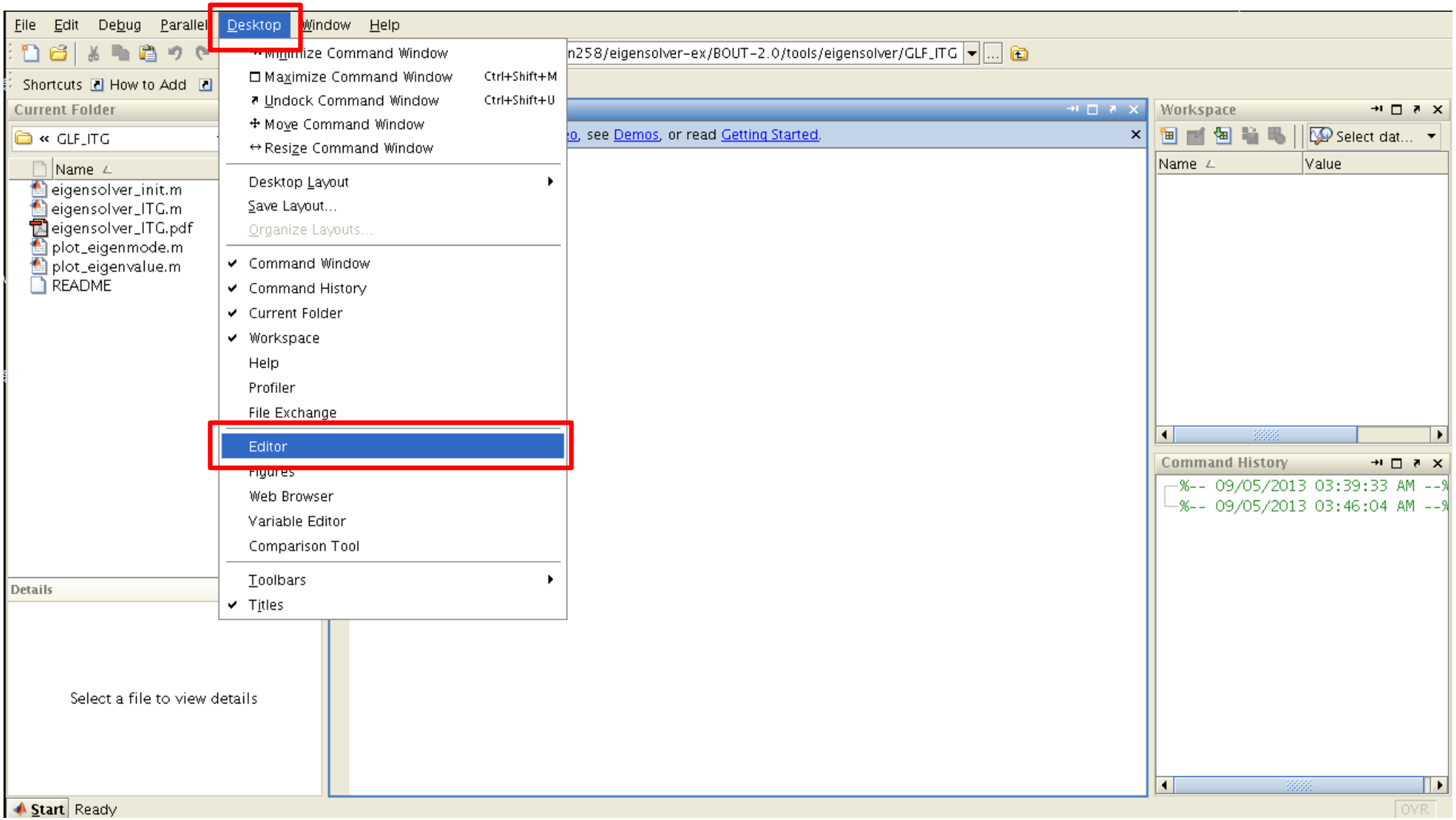
Command History

```
%-- 09/05/2013 03:39:33 AM --%
```

Start Ready

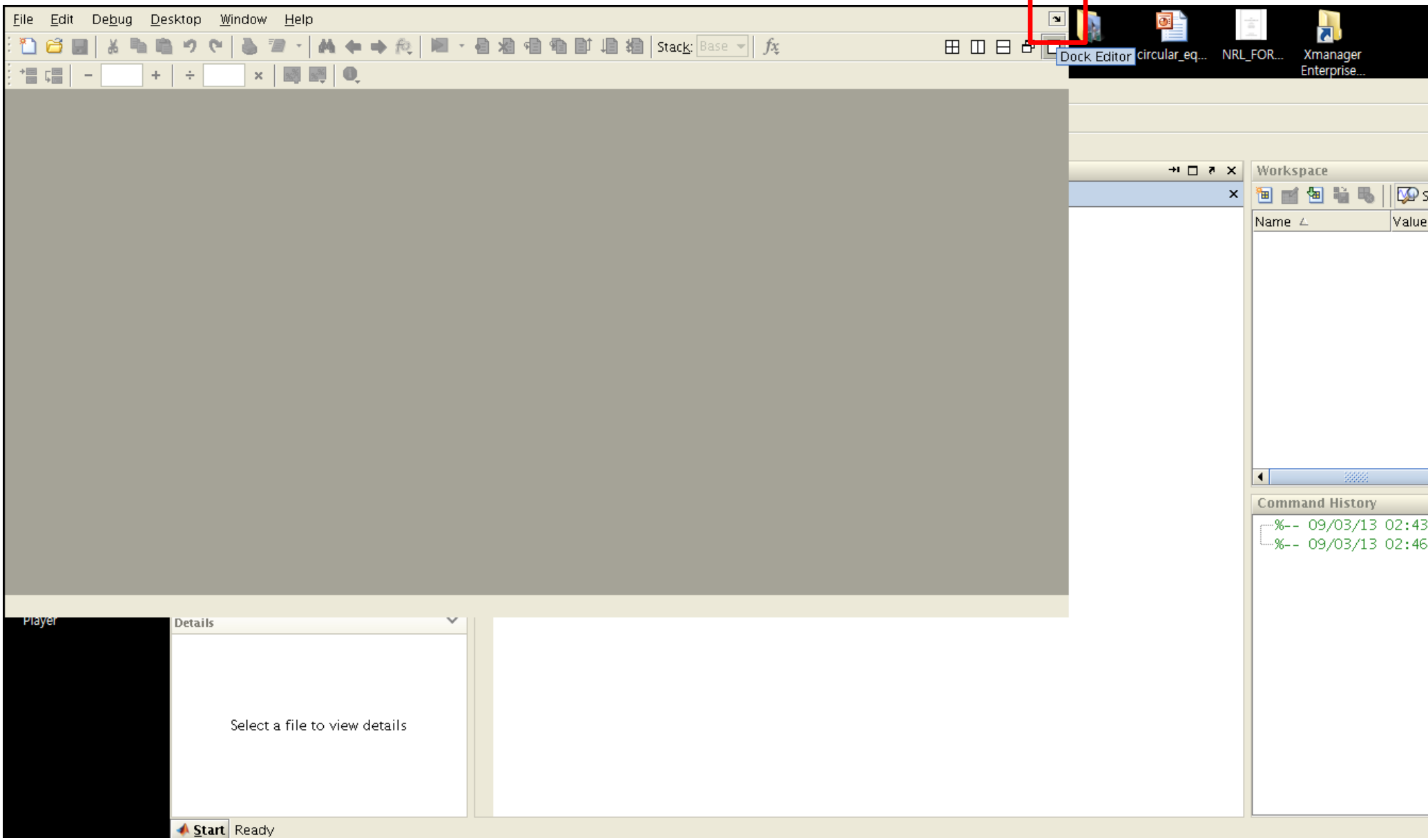
OVR

Open Editor window:



Dock Editor window:

Push this button



File Edit Debug Parallel Desktop Window Help

Current Folder: /global/u2/t/train258/eigensolver-ex/BOUT-2.0/tools/eigensolver/GLF_ITG

Shortcuts How to Add What's New

Current Folder

« GLF_ITG

Name
eigensolver_init.m
eigensolver_ITG.m
eigensolver_ITG.pdf
plot_eigenmode.m
plot_eigenvalue.m
README

Details

Select a file to view details

Editor

Command Window

New to MATLAB? Watch this Video, see Demos, or read Getting Started.

>>

Workspace

Select dat...

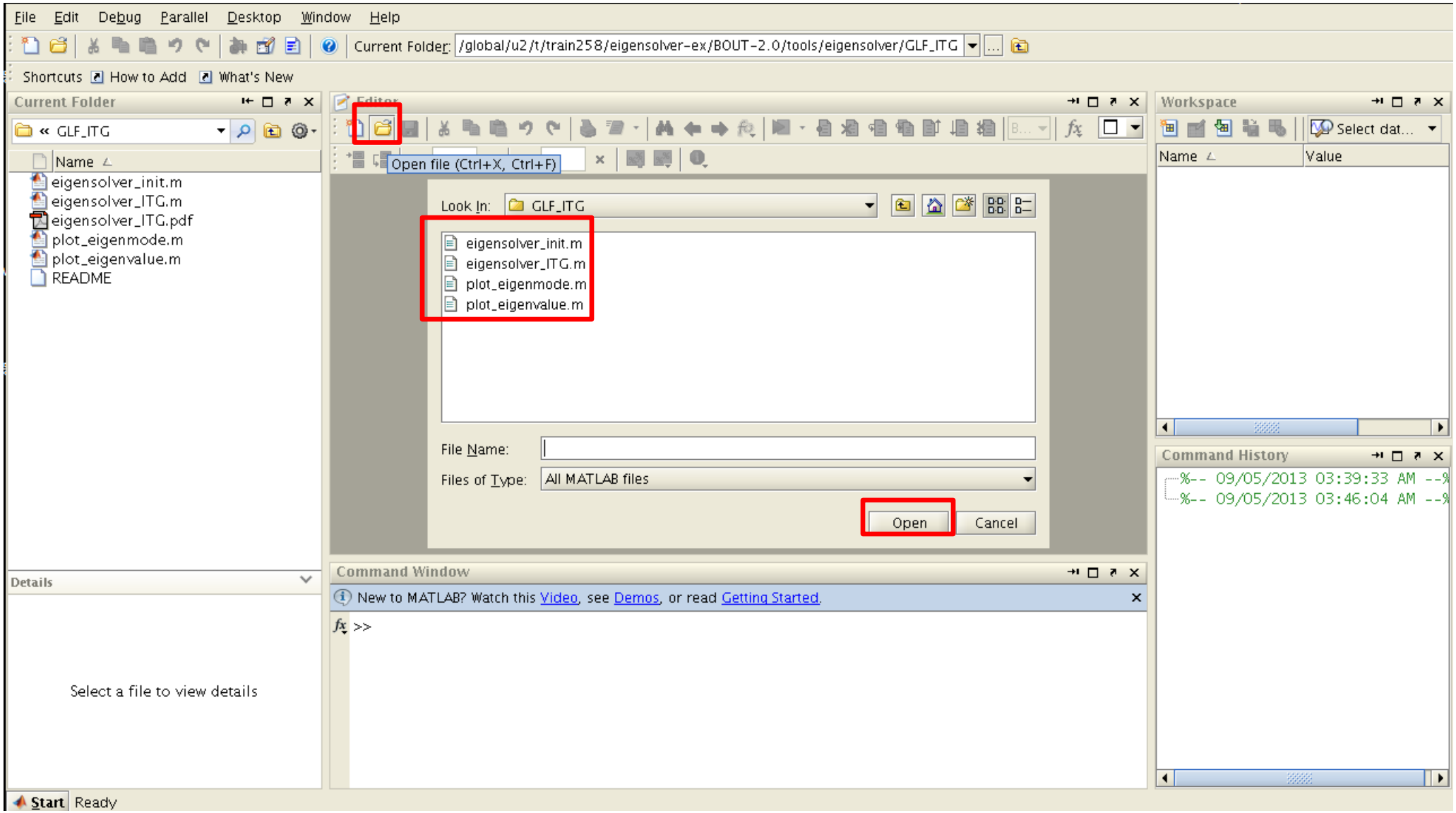
Name	Value

Command History

```
%-- 09/05/2013 03:39:33 AM --%
%-- 09/05/2013 03:46:04 AM --%
```

Start Ready

Open eigensolver_init.m, eigensolver_ITG.m, plot_eigenmode.m, plot_eigenvalue.m :



Run eigensolver_init.m: it takes about 5 min when nmax=15.

File Edit Text Go Cell Tools Debug Parallel Desktop Window Help

Current Folder: /global/u2/t/train258/eigensolver-ex/BOUT-2.0/tools/eigensolver/GLF_ITG

Shortcuts How to Add What's New

Current Folder

« GLF_ITG

Name
- eigensolver_init.m
- eigensolver_ITG.m
- eigensolver_ITG.pdf
- plot_eigenmode.m
- plot_eigenvalue.m
- README

Details

Select a file to view details

Editor - /global/u2/t/train258/eigensolver-ex/BOUT-2.0/tools/eigensolver/GLF_ITG/eigensolv...

Run eigensolver_init.m (F5)

```
7     %==================================================================
8
9 -         jmax=256; % number of radial grids
10 -        nmax=15;  % maximum of toroidal mode number
11 -        nmin=5;   % minimum of toroidal mode number
12 -        ndel=5;   % toroidal mode number spacing
13 -        mmax=150; % maximum of poloidal mode number
14 -        mmin=1;   % minimum of poloidal mode number
15 -        mdel=1;   % poloidal mode number spacing
16 -        kyrhoscut=1.3;  % maximal ktheta*rhoi allowed in the calculation
17 -        xmu1=0.1; % viscous damping coefficient
18 -        xmu2=0.0; % hyper-viscous damping coefficient
19 -        r0=0.2;   % inner boundary
20 -        r1=0.9;   % outer boundary
21
22 -        hermite=1;
23 -        bessel=2;
24 -        basis=bessel;     % choose radial basis function
```

eigensolver_init.m | eigensolver_ITG.m | plot_eigenmode.m | plot_eigenvalue.m

Command Window

New to MATLAB? Watch this Video, see Demos, or read Getting Started.

```
status =

   Calculating radial basis functions:


status =

   ... finding zeros of Bessel functions
```

Workspace

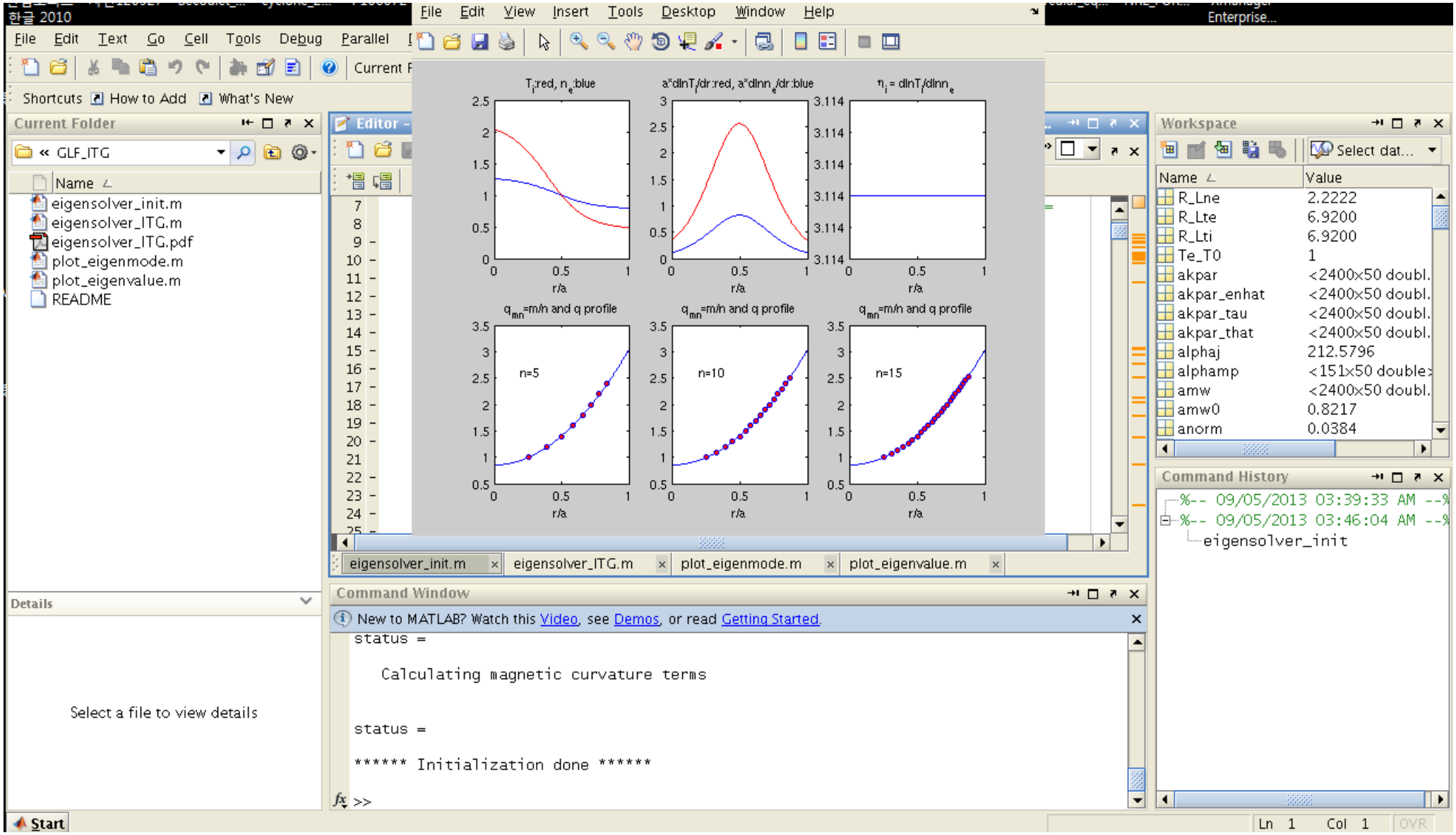
Select dat...

Name	Value

Command History

```
%-- 09/05/2013 03:39:33 AM --%
%-- 09/05/2013 03:46:04 AM --%
    eigensolver_init
```

Start Busy Ln 1 Col 1 OVR


File Edit View Insert Tools Desktop Window Help
File Edit Text Go Cell Tools Debug Parallel
Shortcuts How to Add What's New
Current Folder
GLF_ITG
Name
eigensolver_init.m
eigensolver_ITG.m
eigensolver_ITG.pdf
plot_eigenmode.m
plot_eigenvalue.m
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Select a file to view details
Editor
eigensolver_init.m
eigensolver_ITG.m
plot_eigenmode.m
plot_eigenvalue.m
T_i:red, n_e:blue
a*dlnT_i/dr:red, a*dlnn_e/dr:blue
η_i = dlnT_i/dlnn_e
q_mn=m/n and q profile
n=5
n=10
n=15
r/a
Command Window
New to MATLAB? Watch this Video, see Demos, or read Getting Started.
status =
Calculating magnetic curvature terms
status =
****** Initialization done ******
>>
Workspace
Select dat...
Name
Value
R_Lne 2.2222
R_Lte 6.9200
R_Lti 6.9200
Te_T0 1
akpar <2400x50 doubl.
akpar_enhat <2400x50 doubl.
akpar_tau <2400x50 doubl.
akpar_that <2400x50 doubl.
alphaj 212.5796
alphamp <151x50 double>
amw <2400x50 doubl.
amw0 0.8217
anorm 0.0384
Command History
%-- 09/05/2013 03:39:33 AM --%
%-- 09/05/2013 03:46:04 AM --%
eigensolver_init
Start
Ln 1 Col 1 OVR

Run eigensolver_ITG.m : it takes about 10 min.

File Edit Text Go Cell Tools Debug Parallel Desktop Window Help

Current Folder: /global/u2/t/train258/eigensolver-ex/BOUT-2.0/tools/eigensolver/GLF_ITG

Shortcuts How to Add What's New

Current Folder

« GLF_ITG

Name
- eigensolver_init.m
- eigensolver_ITG.m
- eigensolver_ITG.pdf
- plot_eigenmode.m
- plot_eigenvalue.m
- README

Editor - /global/u2/t/train258/eigensolver-ex/BOUT-2.0/tools/eigensolver/GLF_ITG/eigensolv...

Run eigensolver_ITG.m (F5)

```
%==================================================================
% Solve eigenvalue equation for ITG :
% Ottaviani model used [M. Ottaviani et.al., Phys. Plasmas 6, 3267 (1999)]
%==================================================================

  gyroaverage=1; % turn-on(1) or off(0) gyroaverage
  damping_glf=1; % turn-on(1) or off(0) Landau damping term
  damping_c=0;   % turn-on(1) or off(0) viscous damping term

  l_k=0*ones(1,kmax);
  lmaxn=0*ones(1,nmax/ndel);

  l=0;
  for k=1:kmax
     if(nfb(k)==nmax)
       l=l+1;
     end
  end
  lmax=l;
```

eigensolver_init.m | eigensolver_ITG.m | plot_eigenmode.m | plot_eigenvalue.m

Workspace

Name	Value
R_Lne	2.2222
R_Lte	6.9200
R_Lti	6.9200
Te_T0	1
akpar	<2400x50 doubl.
akpar_enhat	<2400x50 doubl.
akpar_tau	<2400x50 doubl.
akpar_that	<2400x50 doubl.
alphaj	212.5796
alphamp	<151x50 double>
amw	<2400x50 doubl.
amw0	0.8217
anorm	0.0384

Command History

```
%-- 09/05/2013 03:39:33 AM --%
%-- 09/05/2013 03:46:04 AM --%
  eigensolver_init
  eigensolver_ITG
```

Command Window

New to MATLAB? Watch this Video, see Demos, or read Getting Started.

```
status =

   Setting up matrix for eigenvalue equation


status =

   Calculating eigenvalues and eigenvectors
```

Details

Select a file to view details

Start Busy script Ln 8 Col 61 OVR

Run plot_eigenvalue.m;

Current Folder: /global/u2/t/train258/eigensolver-ex/BOUT-2.0/tools/eigensolver/GLF_ITG

Editor - /global/u2/t/train258/eigensolver-ex/BOUT-2.0/tools/eigensolver/GLF_ITG/plot_eigen...

Run plot_eigenvalue.m (F5)

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% plot linear growth rate and real frequency vs. ktheta*rhoi
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
clear ktheta_rhoi
qval=1.4;
rval=0.5;
for n1=nmin:ndel:nmax
    ktheta_rhoi(n1/ndel)=n1*qval/rval*rhos0;
end
R=1.3; a=0.48;  % for cyclone base case
gamfactor=R_Lne*a/R;  % change time unit from a/v_ti to L_ne/v_ti
omgfactor=gamfactor*4;

figure(2)
plot(ktheta_rhoi,gamlin/gamfactor,'-or');
xlabel('k_\theta \rho_i')
ylabel('\gamma  and  \omega/4  (v_{ti}/L_{ne})')
title('Linear growth rate and real frequency')
hold on
plot(ktheta_rhoi,omglin/omgfactor,'-ob');
```

Command Window

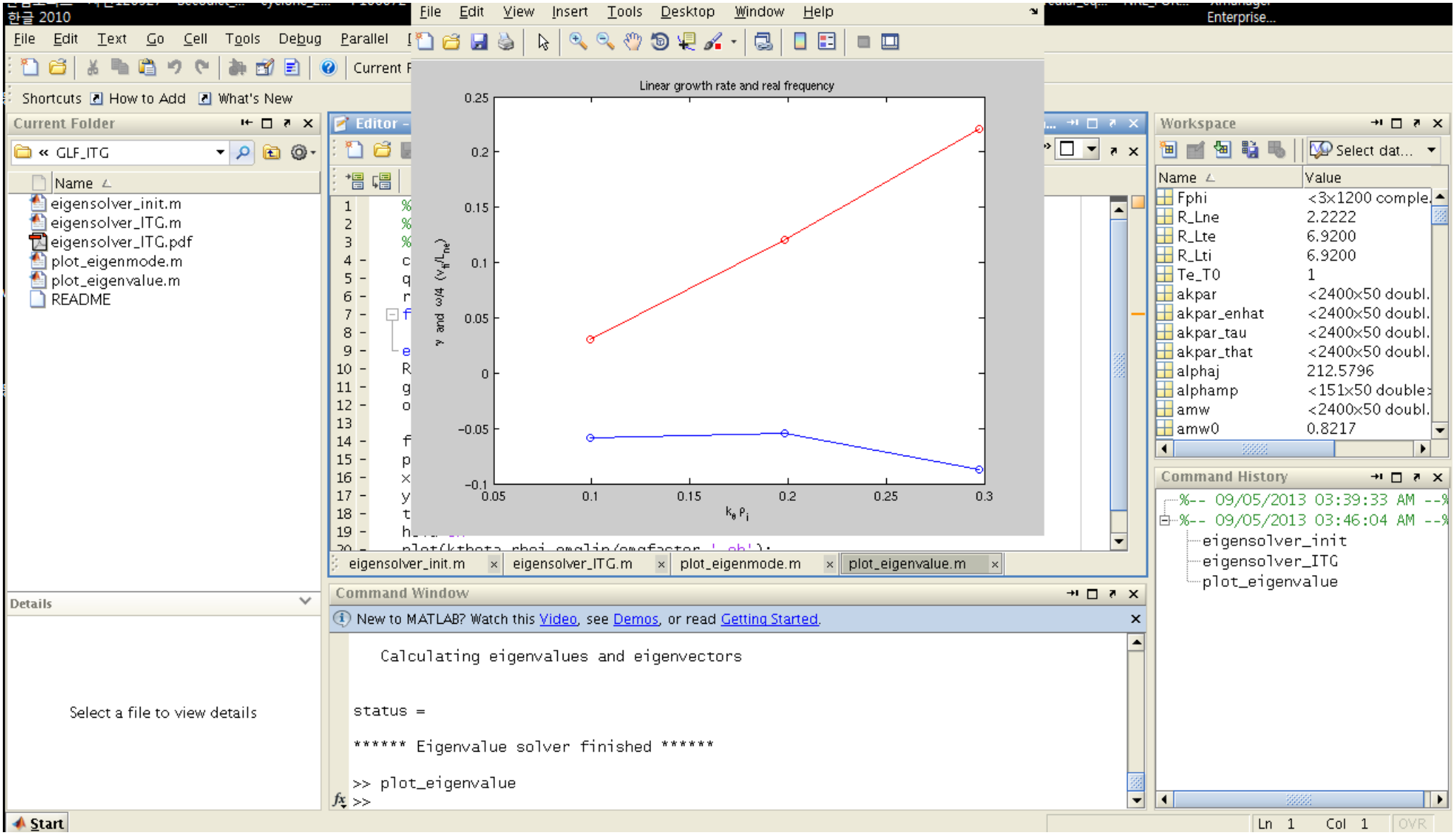
```
status =

   Calculating eigenvalues and eigenvectors


status =

****** Eigenvalue solver finished ******

>>
```


Linear growth rate and real frequency
0.25
0.2
0.15
0.1
0.05
0
−0.05
−0.1
0.05
0.1
0.15
0.2
0.25
0.3
Current Folder
GLF_ITG
Name
eigensolver_init.m
eigensolver_ITG.m
eigensolver_ITG.pdf
plot_eigenmode.m
plot_eigenvalue.m
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Select a file to view details
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eigensolver_init.m
eigensolver_ITG.m
plot_eigenmode.m
plot_eigenvalue.m
Command Window
New to MATLAB? Watch this Video, see Demos, or read Getting Started.
Calculating eigenvalues and eigenvectors
status =
****** Eigenvalue solver finished ******
>> plot_eigenvalue
>>
Workspace
Name
Value
Fphi
<3x1200 comple.
R_Lne
2.2222
R_Lte
6.9200
R_Lti
6.9200
Te_T0
1
akpar
<2400x50 doubl.
akpar_enhat
<2400x50 doubl.
akpar_tau
<2400x50 doubl.
akpar_that
<2400x50 doubl.
alphaj
212.5796
alphamp
<151x50 double
amw
<2400x50 doubl.
amw0
0.8217
Command History
%-- 09/05/2013 03:39:33 AM --%
%-- 09/05/2013 03:46:04 AM --%
eigensolver_init
eigensolver_ITG
plot_eigenvalue

Run plot_eigenmode.m:

Current Folder: /global/u2/t/train258/eigensolver-ex/BOUT-2.0/tools/eigensolver/GLF_ITG

Editor - /global/u2/t/train258/eigensolver-ex/BOUT-2.0/tools/eigensolver/GLF_ITG/plot_eigen...

Run plot_eigenmode.m (F5)

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Contour plot of eigenmode structure for potential
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
theta=-pi:0.01:pi;
mft=mf';
matexp=exp(i*mft(1:kmax)*theta);
wavemap=wave(:,1:kmax);
rcore=r';
x=rcore*cos(theta);
y=rcore*sin(theta);

%for n1=nmin:ndel:nmax

n1=10; % choose toroidal mode number

  nn=n1/ndel;
  figure(2+nn)
  ph=0*ones(kmax,1);
  l=0;
```

You can choose toroidal mode number.

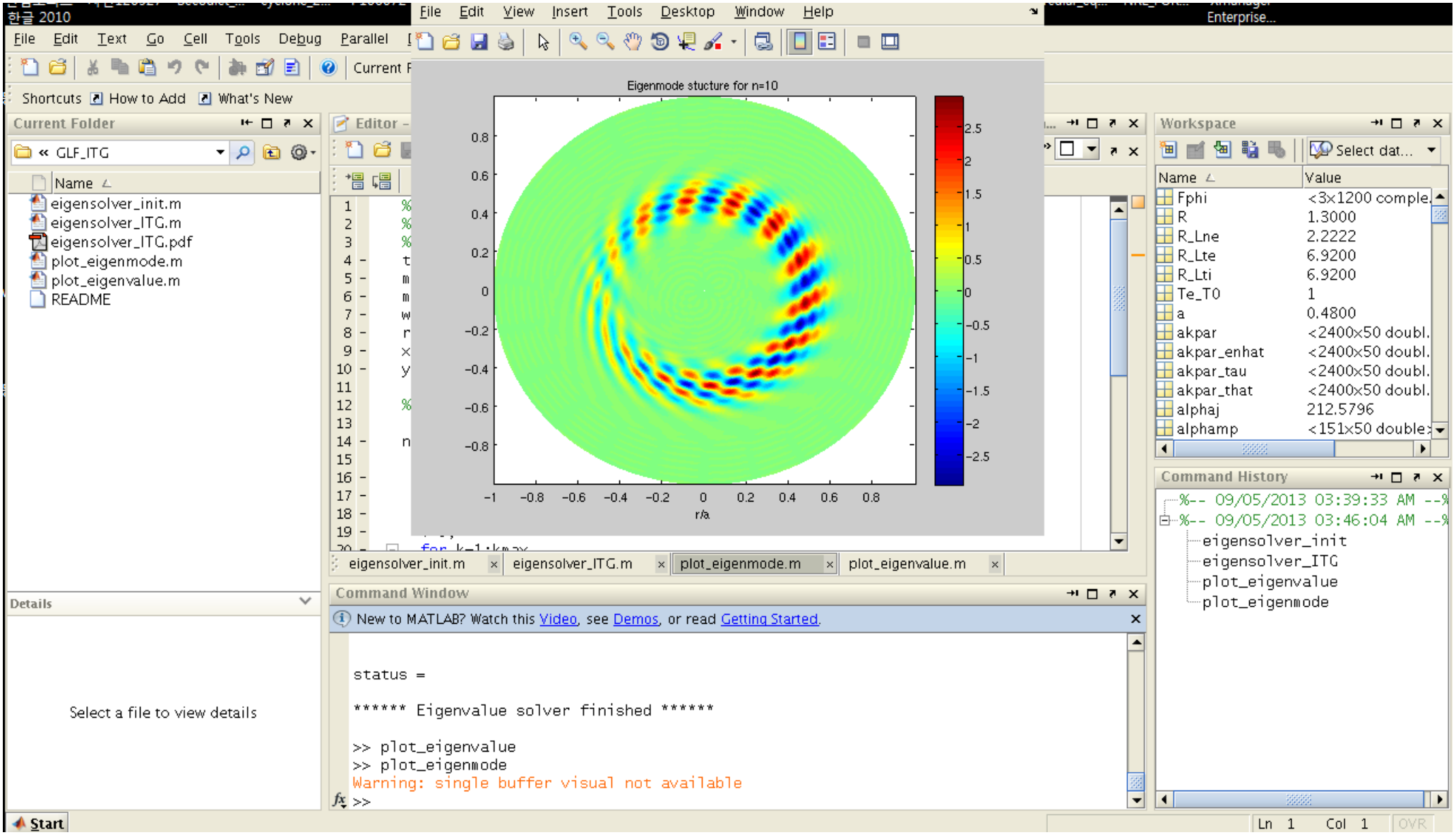
Command Window

```
Calculating eigenvalues and eigenvectors


status =

****** Eigenvalue solver finished ******

>> plot_eigenvalue
>>
```


Eigenmode stucture for n=10
r/a
Current Folder
GLF_ITG
eigensolver_init.m
eigensolver_ITG.m
eigensolver_ITG.pdf
plot_eigenmode.m
plot_eigenvalue.m
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Select a file to view details
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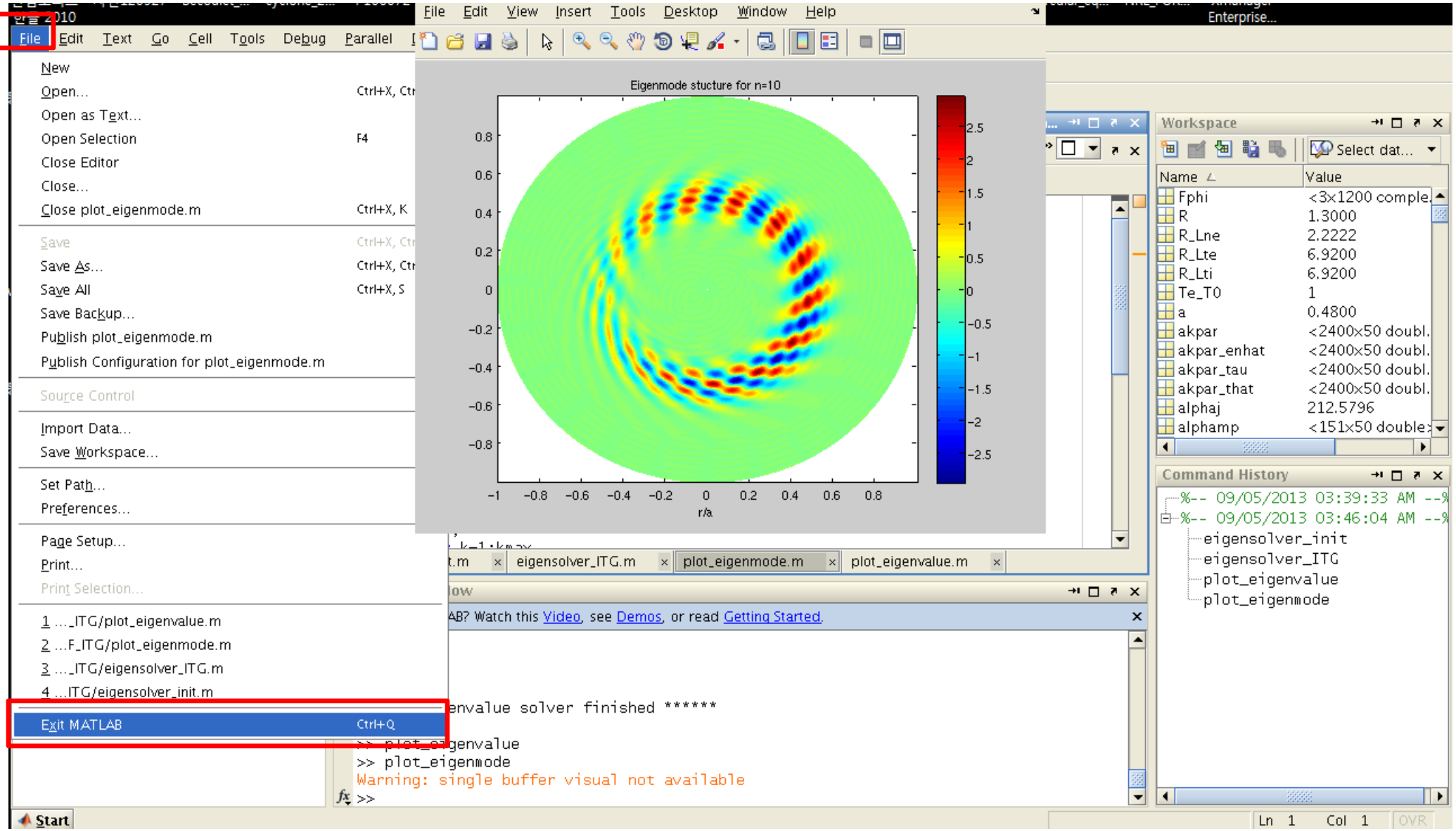
eigensolver_init.m
eigensolver_ITG.m
plot_eigenmode.m
plot_eigenvalue.m
Command Window
New to MATLAB? Watch this Video, see Demos, or read Getting Started.
status =
****** Eigenvalue solver finished ******
>> plot_eigenvalue
>> plot_eigenmode
Warning: single buffer visual not available
Workspace
Fphi <3x1200 comple.
R 1.3000
R_Lne 2.2222
R_Lte 6.9200
R_Lti 6.9200
Te_T0 1
a 0.4800
akpar <2400x50 doubl.
akpar_enhat <2400x50 doubl.
akpar_tau <2400x50 doubl.
akpar_that <2400x50 doubl.
alphaj 212.5796
alphamp <151x50 double
Command History
%-- 09/05/2013 03:39:33 AM --%
%-- 09/05/2013 03:46:04 AM --%
eigensolver_init
eigensolver_ITG
plot_eigenvalue
plot_eigenmode

Exit matlab:


File Edit Text Go Cell Tools Debug Parallel
New
Open... Ctrl+X, Ctr
Open as Text...
Open Selection F4
Close Editor
Close...
Close plot_eigenmode.m Ctrl+X, K
Save Ctrl+X, Ctr
Save As... Ctrl+X, Ctr
Save All Ctrl+X, S
Save Backup...
Publish plot_eigenmode.m
Publish Configuration for plot_eigenmode.m
Source Control
Import Data...
Save Workspace...
Set Path...
Preferences...
Page Setup...
Print...
Print Selection...
1 ..._ITG/plot_eigenvalue.m
2 ...F_ITG/plot_eigenmode.m
3 ..._ITG/eigensolver_ITG.m
4 ...ITG/eigensolver_init.m
Exit MATLAB Ctrl+Q
File Edit View Insert Tools Desktop Window Help
Eigenmode stucture for n=10
r/a
eigensolver_ITG.m
plot_eigenmode.m
plot_eigenvalue.m
Workspace
Select dat...
Name Value
Fphi <3x1200 comple.
R 1.3000
R_Lne 2.2222
R_Lte 6.9200
R_Lti 6.9200
Te_T0 1
a 0.4800
akpar <2400x50 doubl.
akpar_enhat <2400x50 doubl.
akpar_tau <2400x50 doubl.
akpar_that <2400x50 doubl.
alphaj 212.5796
alphamp <151x50 double
Command History
%-- 09/05/2013 03:39:33 AM --%
%-- 09/05/2013 03:46:04 AM --%
eigensolver_init
eigensolver_ITG
plot_eigenvalue
plot_eigenmode
envalue solver finished ******
>> plot_eigenvalue
>> plot_eigenmode
Warning: single buffer visual not available
Start
Ln 1 Col 1 OVR