Magnetic Reconnection in Slab Geometry

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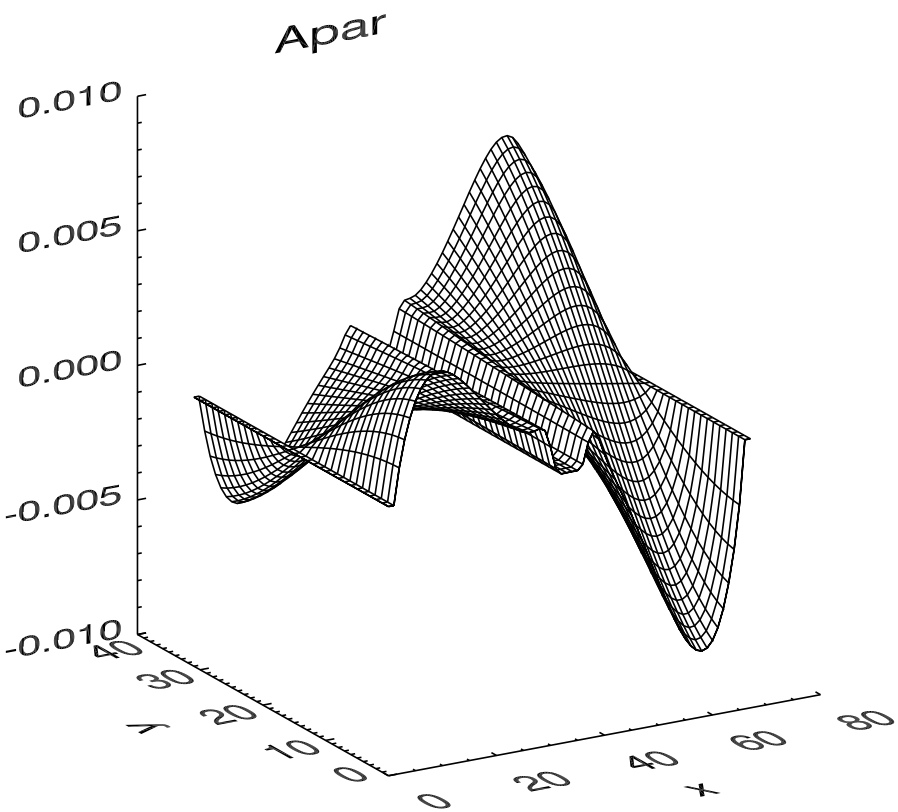
LLNL-PRES-643597

Outline

- Code overview
  - some recent improvements
  - slab grid generator
  - reconnect-2field
  - post-processing

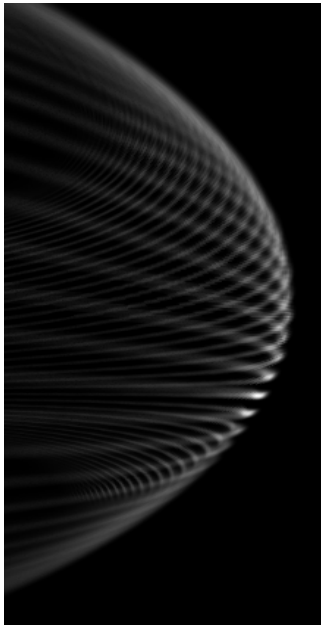
- Benchmarking reconnection dynamics
  - Scaling with dissipation
  - Suppression due to rotation



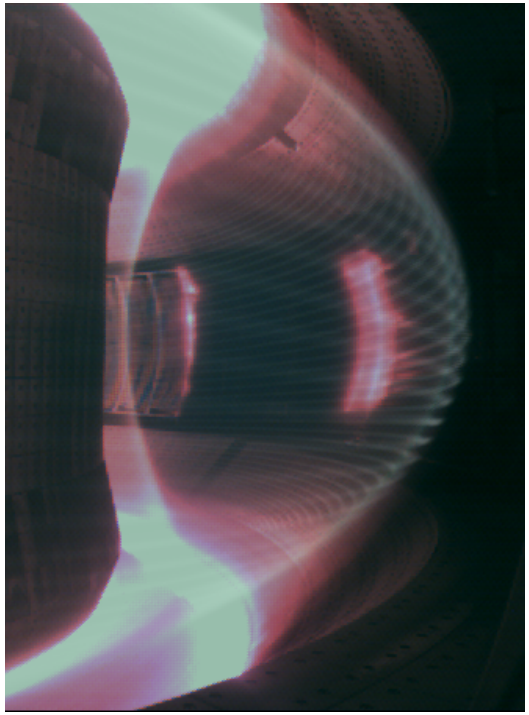
Some improvements to IDL tools since 2011

- I developed some tools to generate output to Bill Meyer's raytracing code
  - `write_polslice, resize_bgrid`

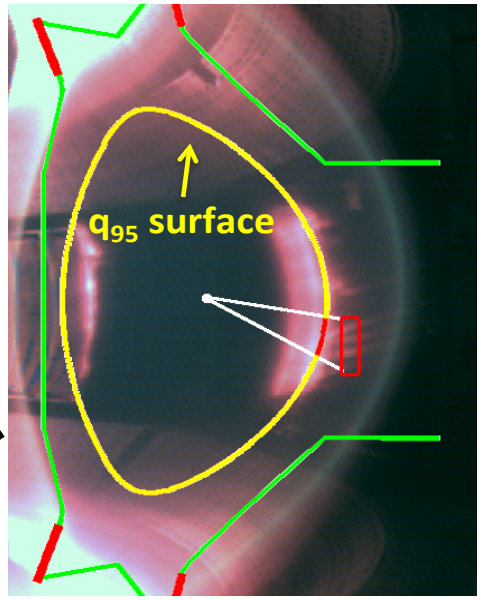
Simulation



Comparison



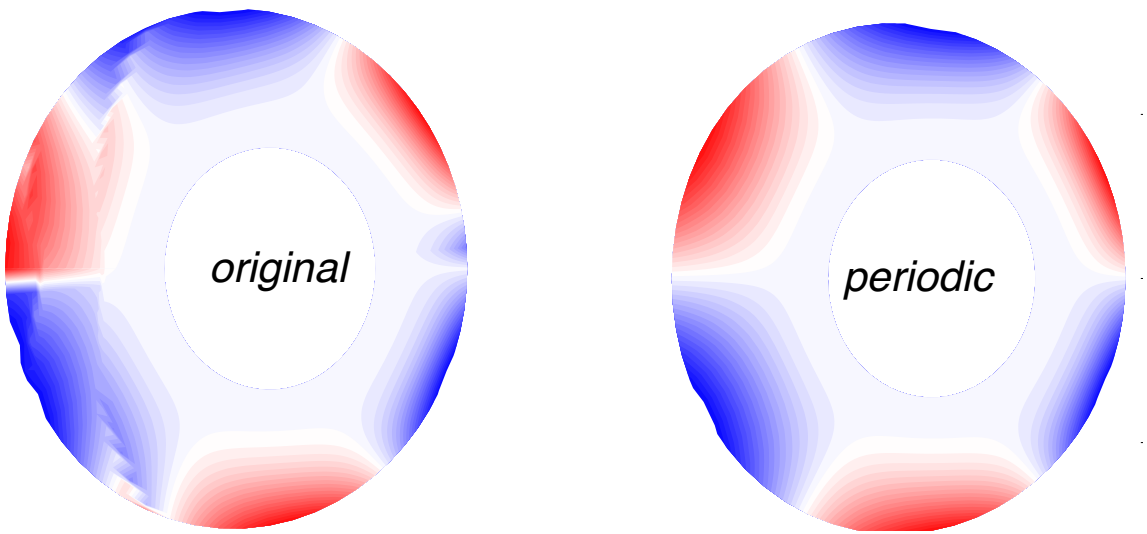
Measurement



T.Y. Xia, et al., 24th IAEA FEC, San Diego, Oct. 2012

Some improvements to IDL processing

- IDL processing improvements
  - Added some useful numerical utilities to **`tools/numlib`**
    - Clean up coding by using `CASE` to test dimension of input variable
  - Fixed error in `zshift` and `plotpolslice` for periodic domains
  - Please contribute!



Some improvements to core code

- **Core BOUT++ improvements (B. Dudson)**
  - Physics-based **preconditioning** for shear Alfven wave greatly improves convergence for large time steps
    - Must carefully set error tolerances to ensure that output is not **garbage**
      - ATOL and RTOL
  - New Option **SymmetricGlobalX = true** ➔ initial profiles are symmetric in X
    - otherwise there is an asymmetry in GLOBALX function!
  - **Laplace inversion** with nonconstant BCs now gives correct result when **NXPE>1**
    - allows simulations with more processors

"Taylor's problem" for resistive reconnection proceeds in a number of different temporal phases*

- **2 Field model**

$$\partial_t A_\parallel = -\nabla_\parallel \phi - \eta J_\parallel \qquad J_\parallel = -\nabla_\perp^2 A_\parallel$$

$$\partial_t U + \nabla \cdot V_E U = V_A^2 \nabla \cdot J_\parallel + \mu \nabla_\perp^2 U \qquad U = mn\nabla_\perp^2 \varphi / B$$

- **Spatial scales***

$$s = aq'/q = aB_y'/B_y = qR/R_s \sim 3.3 \qquad k_y \delta_{Rec} = \left(\tau_H / \tau_R\right)^{1/3} = S_R^{-1/3}$$

$$R_s = B_z / B_y' = k_y / k_\parallel' = qR/s \sim 4.5m \qquad k_y \delta_{RI} = \left(\omega \tau_H^2 / \tau_R\right)^{1/4} = S_R^{-1/4} S_\omega^{-1/4}$$

- **Intrinsic & derived time scales**

$$\tau_A = R_s / V_A = 0.64 \mu s \qquad \tau_{Rec} = \tau_R^{1/3} \tau_A^{2/3} \propto \eta^{-1/3} k_y^{-2/3}$$

$$\tau_R = a^2 / \eta = 4.6ms \qquad \tau_{RI} = 1.8 \tau_R^{3/5} \tau_H^{2/5} \propto \eta^{-3/5} k_y^{-2/5}$$

$$\tau_V = a^2 / \mu$$

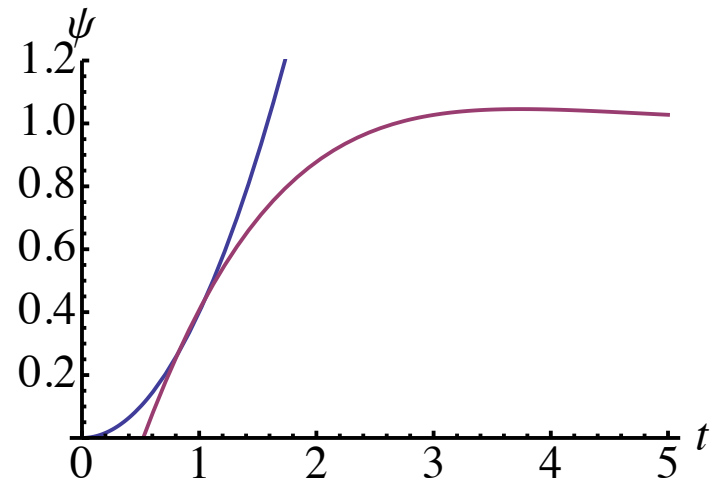
Linear "Taylor's problem" proceeds in 2 distinct phases*

- **Approximate linear analytic solution****

$$\psi \approx \begin{cases} At^2 / 2\tau_R \tau_A & t < \tau_{rec} \\ B\left(1 - \frac{8}{5} e^{-tc/\tau_{RI}} \cos\left(ts / \tau_{RI}\right)\right) & t > \tau_{rec} \\ c + is = e^{i\pi/5} & \end{cases}$$

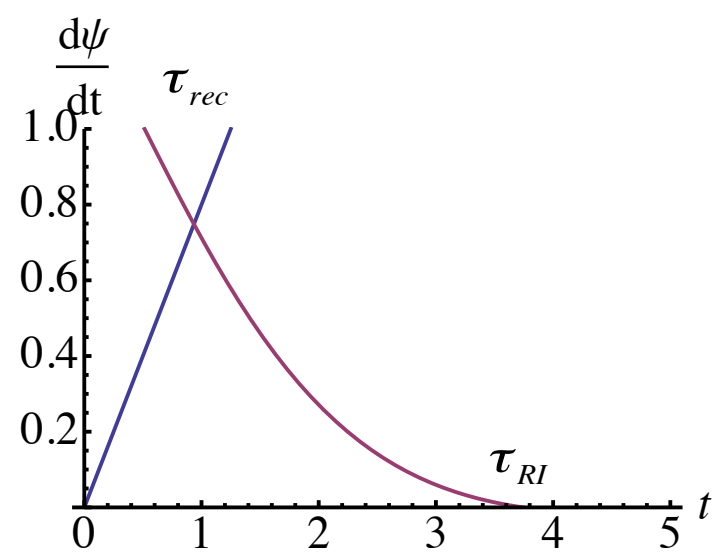
$$\tau_{Rec} = \tau_R^{1/3} \tau_A^{2/3} \propto \eta^{-1/3} k_y^{-2/3}$$

$$\tau_{RI} = 1.8 \tau_R^{3/5} \tau_H^{2/5} \propto \eta^{-3/5} k_y^{-2/5}$$



- **Different scalings for**
  - **time of peak rate = reconnection time τ_{Rec}**
  - **time of peak flux = saturation time τ_{RI}**

$$\max\left(\tau_A \dot{\psi}\right) \sim \tau_A \dot{\psi}\left(\tau_{Rec}\right) \propto \frac{\tau_{Rec}}{\tau_R} = \left(\frac{\tau_A}{\tau_R}\right)^{2/3} \propto \left(\frac{\eta}{k_y}\right)^{2/3}$$

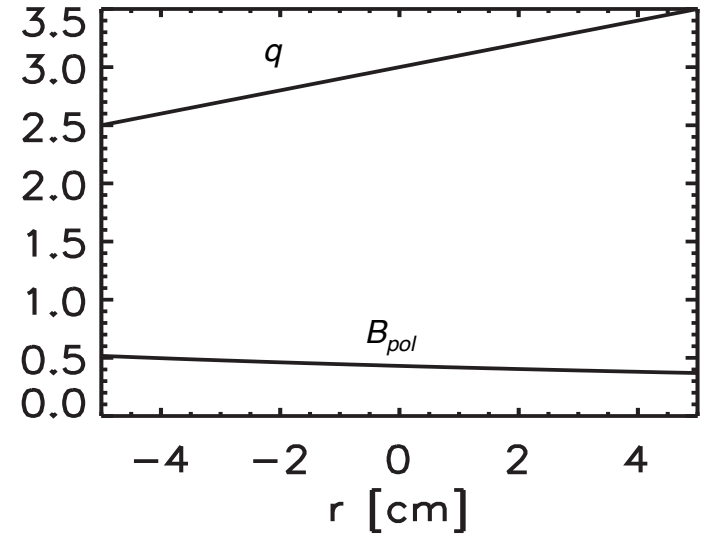


*T.S. Hahm and R. M. Kulsrud, Phys. Fluids **28** (1985) 2412

A. Cole and R. Fitzpatrick, Phys. Plasmas **11 (2004) 3525

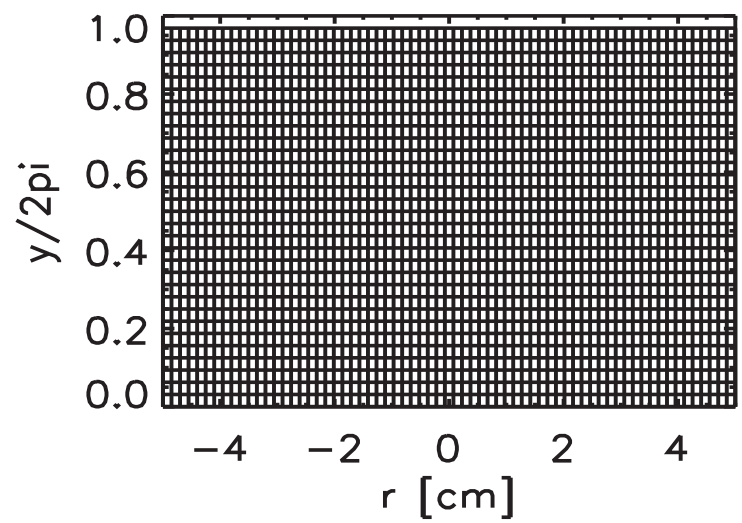
Slab geometry

- Equilibrium
  - $R = 5$ m, $a = 1$ m, $\varepsilon = 0.2$
  - $B_{tor} = 6.46$ T, $n_e = 10^{20}$ m^{-3}
  - $B_{pol}/B_{tor} = \varepsilon/q = 6\text{-}8\times10^{-2}$
  - $q=[2.5,3.5]$
  - $s = aq'/q = 10/3 \sim 3.3$, $Ls = qR/s = 4.5$ m
  - $\Delta r = 0.1$ m, $\Delta\psi = 0.21$ Vs
  - $\rho = 1$ mm, $\rho/\Delta r = 1/100$
  - $\delta r = 0.1$ m$/68 \sim 1.5$ mm $\sim 1.5\ \rho$



- BOUT mesh uses twist-shift periodic coordinates
  - Given orthogonal coordinates: ψ, θ, ζ
  - BOUT coordinates are $x=\psi/\delta\psi$, $y=\theta$, and

$$z = \zeta - \int d\theta \frac{B\cdot\nabla\zeta}{B\cdot\nabla\theta} = \zeta - q\theta + periodic$$



Slab Grid Generator: $BOUT_TOP/tools/slab/slab.pro

```
tools/slab> more slab.pro
; Slab geometry grid generator
;
; Optional keywords:
; =================
;
; output   =   Set output file name
; thin     =   Use thin radial box approximation
;              so Bpxy = constant, but gradient is non-zero
;
; ni  = Ion density in 10^20 m^-3
; Ti  = temperature in eV (Te = Ti)
;
; Rmaj    = Major radius [meters]
; rminor  = Minor radius [m]
; dr      = Radial width of box [m]
; r_wid   = Radial extent, normalised to gyro-radius r_wid = dr / rho_i
;
; q       = Safety factor q = r*Bt/(R*Bp) at middle of box
; dq      = Change in q. Will go from q-dq/2 to q+dq/2
;
; L_T     = Temperature length scale [m]
; eta_i   = Ratio of density to temp. length scales eta = L_n / L_T

PRO slab, output=output, thin=thin, $
          ni=ni, Ti=Ti, $
          Rmaj=Rmaj, rminor=rminor, dr=dr, $
          r_wid=r_wid, $
          q=q, dq=dq, $
          L_T=L_T, eta_i=eta_i, $
          nx=nx, ny=ny
```

Generate a Slab Grid

```
tools/slab> more idlsave.pro
; IDL Version 8.2.2 (linux x86_64 m64)
; Journal File for train213@hopper09
; Working directory: /global/u2/t/train213/BOUT_Workshop_2013/BOUT-2.0/to
b
; Date: Wed Sep  4 14:12:39 2013

nx=2^8+4 & print, nx
;     260
ny=64
file = 'slab_'+str(nx)+'x'+str(ny)+'.nc' &  print, file
;slab_260x64.nc
slab, Rmaj=10,rmin=1,dr=.1, nx=nx, ny=ny, out=file
;Poloidal field varies from 0.184642 to 0.258498
;Writing grid to file slab_260x64.nc
;DONE
g=file_import(file)
help, g
tools/slab>
```

2-field model: $BOUT/examples/reconnect-2field/2field.cxx

```
examples/reconnect-2field> cd $BOUT_TOP/examples/reconnect-2field
examples/reconnect-2field> make clean
examples/reconnect-2field> make
  Compiling  2field.cxx
  Linking 2field
examples/reconnect-2field> ls
2field      2field-new.cxx  2field-precon.cxx  data      makefile-new
2field.cxx  2field.o        BOUT-new.inp       makefile  slab_68x32.nc
examples/reconnect-2field> more 2field.cxx
/*******************************************************************************
 * 2 field (Apar, vorticity) model for benchmarking
 * simple slab reconnection model
 *******************************************************************************/

#include <bout.hxx>
#include <boutmain.hxx>

#include <invert_laplace.hxx>
#include <invert_parderiv.hxx>
#include <initialprofiles.hxx>
#include <bout/constants.hxx>

#include <math.h>
#include <stdio.h>
#include <stdlib.h>

// 2D initial profiles
Field2D Jpar0, Te0, Ni0;

// 3D evolving fields
Field3D Upar, Apar;

// Derived 3D variables
Field3D Phi, Jpar;

// External fields
Field3D Apar_ext, Jpar_ext, Phi0_ext, Upar0_ext;
```

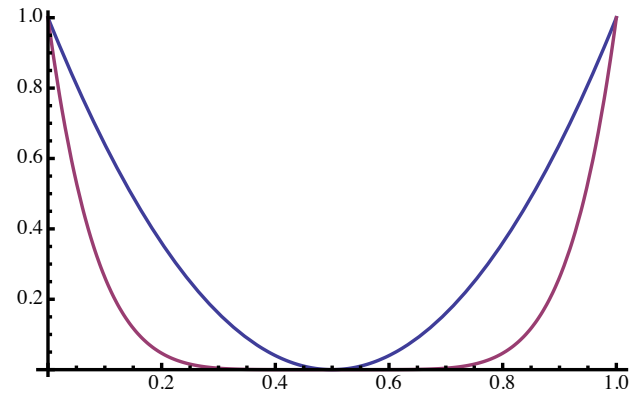
External Fields Stay Constant & Don't Evolve

Apply fixed external field A_{ext}

- Chosen form is resonant for q=3 in center of domain

$$A_{\parallel} = A_{edge}(1-4x(1-x))\sin(3\theta-\zeta)$$

- Radial profile chosen to vanish at the center to avoid reconnection at t=0



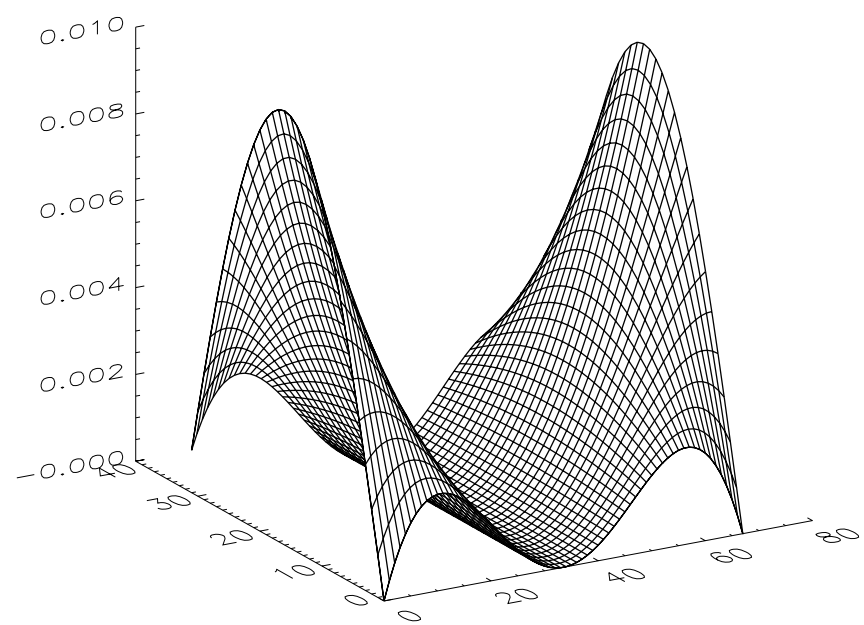
- 2field code treats variables with ordering:

$$A_{\parallel} = \delta(A_{ext} + A_1)$$

$$J_{\parallel} = -\nabla_{\perp}^2 A_{\parallel}$$

$$\varphi = \varphi_{0,ext} + \delta\varphi_1$$

$$U_1 = \nabla_{\perp}^2 \varphi$$

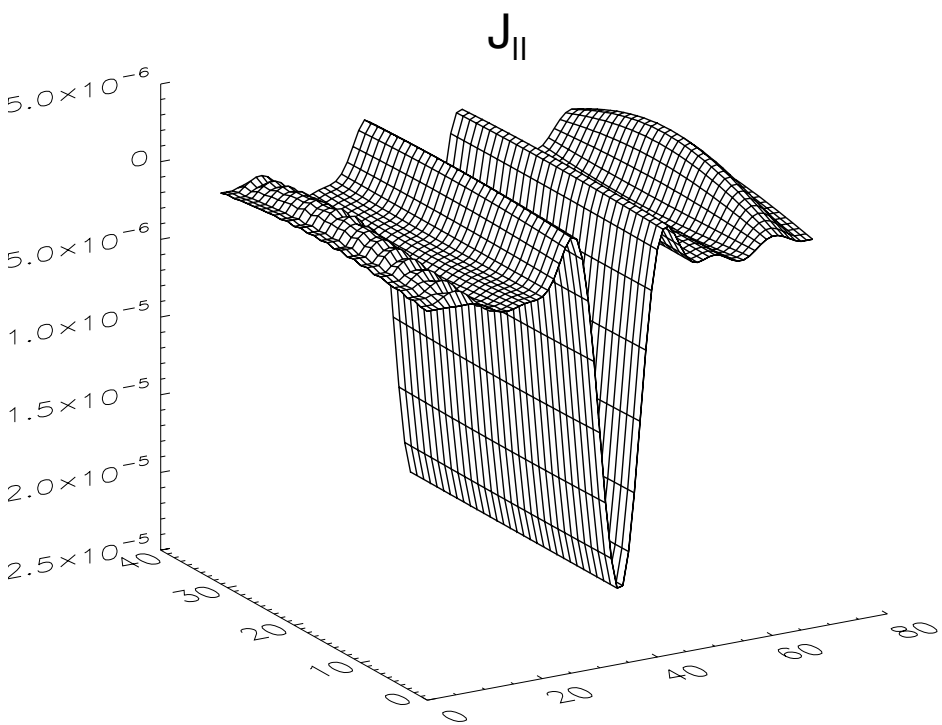


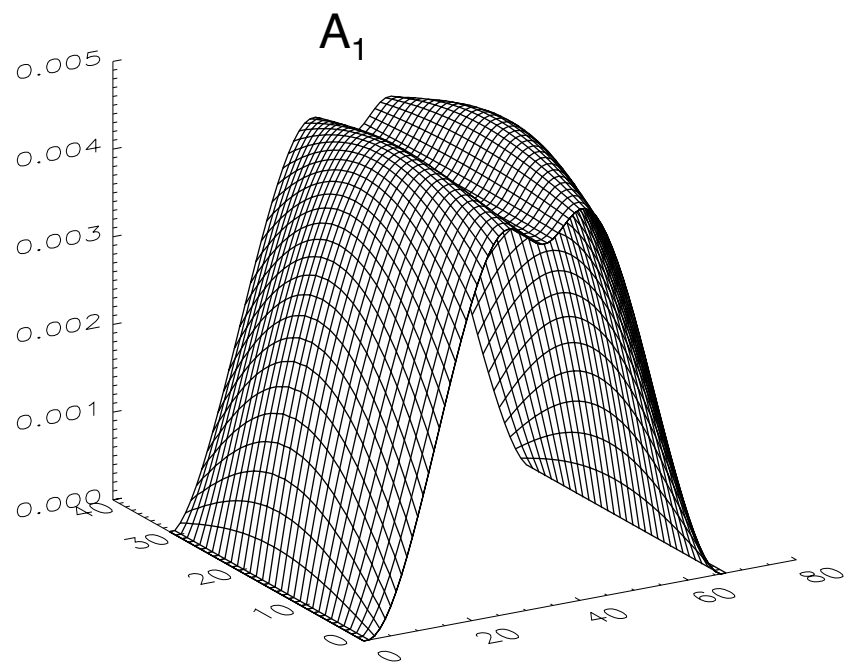
Run a 2field example

```
examples/reconnect-2field> qsub -I -V -q interactive -l mppwidth=2 -l advres=bout.10
qsub: waiting for job 6436873.hopque01 to start
qsub: job 6436873.hopque01 ready

ModuleCmd_Switch.c(172):ERROR:152: Module 'PrgEnv-pgi' is currently not loaded
t/train213> cd $BOUT_TOP/examples/reconnect-2field/
examples/reconnect-2field> aprun -n 2 ./2field

BOUT++ version 1.10
Revision: 546769c683863172c675d673f10c106ff9f5a2dc
Code compiled on Sep  4 2013 at 10:15:12

B.Dudson (University of York), M.Umansky (LLNL) 2007
Based on BOUT by Xueqiao Xu, 1999

Processor number: 0 of 2

pid: 19920

Compile-time options:
        Checking enabled, level 2
        Signal handling enabled
        PDB support disabled
        netCDF support enabled
        Parallel NetCDF support disabled
        OpenMP parallelisation disabled
Reading options file data/BOUT.inp
        Option /append = false  (default)
        Option /dump_format = nc (data/BOUT.inp)
        Option /staggergrids = false (data/BOUT.inp)
```

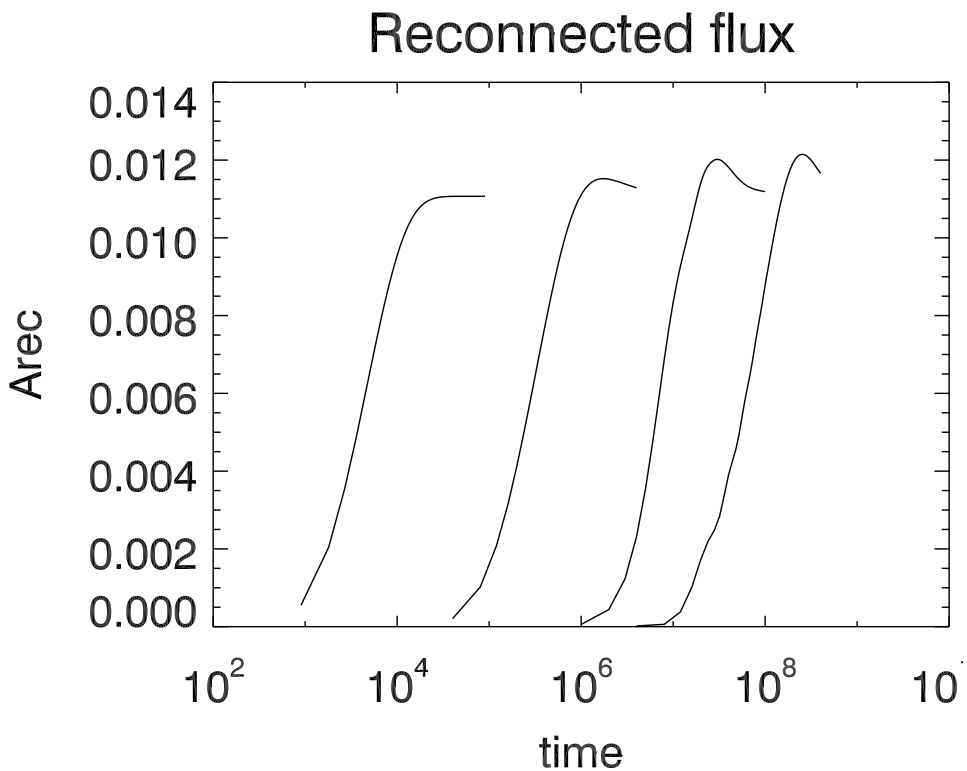
Examine data

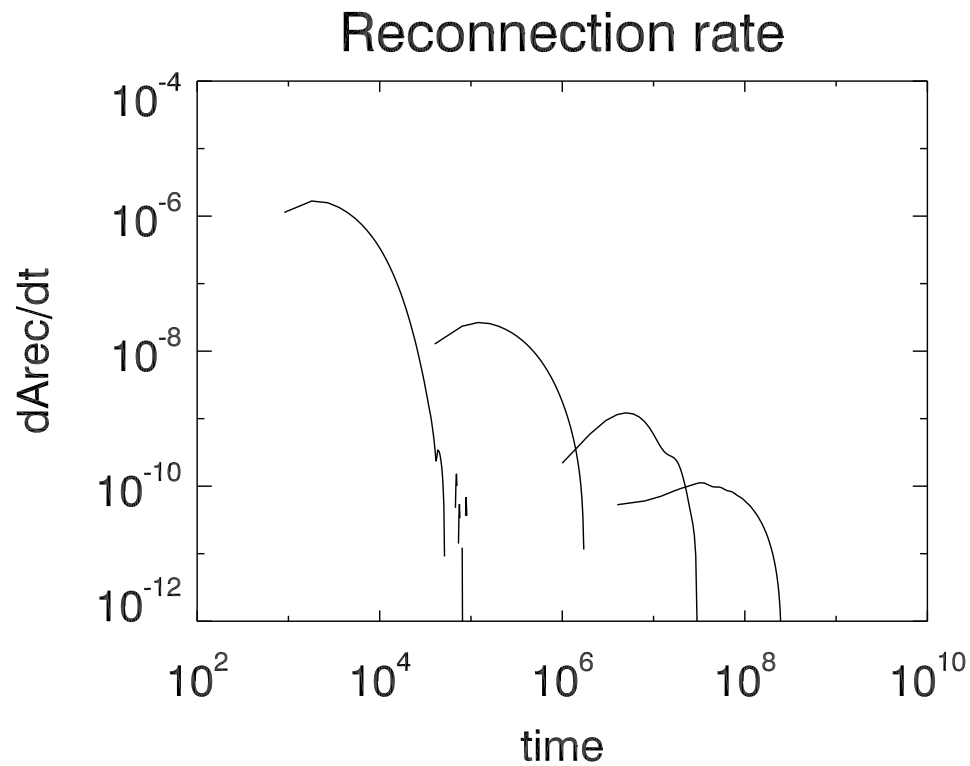
```
aext=collect(var='Apar_ext',path='data')
surface, reform(aext[*,*,4,*])
apar=collect(var='Apar',path='data')
showdata, reform(apar[*,*,4,*])
;chars=       2

examples/reconnect-2field>
```





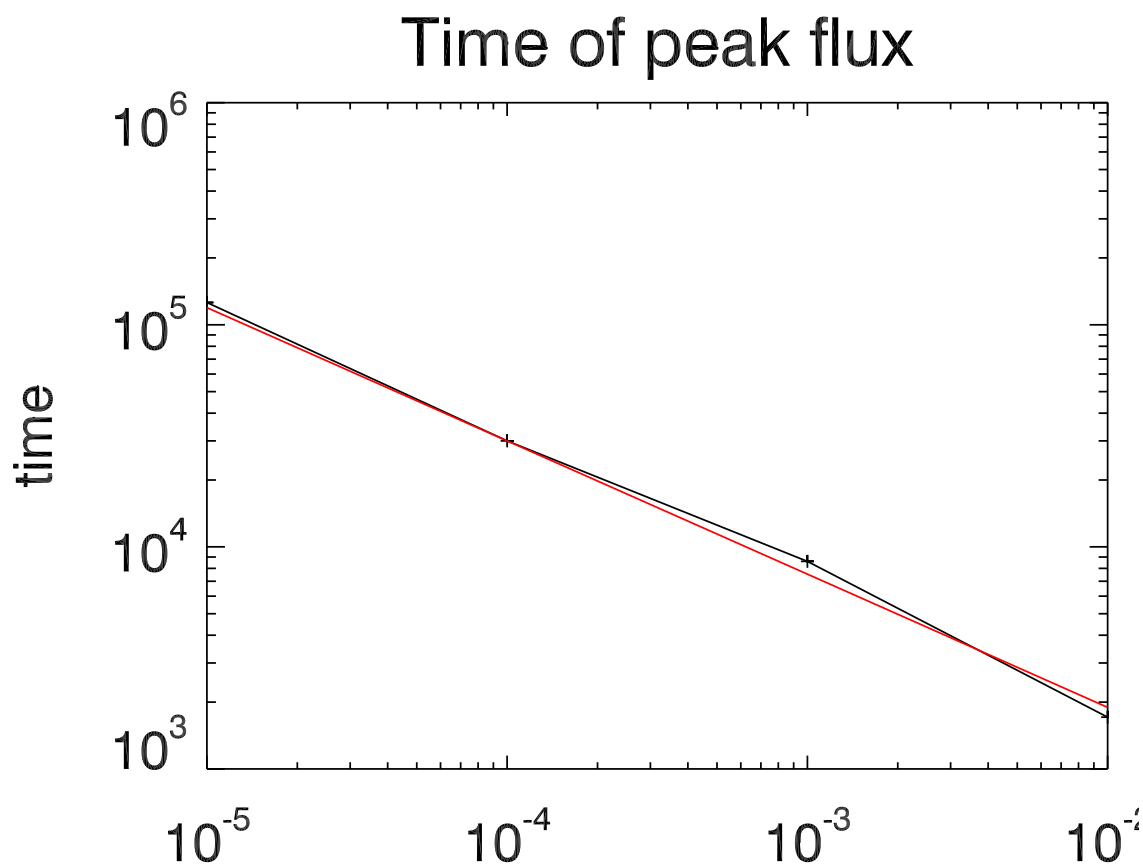
Evolution of reconnected flux for resistive model





- Reconnected flux generally peaks & relaxes as expected
- Initial growth is quadratic in time: t^2
- Resistivity range $\eta/D_B = 10^{-6}$-10^{-4}

Intrinsic time-scales demonstrate theoretical resistivity scaling



- *Time of peak flux displays resistive-inertial reconnection time:* $\tau \sim \eta^{-3/5}$

Linear reconnection theory: visco-resistive MHD

reviewed in Fitzpatrick PoP (1998)

- **2-field plasma model** (at low beta)

$$\partial_t A + \nabla_{\parallel}\phi = \eta \nabla_{\perp}^2 A$$

$$\left(\partial_t + v_E \cdot \nabla\right)\nabla_{\perp}^2 \phi = -V_A^2 \nabla_{\parallel} \nabla_{\perp}^2 A + \mu_i \nabla_{\perp}^4 \phi$$

$$\mu_s \rightarrow \mu_s / m_s n_s$$

$$\eta \rightarrow \eta_{\parallel} / \mu_0$$

- **2 reconnecting cases:** constant ψ

  2 ideal cases: not constant $\psi(r_s)=0$

Regime	Δ	δ	$\gamma\tau_A$
RI: inertia + resistivity	$2.1 S_R^{3/4}\Gamma^{5/4}$	$\left(\Gamma/S_R\right)^{1/4}$	$0.64 S_R^{-3/5}$
VR: viscosity + resistivity	$2.1 S_R^{5/6} S_V^{-1/6}\Gamma$	$S_R^{-1/6} S_V^{-1/6}$	$0.48 S_R^{-5/6} S_V^{1/6}$
VI: viscosity + inductance	$4.6 S_V^{1/4}\Gamma^{-1/4}$	$\left(\Gamma/S_V\right)^{1/4}$	$450 S_V$
I: inertia + inductance	$\pi\Gamma^{-1}$	$1/\Gamma$	$1/\pi$

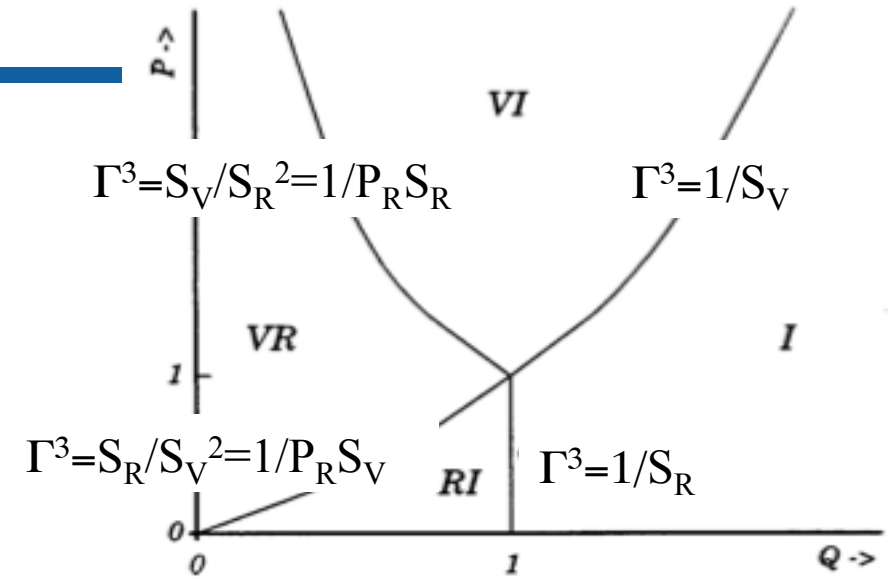


FIG. 1. A schematic diagram showing the extent of the four linear response regimes in normalized viscosity, P, versus normalized "slip frequency," Q, space. The four regimes are the *visco-resistive* regime (VR), the *resistive-inertial* regime (RI), the *visco-inertial* regime (VI), and the *inertial* regime (I).

- **Dimensionless #'s**

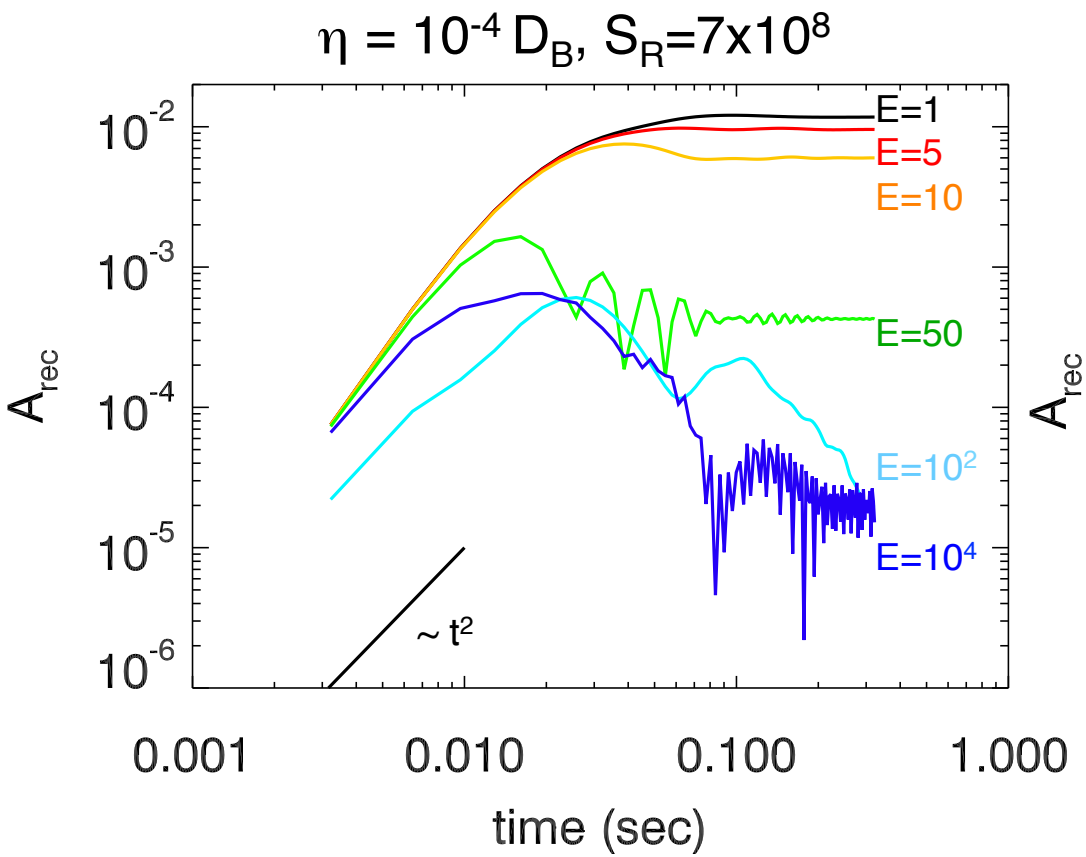
$$S_R = \tau_R / \tau_A = L v_A / \eta$$

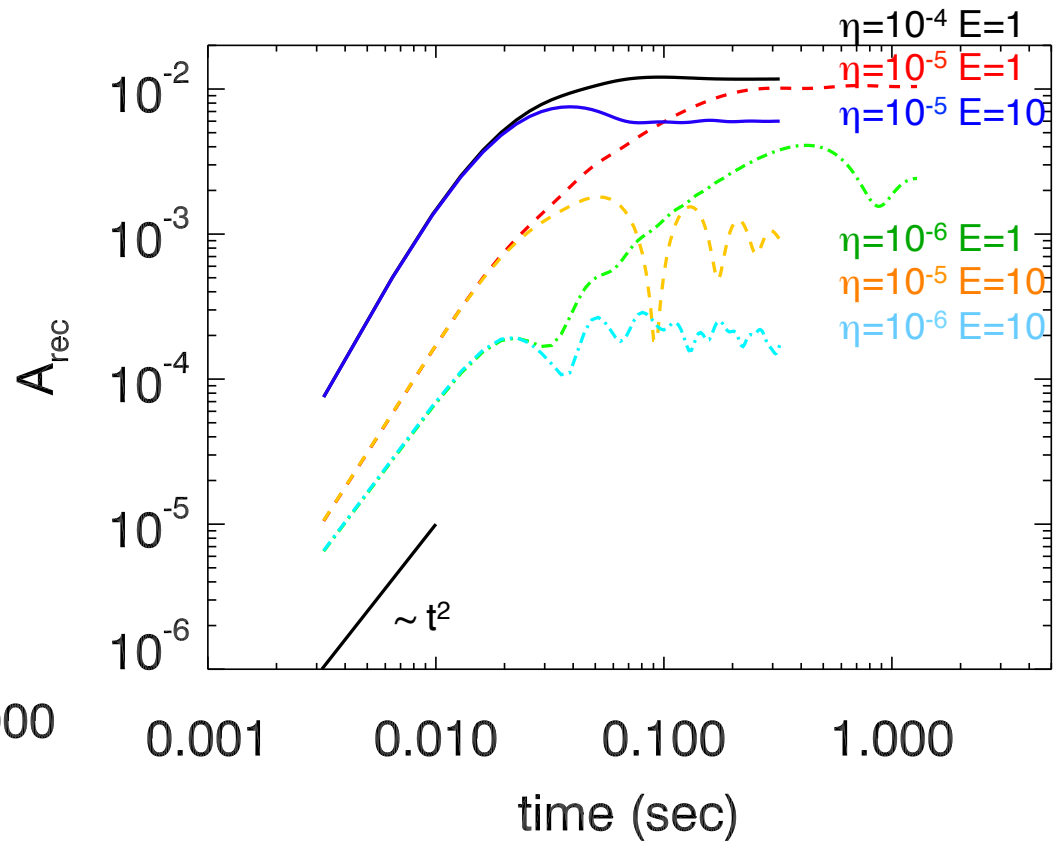
$$S_V = \tau_V / \tau_A = L v_A / \mu_i$$

$$P_R = \tau_R / \tau_V = \mu_i / \eta$$

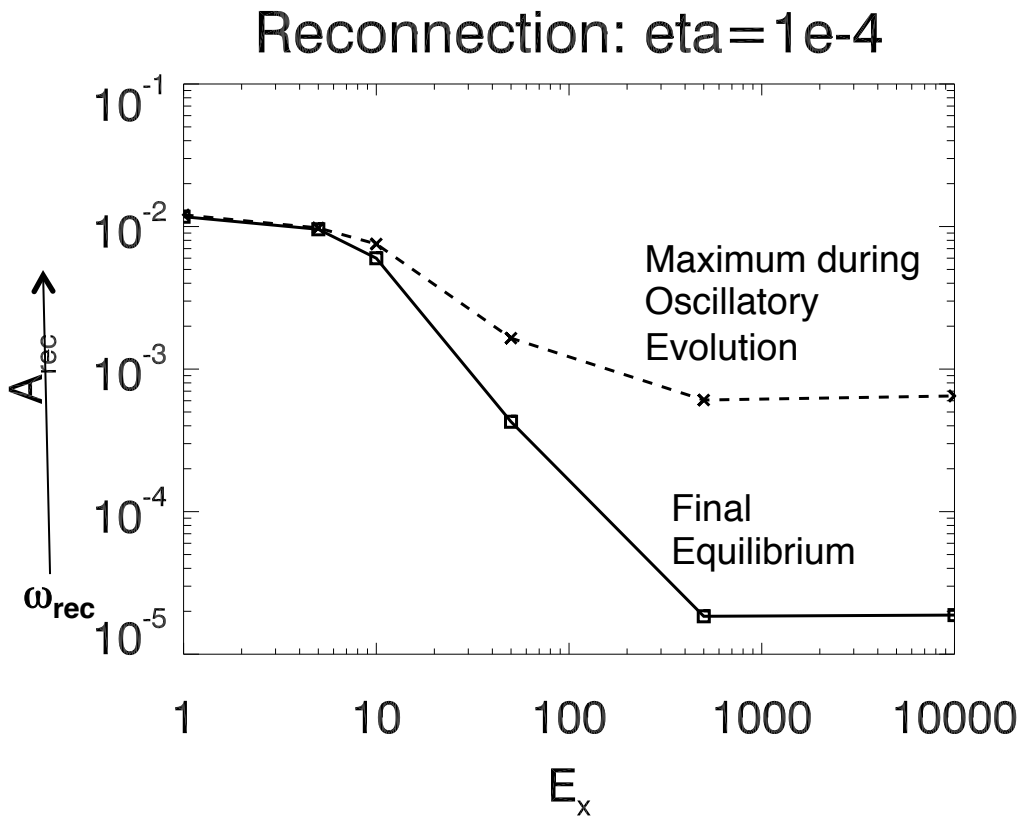
$$\Gamma = -i\omega\tau_A \qquad Q_R = \omega\tau_A S_R^{1/3}$$

Perpendicular rotation suppresses reconnection: $Phi_0 = -E_0\ x$



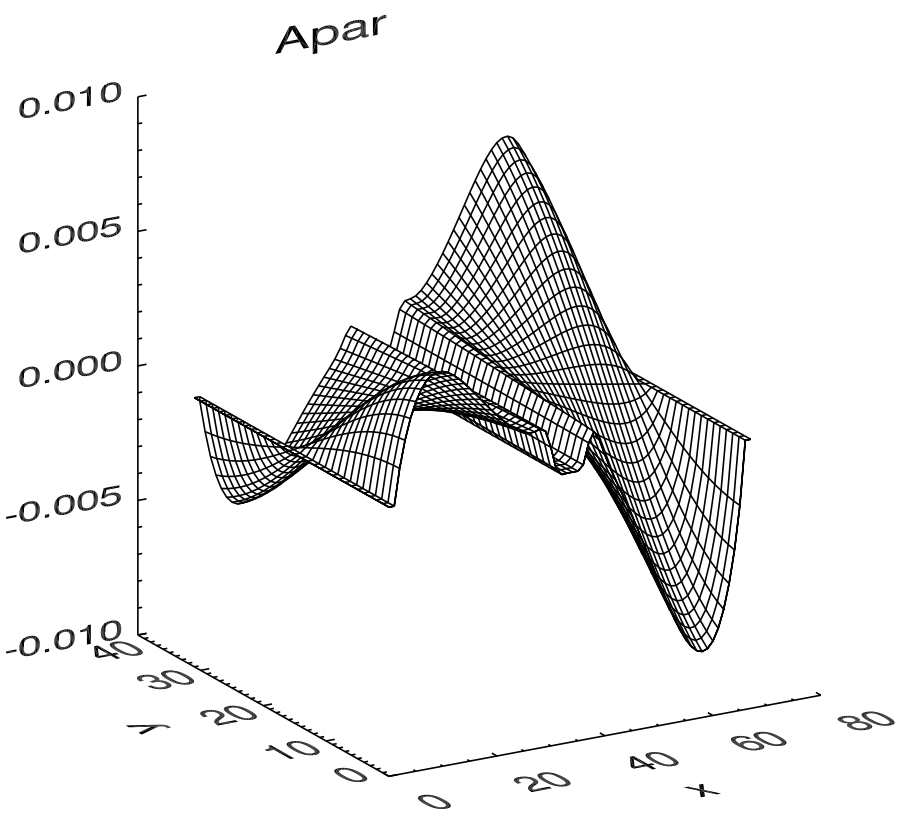


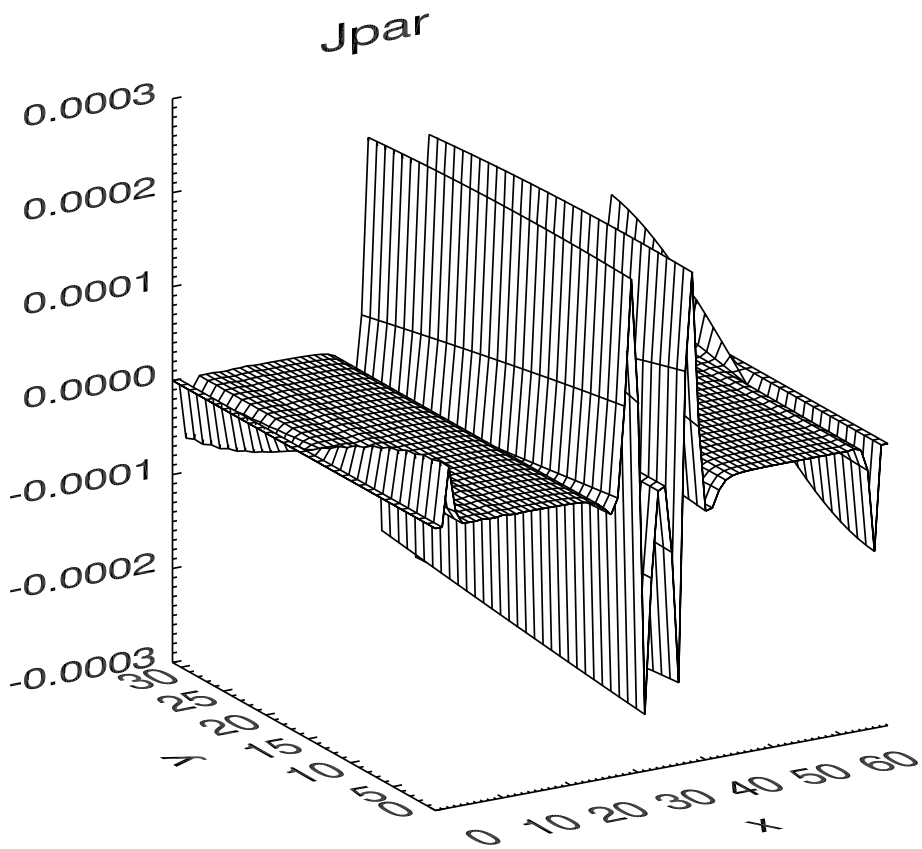
Suppression vs. Rotation: $\eta = 10^{-4} D_B$, $S_R = 7 \times 10^8$



- Reconnection rotation rate measured to be $\omega_{rec} \sim 10\ \omega_{E0} \sim 25$ krad/s
  - close to expected value $\omega_{rec} \sim S_R^{1/3}\ \omega_A \sim 23$ krad/s
- Phase difficult to measure accurately due to oscillations

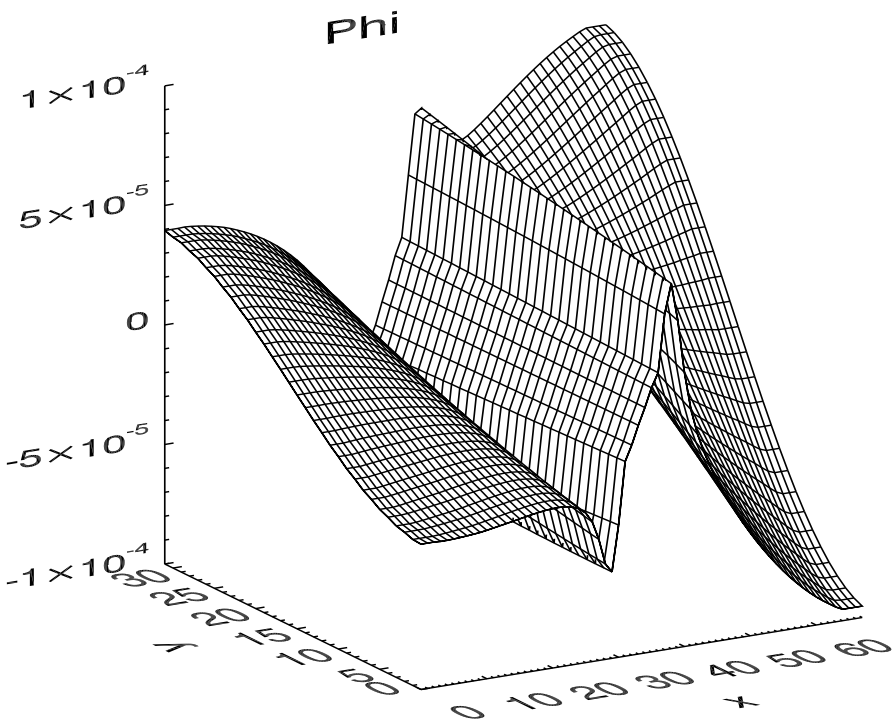
Alfven resonance structure can be clearly seen at low resistivity & high rotation [phi=5,eta=1e-7]

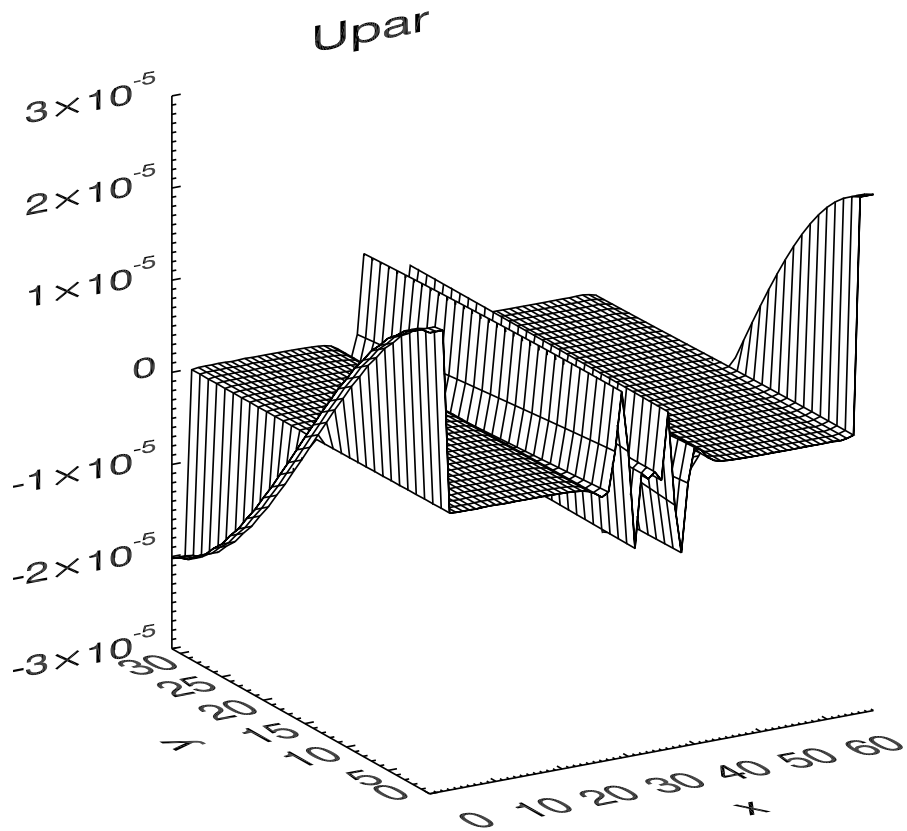




- Alfven resonance spacing $x_A \sim \omega_E/\omega_A k_y \sim 5.7$ mm ~ 4 grid points

Perpendicular rotation suppresses reconnection: $Phi_0 = -E_0\ x$

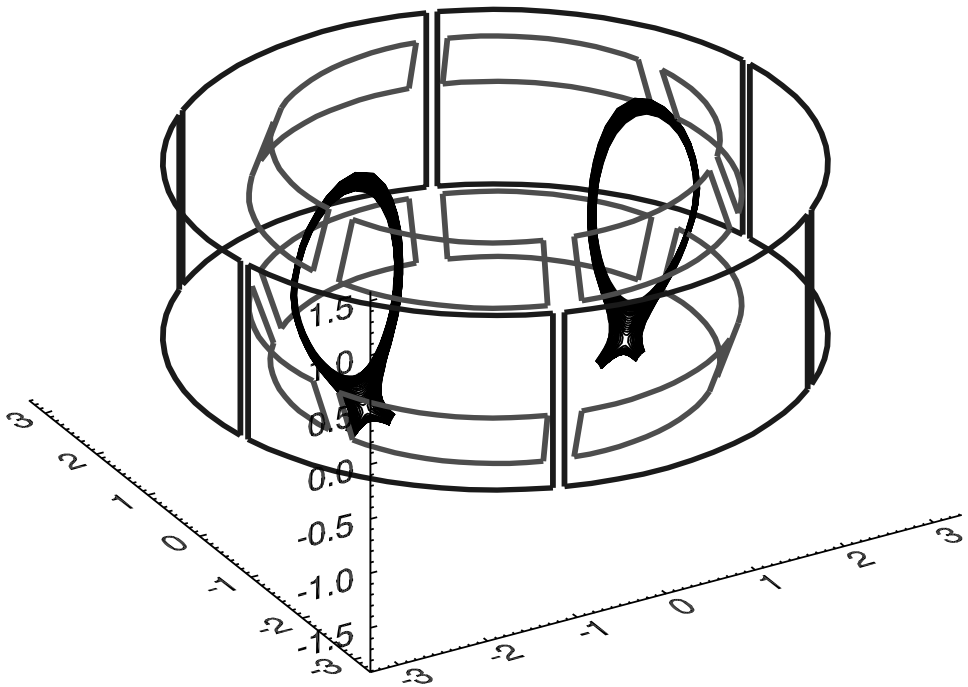




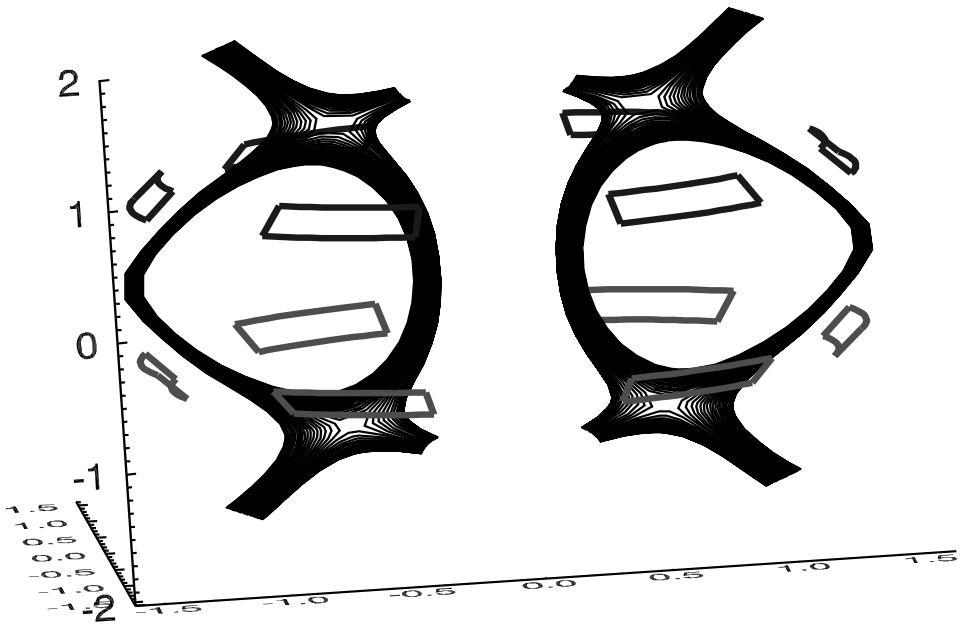
- Alfven resonance spacing $x_A \sim \omega_E/\omega_A k_y \sim 5.7$ mm ~ 4 grid points

Moving toward realistic geometry: Implementation of 3D coil models improved

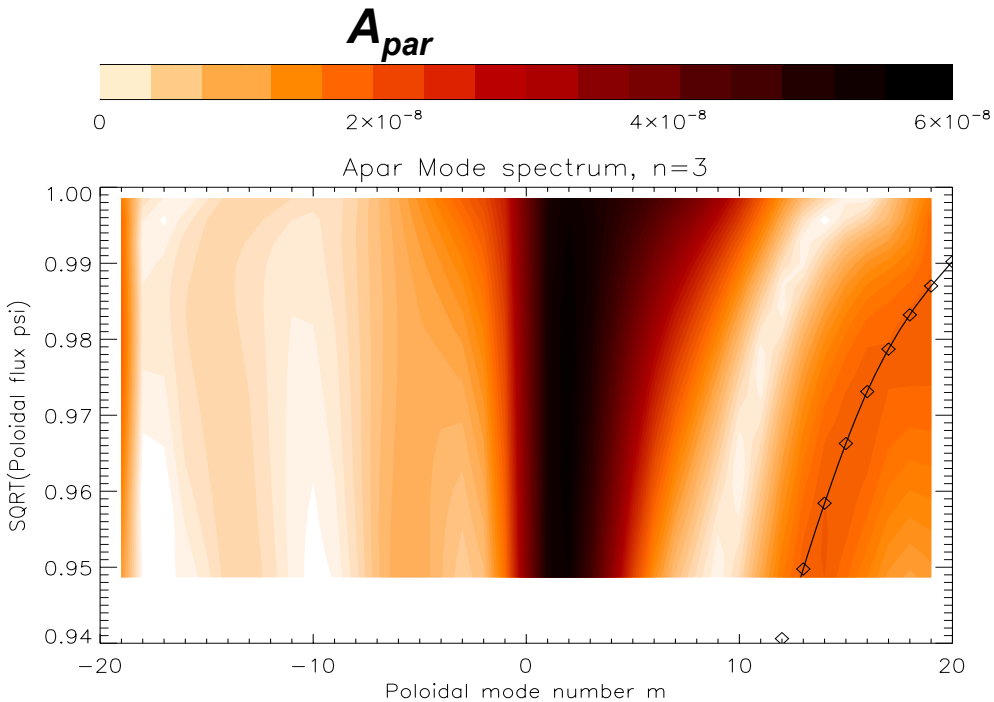
- **DIII-D: I-coils & C-coils**

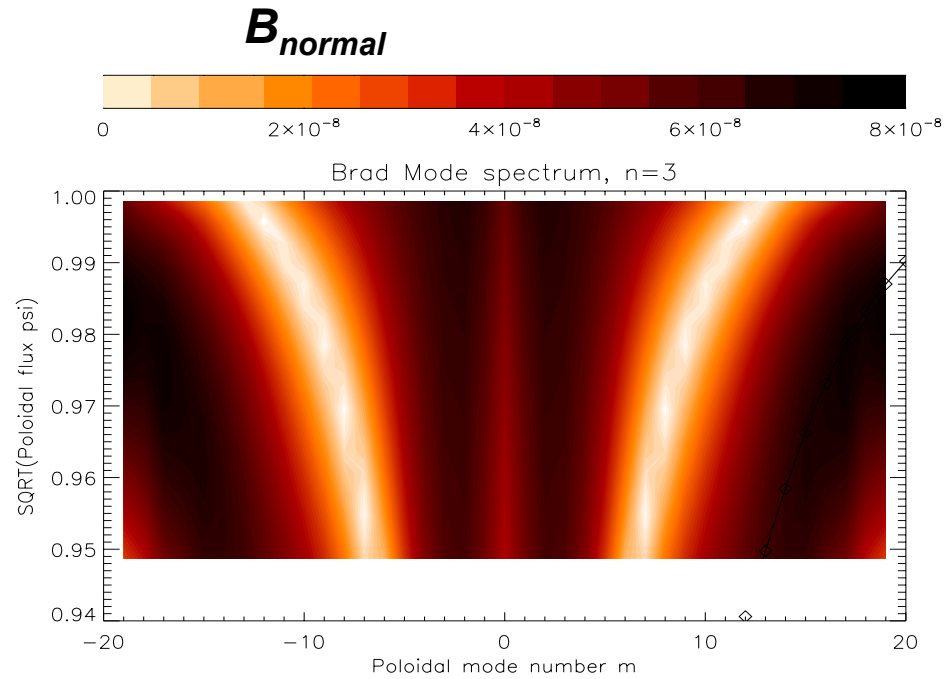


MAST: control coils



Magnetic field spectrum for MAST coil geometry





- **Computed by representing coil fields as a superposition of fields due to individual line segments[1]**

[1]Hanson & Hirshman, Phys. Plasmas 9, 4410 (2002)

Conclusions

- **Improvements made to allow robust treatment of BCs and coils**
  - **Realistic coil geometry implemented & will be benchmarked vs. other codes**

- **Linear & nonlinear reconnection benchmarks have been performed**
  - **Slab models of 2-field reconnection are being benchmarked vs. theory**
  - **Studies of hyper-resistive reconnection physics show different scaling laws**

- **Also: Ideal MHD boundary conditions have been implemented**
  - **Smoother relaxation to equilibrium, less generation of standing wave with large edge currents**

Exercises

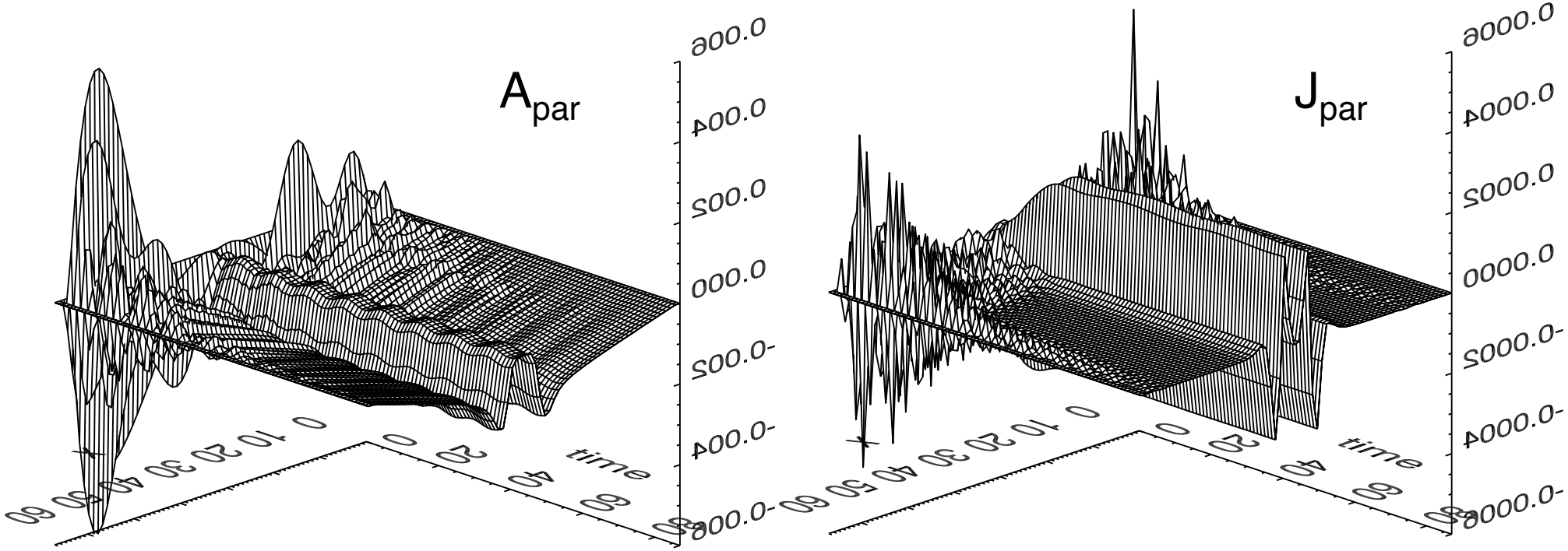
- Make slab grid
  - cd $BOUT_TOP/tools/slab
  - idl
    - slab, Rmaj=5, rmin=1, dr=0.1, nx=68, ny=32, output='myslab_68x32.nc'
    - exit
- Run an example
  - cd $BOUT_TOP/examples/reconnect-2field
  - git pull
  - make
  - >> edit data/BOUT.inp ; set parameters of interest
  - cp $BOUT_TOP/tools/slab/myslab_68x32.nc .
  - qsub –IV –q interactive -l mppwidth=16 –l advres=bout.10
  - cd $PBS_O_WORKDIR
  - aprun –n 16 ./2field –d data
  - idl ; examine data
    - apar_ext=collect(var='Apar_ext',path='data')

Exercises

- Examine data
  - idl ; examine data
    - apar_ext=collect(var='Apar_ext',path='data')
    - apar=collect(var='Apar',path='data')
    - jpar=collect(var='Jpar',path='data')
    - surface, reform(apar_ext[*,*,16])
    - showdata, reform(apar[*,*,16,*])

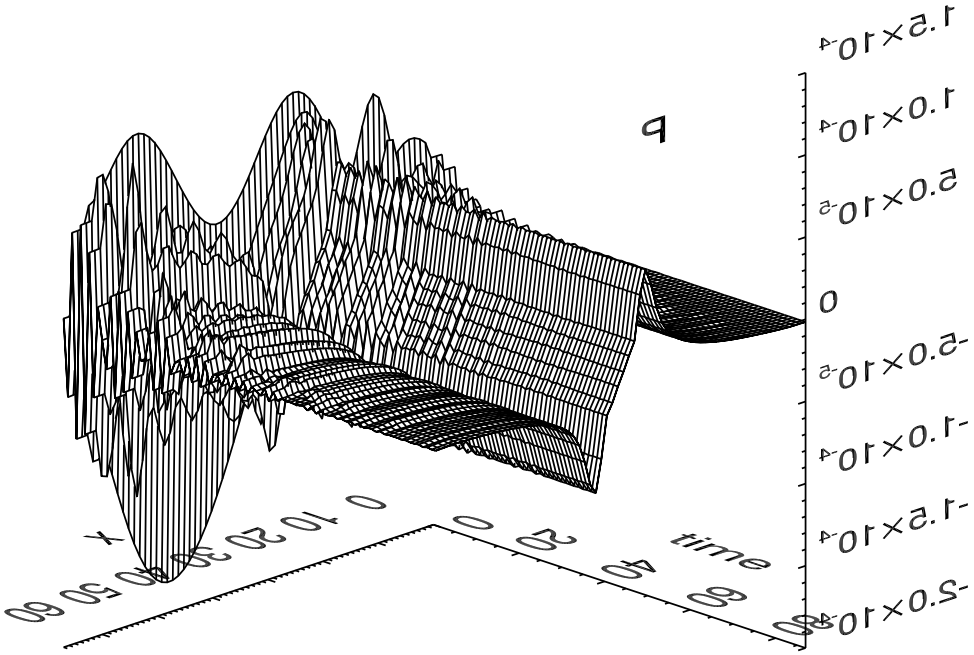
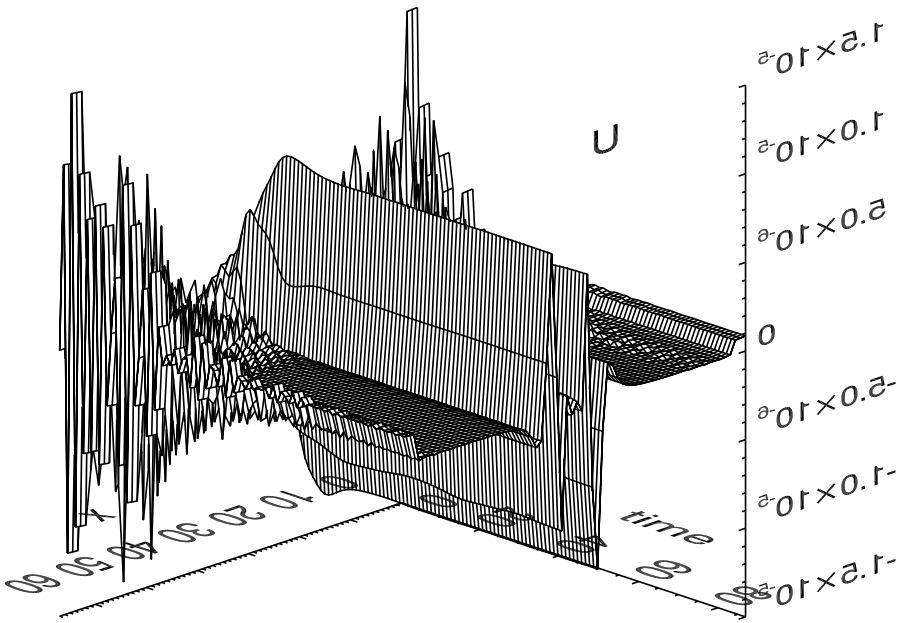
- Run an example with preconditioning on/off
- Investigate behavior with changes to RTOL and ATOL

Alfven resonance structure can be clearly seen at low resistivity & high rotation [phi=5,eta=1e-7]



Alfven resonance structure can be clearly seen at low resistivity & high rotation [phi=5,eta=1e-7]

Phi



U_{par}