Perspectives on Problems in Turbulence and Multi-Scale Interaction for BOUT++

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Outline

- Some thoughts on BOUT++'s niche in the ecology of simulation codes

➔

- OV of selected physics problems to which BOUT++ can contribute. Focus on key issues
- Physics capabilities BOUT++ should acquire to address the problems

N.B. Style is that of a "big picture" OV.

On BOUT's Niche

- At a coarse grained level, 3 types of simulation codes appear in MFE
  - "MHD" solves full MHD equations with high fidelity geometry; since $t = -\infty$.
  - "Multi-Fluid / Gyro-Fluid" (includes BOUT++) solves more detailed (including kinetic mock-ups) with emphasis on dynamics; since mid 80's, neglected since ~ 2000
  - "Gyrokinetic" — δf solves GK equation, emphasis on kinetic effects; full-f, flux driven since late 90's, flux drive ~ 2010

On BOUT's Niche, cont'd

- In terms of lessons learned, from published work:
  - Gyrokinetic codes good at: quasi-stationary, $K \leq 1$ simulations; usually of micro-scales, meso-scales e.g.
    - Dimits shift, zonal flows
    - core local χ with "full physics"
    - ExB staircase (GYSELA)

K = Kubo number

$$K \sim \frac{\tilde{v}\tau_c}{\Delta_c}$$

  - But… Many important and interesting problems are non-stationary, strongly nonlinear, qualitatively multi-scale

➔ Logical to approach by GF/BOUT !

Problem Topics for BOUT – A Selected List

- ELM dynamics
- 3D Multi-Scale physics – RMP, NTM
- Transport bifurcations
  - L→H transition, back transition ➔ ELM
  - ITB
- Edge ←→ Core ←→ SOL coupling ➔ especially rotation ←→ L-H

ELM Dynamics

- Conventional wisdom links ELM to linear theory of ideal MHD Peeling-Ballooning mode
- Begs the questions:
  - What is the agent of dissipation, fast reconnection, transport which allows crash? (addressed by Xu, 2010)
  - What determines crash vs turbulent state? (see P.W. Xi, this meeting)
  - What of subcritical bifurcation scenario?

ELM Dynamics, cont'd

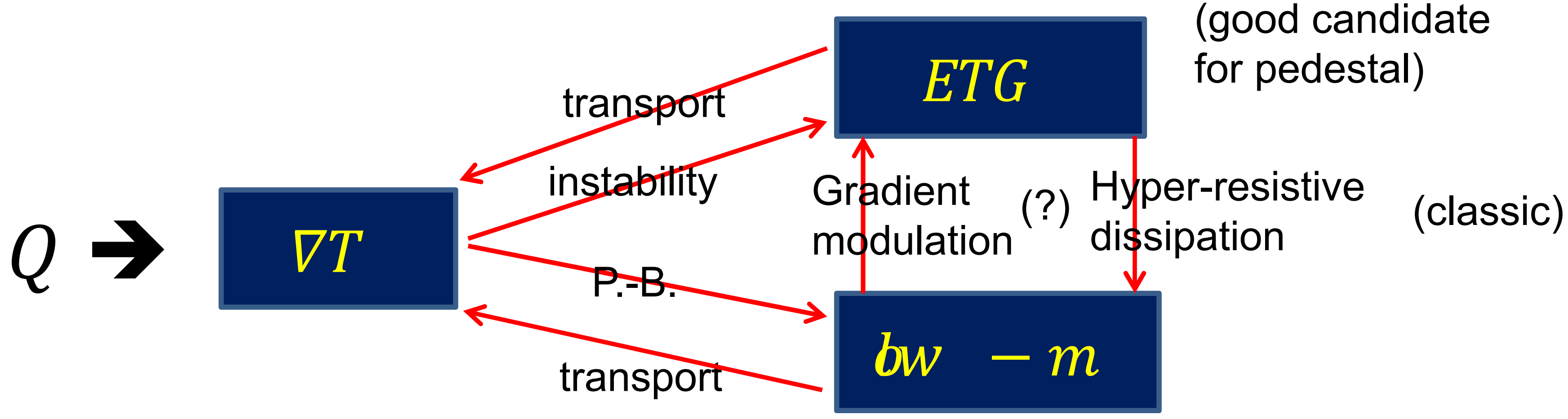
- Xu et al 2010:
  - inclusion of *hyper-resistivity* in Ohm's Law, comparable to χ_e

    i.e. $\frac{\partial A_\parallel}{\partial t} = -\nabla_\parallel \phi + \eta \nabla_\perp^2 A_\parallel - \eta_H \nabla_\perp^4 A_\parallel$ → diffusion of current
    → from Kaw, et al '77, forward

    - P.-B. triggers fast magnetic reconnection
    - Hyper-resistivity ➔ dissipation of current sheet, needed for crash
    - Realistic ELM sizes (5 ~ 10% W)
  - Begs the questions:
    - What is the physics origin of hyper-resistivity? $\eta_H \leftrightarrow \chi_e$ ➔ micro-turbulence! i.e. ETG, KBM etc.
    - Need treat evolution of micro-turbulence and P-B consistently ➔ address feedback of P-B on micro-turbulence drive

ELM Dynamics, cont'd

- Multi-Scale ELM and Pedestal Transport Problem



  – Origin of η_H? ➔ $\langle \tilde{V}_r \tilde{J}_\parallel \rangle \leftrightarrow$ flux of current, due turbulence
    - $\eta_H \sim \chi_J \sim \chi_e$ similar to well known $\chi_\phi \sim \chi_i$ trend in ITG (see Singh, Jhang, P.D. 2013)
    - Small scale acts as effective dissipation for large
  – Feedback? ➔ low-n P-B mode will react on driving ∇T_e ➔ local gradient drive reduction (c.f. Holland, P.D.; P.D., Singh)
  – How does crash emerge? Crash vs Turbulence? - Xi, et al has fixed η_H

Comments

- Problem of nonlinear P-B evolution is intrinsically multi-scale, to dispose of current sheets.
- Need progress beyond fixed number, expression ➔ hyper-resistivity is old idea, yet feedback of P-B on turbulent gradient drive is not addressed.
- Problem reduction ➔
  - treat turbulence at level of wave kinetics or intensity equation, with low m contribution to effective total gradient, etc.
  - express stresses in terms of high m intensity

➔ Subgrid Model ?!

Comments, cont'd

- Defines dynamic sub-grid scale model for P-B: MHD + stress modeling + ETG intensity field ➔ could be input into low m calculation
- Must be validated by multi-scale simulation (coupled ETG + P-B): challenging
- Essential that system be flux-driven ➔ profiles evolve dynamically
- Interesting question (Drake, '94): Is hyper resistivity triggered as consequence of current sheet formation $\leftrightarrow$ ∇J-driven analogue PSF?

Multi-Turbulence Dynamics 3D (RMP, NTM): *Some Basic Ideas*

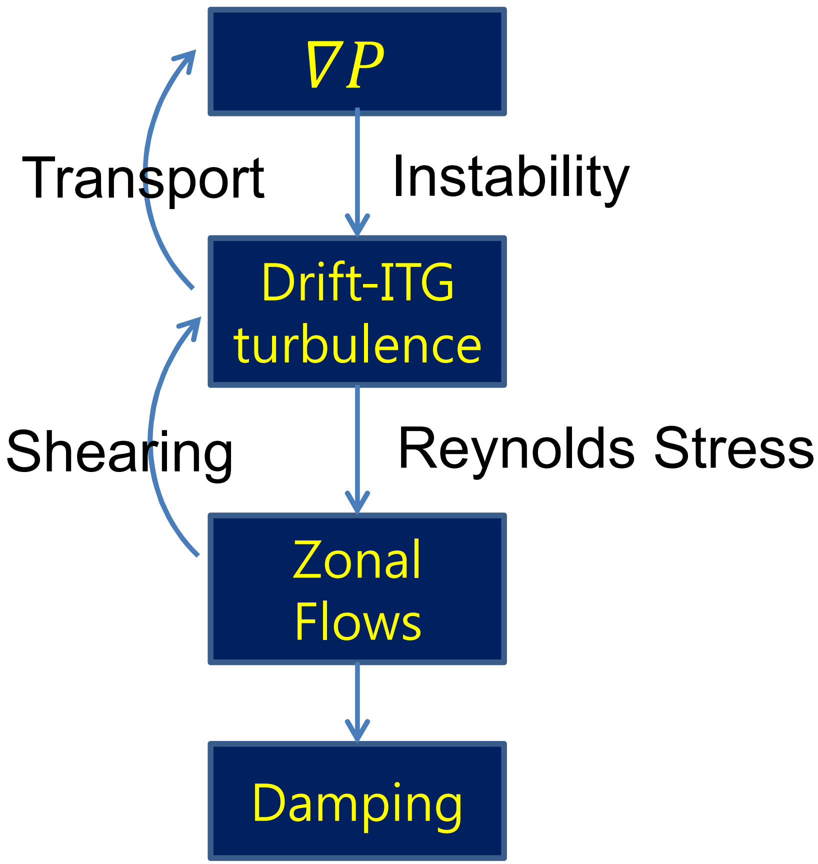
- Base state (i.e. $\delta B_r = 0$) is already multi-scale and self-regulating (i.e. interacting)

Micro-Scale	Meso-Scale	Macro-Scale
$l \sim \Delta_c$ Drift-ITG, etc.	$l \sim (\Delta_c L)^{1/2}$ Zonal flows, fields, GAM, …, non-resonant convective cells	$\rho \sim L$ Profiles, corrugations, mean flows

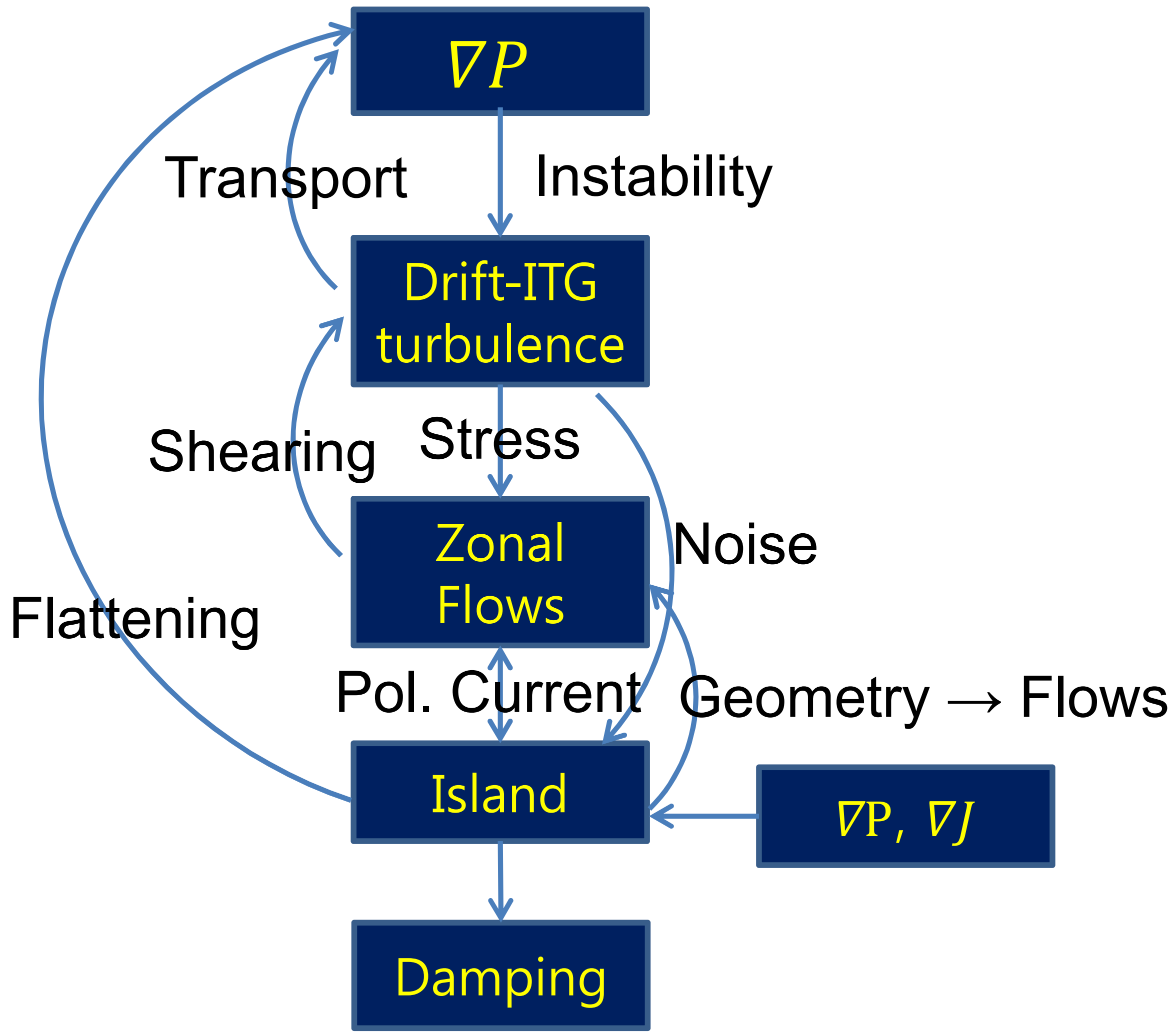
- In pedestal, idealized scale separation is severely compressed
- Micro-meso-macro ranges strongly, nonlinearly coupled

Introduction of Helical Perturbation *Hideously* Complicates Feedback Loops

2D Configuration



3D Configuration, NTM



➔ Attractive Approach:
consider sequence of simplified paradigm problems

Paradigm Problems: Drift-Zonal Turbulence + RMP

- Static, prescribed δB_r helical perturbation
- Seek:
  - response of flows, electric field
  - impact on turbulence, transport

(Leconte, P.D. ‘11, ‘12 and submitted)

i.e. how does RMP modify the flow-fluctuation feedback loop?

Major issues:

- magnetic coupling of zonal modes, flows ➔ penetration
- mean-zonal electric field interplay
- force balance: electron vs ion competition

- Model: H-W + $\delta B\,(r, \theta, \phi)$ ➔ need expand

Paradigm Problems: Drift-Zonal Turbulence + RMP

- Fundamental Mechanism:
  - N.B. Y. Xu, et. al. demonstrated RMP effect on LRC and (likely) zonal shears
  - Key: Radial current induced by δB_r

$$\langle J_r \rangle = \langle \frac{\delta B_r}{B_0} \delta J_{\parallel} \rangle$$

Current induced by RMP (i.e. tilting)

RMP field

Mesoscale radial current (parallel tilted lines)

Radial drive Zonal scale

$$\delta J_{\parallel} = -\frac{1}{\eta} \nabla_{\parallel} (\phi - n) - D_{\parallel} \frac{\delta B_r}{B_0} \left(\frac{d}{dr} (\langle \phi \rangle_z - \langle n \rangle_z\right)$$

Linear current in H-W

RMP perturbation

$$D_{\parallel} = v_{th}^2 / \nu_{e,e}$$

Paradigm Problems: Drift-Zonal Turbulence + RMP

- *Zonal Flows ↔ Charge Balance*

$$\frac{dQ}{dt} = -S_o\left[\langle \tilde{v}_r\ \tilde{\rho}_{pol} \rangle\right] - \langle \frac{\widetilde{\delta B_r}}{B_0} \delta J_{\parallel}^{RMP} \rangle] - \mu_{col}\ Q$$

$\langle J_r \rangle$ - radial current

$Q \equiv$ polarization charge (i.e. vorticity)

Turbulent Reynolds stress $\langle \tilde{v}_r\ \tilde{\rho}_{pol} \rangle = -\partial_r \langle \tilde{v}_r \tilde{v}_\theta \rangle$

↓

Polarization charge advection

Magnetic stress induced by RMP

↓

flow along tilted lines

And similarly:

Particle flux (turbulence)

Electron particle flux along tilted lines

$$\frac{dN}{dt} = -S_o[\langle \tilde{v}_r\ \tilde{n} \rangle] - \langle \frac{\widetilde{\delta B_r}}{B_0} \delta J_{\parallel}^{RMP} \rangle]$$

N ≡ particle #

Paradigm Problems: Drift-Zonal Turbulence + RMP

- *More comments:*
  - RMP effect in Q, N equations same

  ➔ δB_r directly couples zonal perturbations in density, vorticity
  - Limiting cases:
    - Weak RMP ➔ modest correction to usual Z.F. dynamics
    - Strong RMP ➔ electron force balance on mesoscales

    i.e. $\langle E_r \rangle_{zonal} \sim -\frac{T_e}{|e|}\frac{\partial}{\partial_r}\langle n \rangle_{zoanl}$
  - How strong is ‘strong’ and how weak is ‘weak’? ➔ dynamical model needed! [N.B. Dynamical models need to guide simulations!]
  - T vs N distinction ? ➔ need extend model

Paradigm Problems: Drift-Zonal Turbulence + RMP

- *Dynamical Model*
  - extend coupled zonal mode – turbulence model to case with δB_r
  - 'Predator-Prey' style calculation

Zonal friction

$$\frac{\partial}{\partial t} V_{Eq} + \mu V_{Eq} - \alpha\varepsilon(\phi_q - c n_q) = -\frac{D_R}{\rho_s^2}(\phi_q - n_q)$$

Zonal ExB — coupling

$$\frac{\partial}{\partial t} n_q + D_n q^2 \varepsilon n_q = -D_R(\phi_q - n_q)$$

Zonal density

$$\frac{\partial}{\partial t}\varepsilon = \gamma\varepsilon - \alpha\varepsilon\,|\phi_q - c n_q|^2 - \gamma_{NL}\varepsilon^2$$

DW energy — Growth/input

q = zonal mode
Radial wave #

$$D_R = D_{\parallel}\langle \delta B_r^2 \rangle / B_0^2$$

D_n =Unperturbed diffusivity (CDW)

Paradigm Problems: Drift-Zonal Turbulence + RMP

- *Model*, cont'd
  - RMP regimes:
    - 'weak' (analogous fluid in H-W): $D_{\parallel}\langle\delta B_r^2\rangle/B_0^2 < \rho_s^2\mu$
    - 'strong' (analogous adiabatic in H-W): $D_{\parallel}\langle\delta B_r^2\rangle/B_0^2 > \rho_s^2\mu$
  - 'Strong' regime is more relevant ➔ drift – Z.F. system is in state of near electron force balance on mesoscales
  - What happens?
    - Enhanced Z.F. damping
    - Zonal density drive: mesoscale $\langle n\rangle' > 0$, opposite to initial $n_0' < 0$, implying trend toward *flattening!* ➔ *Is this 'pump-out' ?!*

BOUT Projects

- Explore basic model, simple extension
- Consequences for penetration (c.f. Beyer, 2013)
- Needed ingredients:
  - sources (T, N) and gradient evolution
  - mean poloidal, toroidal flows
  - extend LF models to treat transport in stochastic fields (esp. collisionless regimes)
- RMP-driven convective cells and their coupling to flows are a key question

NTM Problem

- From the perspective of turbulence, NTM problem encompasses RMP problem and brings the problems of
  - turbulence feedback on δB_r

  - Island competes with turbulence to carry the heat flux
  - Island, flows, turbulence all strongly coupled

➔ flux driven simulations needed!

➔ NTM problem is one of a slow transport bifurcation and evolution from turbulent state with mean axisymmetry to one with helical symmetry + island. (N.B. slow = indep. τ_A)

NTM Problem, cont'd

- *Formulating the NTM problem*
  - Drive: bootstrap current ($\Delta' < 0$)
  - Key: polarization currents – flows, E-field

Rutherford ➔

$$\frac{\tau_R}{r_0^2}\frac{dW}{dt} = \Delta' + \Delta_{BS}(W,\omega) + \Delta_{pol}(W,\omega)$$

Polarization current drive → a murky mess…

Bootstrap drive

$$\frac{d\omega}{dt} = F_t(W,\omega)$$

W = Island width
ω = Island rotation frequency

Δ_{BS}: turbulence + island feedback on profiles $\Delta_{BS} = \sqrt{\epsilon}\left(\frac{L_q}{L_p}\right)\beta_p \frac{W}{W^2+W_T^2}$

Δ_{pol}: density, temperature flattening, electric field, electron vs ion competition $\leftrightarrow$ Akin RMP

NTM Problem, cont'd

- *Turbulence Issues*
  - What is threshold island size?

    cw: conduction by island beats turbulent transport

    parallel heat conduction $\chi_{\parallel}$

    $\left(\frac{k_\theta^2 W^4}{L_s^2}\right)\chi_{\parallel} > \chi_{\perp,amb}$ (Fitzpatrick) → Not self-consistent!

  - What is impact of polarization currents?

    cw:

    - Momentum transport (ion and electron) is crucial ➔ into which species is island frozen?
    - Current theoretical treatments only insert ad-hoc χ_ϕ, etc. Island interaction with flow generation mechanism poorly understood
    - Profile flattening complex issue: acoustics, $\langle E_r \rangle$ effects, trapped particles?
    - Are zonal modes a player?

NTM Problem, cont'd

- *An Illustrative Toy Model* (P.D., Ishizawa, unpublished)
  - Complementary to RMP studies, retain Δ_{BS} but ignore island ↔ flow interaction

$$\frac{\tau_R}{a}\frac{dW}{dt} = a\Delta' + \frac{a}{B_0}\delta J_{BS}, \quad \delta J_{BS} = \sqrt{\epsilon}\,\frac{n_0}{B_\theta}\,\delta\left(\frac{dT_e}{dr}\right)$$

Bootstrap drive

Turbulence intensity

Fixed flux: $\frac{dT_e}{dr} = -Q_e/[\frac{\chi_\parallel k_\theta^2}{L_s^2} W^4 + \chi_T \epsilon + \chi_{resid}\ ]$

turbulent transport

Island transport

And $\frac{\partial \epsilon}{\partial t} = \gamma\left[\frac{R}{L_T} - \frac{R}{L_{T\,crit}}\right]\epsilon - \tau\epsilon^2 - \alpha V'^2_E \epsilon$ ➔ intensity

Profile drive

$$\frac{\partial V_E'^2}{\partial t} = \alpha (V_E')^2 \epsilon - \mu (V_E')^2$$ ➔ Z.F. evolution

NTM Problem, cont'd

- *Comments*
  - Straightforwardly extendable to include island effects on flows (ala' Leconte, P.D.)

  ∴ With Δ_{pol} model, would have minimally complete feedback structure

  - Model ➔ $\frac{W_{crit}}{a} \sim \chi_{\parallel}^{-\frac{1}{3}} \beta^{-\frac{1}{3}}$ ➔ differs from CW

    NB: ZF shear enters critical island size consideration!

  - Extended to 1D ➔ turbulence spreading into island !?

  ➔ Role of nonlinear interaction of island with environs?

NTM Problem, cont'd

- *Partial Summary*
  - Interesting simulation work on NTM problem ongoing (i.e. Poli, et. al.; Militello, et. al.) but analysis and modeling have yet to forge beyond ad-hoc transport coefficient level and frozen profiles, islands. Reduced models necessary to extract physics from simulation.
  - Simulations should progress from relatively simple models.
  - Many aspects of RMP [+ turbulence + flows] problem appear exportable to NTM problem. BOUT group should coordinate study of these two topics.
- Cautionary note:
  - Web of feedback loops in NTM problem much denser than for RMP problem…

Transport Bifurcations

- L-H Transition, see also G. Park, this meeting
- ITB, see also S.S. Kim, APS BOUT session

Re: L→H Transition:

- classic, 31 year old problem
- renewed interest re: ITER threshold, back-transition
- significant progress driven by experiment and analytic theory

BUT:

- No contribution from simulations, despite dedicated efforts on edge codes
- BOUT group can fill this void!

Transport Bifurcations, cont'd

- For Motivation: "Competition is the spice of life…"
- Theory Festival '13: Chone, Beyer (brief presentation)
- Electrostatic resistive ballooning model (BOUT 2-field) n.b. no GAM
- Flux driven ➔ ∇P evolves
- Mean flow shear defined by standard neoclassical expression

$$\partial_t \nabla_\perp^2 \phi_{0,0} = \cdots + \mu(\nabla^2 \phi_{neo} - \nabla^2 \phi_{0,0})$$

Radial structure of $\mu(r)$ significant

- Obtained clear ETB transition, some possible LCO activity

Comments

- It can be done!
- Considerable room for analysis improvement, extension even within highly simplified model
- ETB, not L→H, as P is 1-field ➔ no separation of n, T
  ➔ expanded H-W model and/or fluid ITG are logical extensions
- BOUT has unique capability to address possible SOL flow impact on transition physics ➔ edge flow shear, Er boundary condition

Some More Questions

- Physics of transition:
  - structure of ExB flow
  - transition: γ vs $E \times B$ flow or $R_T = -\langle \tilde{V}_r \tilde{V}_\perp \rangle \langle V_E \rangle' \ / \ \gamma_{eff} \langle \tilde{V}^2 \rangle$

    (c.f. Manz '12)
  - role of LCO
- Q_{crt} vs μ, μ dependence $\leftrightarrow$ $P_{crt}\ (n)$
- Back Transitions: an opportunity to lead experiments
- SOL flow effects – c.f. Fedorczak et al; LaBombard

Edge-Core-SOL Coupling

- A huge topic
  - Merits talk in itself
  - Closely related to intrinsic rotation and L→H transition
- Requires:
  - SOL region, with open lines; Plate boundary condition (BOUT is ideal!)
  - good GF model for core plasma
  - toroidal, poloidal flows
  - particle, heat, momentum sources
- Encompasses both quasi-stationary and dynamic phenomena

Some Issues:

- SOL-Core Flow Interaction
  - SOL flow penetration of core? (LaBombard)
  - Boundary condition? (Fedorczak, et al)
- Impact of SOL flows on L→H transition
- SOL flows and intrinsic rotation?
- Is “short fall problem” explained by inward spreading of turbulence from SOL, boundary (c.f. Kadomtsev) or outward avalanching, wave breaking (Mattor, P.D.; Gurcan et al)

N.B. GK can’t address “inward” scenario.

Physics Capabilities for BOUT: A Wish List

- Dynamically evolving profiles → heat (flux drive), also particles, momentum
- G-L "full physics" (including EM):
  - G-L system for ions, including drift resonances
  - D-L system for electrons, including trapped species
- SOL physics and transition region
- Likely will require dynamic sub-grid models for several applications

Some Thorny Problems

- Low collisionality heat transport in stochastic fields

➔ Kadomtsev, Pogutse '78 outlines non-trivial issues in collisional   limit

- Poloidal flow, zonal mode, GAM damping, screening
- Er transition layer between core, SOL
- NTV model for 3D phenomena

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