Gyrofluid Simulations using TRB Code

S. S. Kim[1], Hogun Jhang[1], P. H. Diamond[1,2],
L. Terzolo[1], S. Yi[1], T. S. Hahm[3]

[1] WCI Center for Fusion Theory, NFRI, Korea
[2] CMTFO and CASS, UCSD, USA
[3] Seoul National University, Korea

BOUT++ Workshop
16 Sep. 2011

NFRI National Fusion Research Institute

Contents

- Introduction to gyrofluid model
  - Gyrofluid equation
  - Closure model
  - Introduction to TRB code

- Gyrofluid simulations using TRB
  - Internal transport barrier (ITB) dynamics:
    Intrinsic rotation, hysteresis and back transition

Introduction to gyrofluid theory

- Where does it stand?

Boltzmann

- Strongly magnetized plasma (e.g. tokamak), low frequency fluctuations $\omega << \Omega_i$
- Reduction from 6D to 5D kinetic equation

Gyrokinetic

- Reduction of GK eqs to fluid moments (3D)
- Sophisticated closure to model wave-particle interaction and FLR effects

Gyrofluid

- Reduction to single fluid model
- Simple closures

MHD

Fluid equation

Moment equation

$$\frac{\partial \mathbf{M}^{(N)}}{\partial t} + \nabla \cdot \mathbf{M}^{(N+1)} = \mathbf{S}_E^{(N-1)} + \mathbf{S}_B^{(N)} + \mathbf{S}_c^{(N)}$$

Boltzmann equation

$$\frac{\partial f}{\partial t} + \mathbf{v} \cdot \nabla f + \frac{e}{m}\left(\mathbf{E} + \frac{1}{c}\mathbf{v} \times \mathbf{B}\right) \cdot \nabla_{\mathbf{v}} f = C$$

$$\mathbf{M}^{(N)} = \int d^3v f \underbrace{v_\alpha v_\beta .. v_\gamma}_{N}$$

Particle conservation:

$$\frac{\partial n}{\partial t} + \nabla \cdot (n\mathbf{V}) = 0$$

Momentum conservation:

$$\frac{\partial mn\mathbf{V}}{\partial t} + \nabla \cdot (p\mathbf{I} + \mathbf{\Pi} + mn\mathbf{VV}) = en\left(\mathbf{E} + \frac{1}{c}\mathbf{V} \times \mathbf{B}\right) + \mathbf{F}_c \qquad \text{where } \mathbf{F}_c = \int m\mathbf{v}C(f)d^3v$$

Energy conservation:

$$\frac{\partial}{\partial t}\left(\frac{3}{2}p + \frac{1}{2}mnV^2\right) + \nabla \cdot \left[\mathbf{q} + \left(\frac{5}{2}p + \frac{1}{2}mnV^2\right)\mathbf{V} + \mathbf{V} \cdot \mathbf{\Pi}\right] = en\mathbf{E} \cdot \mathbf{V} + \mathbf{F}_c \cdot \mathbf{V} + Q_c$$

$$\text{where } Q_c = \int \frac{1}{2} m v'^2 C d^3v$$

$\vdots$

NFRI National Fusion Research Institute

Gyrofluid equation

- Gyrokinetic equation for $\bar{f}(\vec{X}, \mathrm{v}_\parallel, \mu, t)$

$$\frac{\partial}{\partial t}\bar{f} + \frac{d\vec{X}}{dt}\cdot\frac{\partial}{\partial \vec{X}}\bar{f} + \frac{d\mathrm{v}_\parallel}{dt}\frac{\partial}{\partial \mathrm{v}_\parallel}\bar{f} = 0$$

$$\frac{d}{dt}\vec{X} = \mathrm{v}_\parallel \hat{b} + \frac{\hat{b}}{B}\times\nabla\langle\Phi\rangle$$

$$\frac{d}{dt}\mathrm{v}_\parallel = -\hat{b}\cdot\nabla\langle\Phi\rangle$$

$$\langle\Phi\rangle = \sum_{\mathbf{k}}\Phi_{\mathbf{k}} e^{i\mathbf{k}\cdot\mathbf{X}}\langle e^{i\mathbf{k}\cdot\boldsymbol{\rho}}\rangle = \sum_{\mathbf{k}}\Phi_{\mathbf{k}} e^{i\mathbf{k}\cdot\mathbf{X}} J_0(k_\perp\rho)$$

- Gyrokinetic Poisson Equation

$$-\frac{\rho_i^2}{\lambda_{Di}^2}\nabla_\perp^2\Phi = 4\pi e(\bar{n}_i - n_e) \quad \Rightarrow \quad \bar{n}_i = n - n_{pol}$$

where $n_{pol} = \dfrac{ne}{m_i\omega_{ci}^2}\nabla_\perp^2\Phi = n\dfrac{\hat{b}\cdot\nabla\times\mathbf{V}_E}{\omega_{ci}}$

- Gyrocenter-fluid moment equation

$$\frac{\partial}{\partial t}\bar{n}_i + \frac{\partial}{\partial \vec{X}}\cdot\left(\bar{n}_i V_\parallel \hat{b} + \bar{n}_i\frac{\hat{b}}{B}\times\nabla\{\langle\Phi\rangle\}\right) = 0$$

where $\bar{n}_i = \int \bar{f}_i d^3\mathrm{v}$

$$V_\parallel = \frac{1}{\bar{n}_i}\int \bar{f}_i \mathrm{v}_\parallel d^3\mathrm{v}$$

$$\{\langle\Phi\rangle\} = \frac{1}{\bar{n}_i}\int \bar{f}_i\langle\Phi\rangle d^3\mathrm{v} \approx \Phi + \frac{\rho_i^2}{2}\nabla_\perp^2\Phi$$

- Particle-fluid moment equation

$$d_t^E(n - n_{pol}) = -n\nabla_\parallel V_\parallel + \frac{\hat{b}\times T_i\nabla n}{2B}\cdot\nabla\frac{n_{pol}}{n}$$

where $d_t^E = \dfrac{\partial}{\partial t} + \mathbf{V}_E\cdot\nabla$

NFRI National Fusion Research Institute

Derivation of gyrofluid equation in TRB

- Gyroviscous cancellation

$$m_i n \frac{d}{dt}\mathbf{V}_d + \nabla\cdot\mathbf{\Pi}_g \approx \nabla\chi - m_i n\left(\mathbf{V}_d\cdot\nabla V_\parallel\right)\mathbf{b} \quad \text{where} \quad \chi = \frac{p\mathbf{b}\cdot\nabla\times\mathbf{V}_\perp}{2\omega_c}$$

(Hazeltine & Meiss, Phys. Rep. '85)

- Momentum balance

$$m_i n \frac{d}{dt}\mathbf{V} + \nabla\cdot\mathbf{\Pi}_g = en\left(\mathbf{E} + \frac{1}{c}\mathbf{V}\times\mathbf{B}\right) - \nabla p_i - \nabla\cdot\mathbf{\Pi}_\parallel$$

$$\mathbf{V}_\perp = \mathbf{V}_E + \mathbf{V}_d + \mathbf{V}_{pol}, \quad \mathbf{V}_{pol} = \frac{\mathbf{b}}{\omega_{ci}}\times\frac{d}{dt}\mathbf{V}_E$$

$$m_i n d_t^E V_\parallel = enE_\parallel - \nabla_\parallel p_i - \mathbf{b}\cdot\nabla\cdot\mathbf{\Pi}_\parallel$$

- Relation between polarization density and drift

$$\nabla\cdot\mathbf{V}_{pol} \cong -\frac{d}{dt}\frac{e}{m_i\omega_{ci}^2}\nabla_\perp^2\Phi = -\frac{d}{dt}\frac{n_{pol}}{n} \quad \Rightarrow \quad \frac{\partial n_{pol}}{\partial t} + \nabla\cdot\left(n_0\mathbf{V}_{pol} + n_{pol}\mathbf{V}_0\right) = 0$$

- Continuity equation

$$\frac{\partial n}{\partial t} + \mathbf{V}_E\cdot\nabla n + n\nabla\cdot\mathbf{V}_{pol} + n\nabla_\parallel V_\parallel = -n\nabla\cdot\mathbf{V}_E - \nabla\cdot\left(n\mathbf{V}_d\right) = n\left(\mathbf{V}_E + \mathbf{V}_d\right)\cdot\left(\boldsymbol{\kappa} + \nabla\ln B\right)$$

$$\Rightarrow \quad d_t^E\left(n - n_{pol}\right) = -n\nabla_\parallel V_\parallel + n\mathbf{V}_d\cdot\nabla\frac{n_{pol}}{n} + n\left(\mathbf{V}_E + \mathbf{V}_d\right)\cdot\left(\boldsymbol{\kappa} + \nabla\ln B\right)$$

Closure model for Landau damping

- General form

$$\mathbf{b}\cdot\nabla\cdot\mathbf{\Pi}_{\parallel} = -m_i n_0 \mu_{\parallel} \nabla_{\parallel}^2 \tilde{V}_{\parallel} - \lambda_1 n_0 \nabla_{\parallel} \tilde{T}_i$$

$$q_{\parallel} = -\lambda_2 p_{i0} \tilde{V}_{\parallel} - \chi_{\parallel} n_0 \nabla_{\parallel} \tilde{T}_i$$

The kinetic interactions are modelled in terms of the effective diffusion operator (Diamond et.al., Modern Plasma Physics vol. I).

- Lee - Diamond model (PF'86)

$$\mu_{\parallel} = \min\left(\frac{\mathrm{v}_{\mathrm{Ti}}^2}{\nu_{ii}}, \frac{\mathrm{v}_{\mathrm{Ti}}^2}{\omega}\right)$$

- Waltz model (PF'88)

$$\mu_{\parallel} = \chi_{\parallel} = \min\left(\frac{\mathrm{v}_{\mathrm{Ti}}}{k_{\parallel}}, \frac{\mathrm{v}_{\mathrm{Ti}}^2}{\omega}\right)$$

- Hammett - Perkins model (PRL'90)

$$\mu_{\parallel} = 0, \ \chi_{\parallel} = \frac{2}{\sqrt{\pi}} \frac{\mathrm{v}_{\mathrm{Ti}}}{|k_{\parallel}|} \quad \Rightarrow \quad q_{\parallel}(z) = -\left(\frac{2}{\pi}\right)^{3/2} n_0 \mathrm{v}_{\mathrm{Ti}} \int_0^{\mathrm{v}_{\mathrm{Ti}}/\omega} dz' \frac{\tilde{T}_i(z+z') - \tilde{T}_i(z-z')}{z'}$$

- Chang - Callen model (PFb'92)

$$\mu_{\parallel} = -\frac{i2\mathrm{v}_{\mathrm{Ti}}}{5\pi k_{\parallel}} Z\left(\frac{3}{5}\zeta\right), \quad \chi_{\parallel} = -\frac{i9\mathrm{v}_{\mathrm{Ti}}}{5\pi k_{\parallel}} Z\left(\frac{36}{25}\zeta\right), \quad \lambda_1 = -\frac{1}{5} Z'\left(\sqrt{\frac{3}{10}}\zeta\right) \cong \lambda_2, \text{ where } \zeta = \frac{\omega}{k_{\parallel}\mathrm{v}_{\mathrm{Ti}}}$$

Gyrofluid equation in TRB

- Vorticity equation (Garbet et. al., PoP '01)

$$\left(d_t^E - D\right)\Omega = -n\nabla_\parallel V_\parallel + n\mathbf{V}_d \cdot \nabla\left(\frac{\tilde{n} - \Omega}{n}\right) + n\left(\mathbf{V}_E + \mathbf{V}_d\right)\cdot\left(\boldsymbol{\kappa} + \nabla \ln B\right) - d_t^E n_0$$

$$\text{where } \Omega = \tilde{n} - n_{pol} : \text{generalized vorticity, } n_{pol} = \frac{ne}{m_i \omega_{ci}^2}\nabla_\perp^2 \Phi, \quad \tilde{n} = n_0 \frac{e\tilde{\Phi}}{T_e} : \text{ adiabatic response}$$

- Ion parallel velocity equation

$$\left(d_t^E - D\right)V_\parallel = -\frac{e}{m}\nabla_\parallel \Phi - \frac{1}{mn}\nabla_\parallel p_i + S_V$$

- Ion pressure equation

$$\frac{3}{2}\left(d_t^E - D\right)p_i = -\nabla_\parallel q_\parallel + \mathbf{q}_\wedge \cdot \left(\boldsymbol{\kappa} + \nabla \ln B\right) + \frac{5}{2}T_i d_t^E n + S_p$$

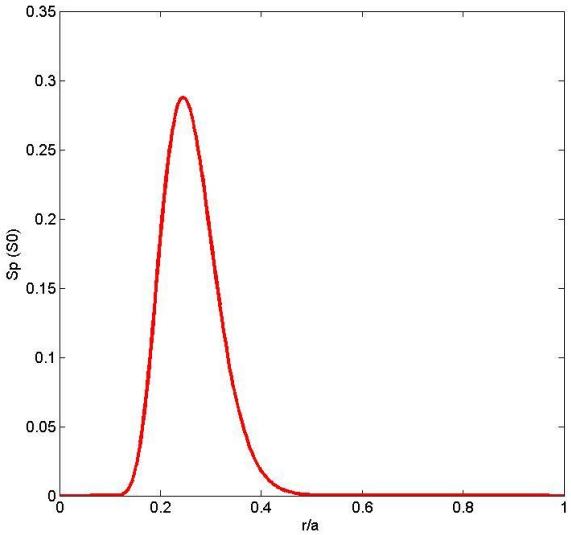
$$\text{where } \nabla_\parallel q_\parallel = \sqrt{\frac{8T_{i0}}{\pi m_i}}\left|\nabla_\parallel\right|\tilde{p}_i : \text{Hammett - Perkins closure, } \quad \mathbf{q}_\wedge = \frac{5p_i}{2m_i\omega_{ci}}\mathbf{b}\times\nabla T_i.$$

NFRI National Fusion Research Institute

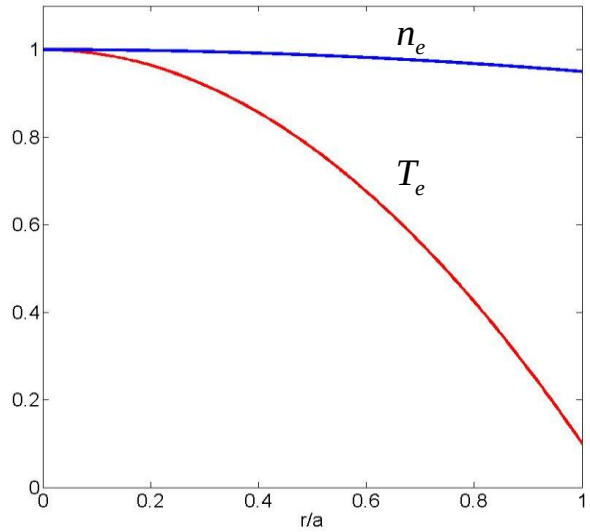
Simulation model in TRB

- **Electrostatic turbulence** with heat and momentum sources
- Fixed density and q-profile with reversed shear
- No-slip boundary condition on $V_{||}$
- **Flux driven**, **self-consistently evolving profiles**
- Spectral method : $F = \sum_{mnp} F_{mnp}(t) W_{mnp}(r) e^{i(m\theta - n\phi)}$
- Only resonant modes are retained.

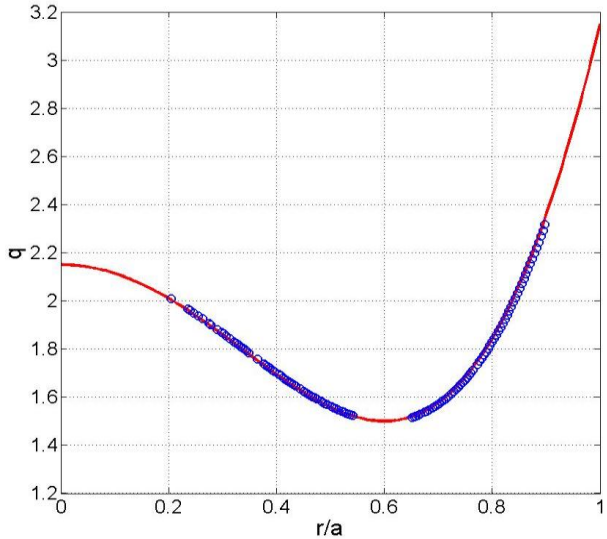
Heating profile



Density and Te profiles



q-profile



Numerical scheme in TRB

- Spectral method

$$F = \sum_{mnp} F_{mnp}(t) W_{mnp}(r) e^{i(m\theta - n\zeta)},$$

$$W_{mnp}(r) = \frac{1}{\sqrt{2 r w_{mn} \nu_p}} H_p(x) e^{-x^2/2} \text{ for } (m,n) \neq (0,0) \text{ modes } \tilde{F}, \quad \text{where } x = \frac{r - r_{mn}}{w_{mn}}, \quad \nu_p = 2^{p/2} \Gamma(p+1)^{1/2} \pi^{1/4}$$

$$W_{mnp}(r) = \frac{1}{\sqrt{2r} J_1(\alpha_{1p})} J_0\left(\frac{\alpha_{0p}}{a} r\right) \text{ for } (m,n) = (0,0) \text{ modes } F_{eq}$$

$$\frac{\partial F_k}{\partial t} = \sum_{k'k''} G_{kk'k''} \varphi_{k''} F_{k'} + \sum_{k'} M_{kk'} F_{k'} + S_k \quad \text{where } k = mnp, \ (m,n) = (m' + m'', n' + n'')$$

- Nonlinear interaction: we exclude coupling terms if

$$\text{Effective distance between modes} >> \text{Average width of modes}$$

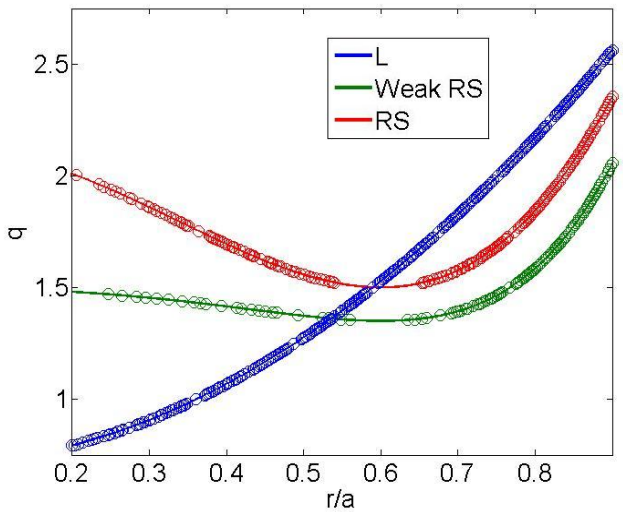
$$\Delta r_{eff} = \frac{\Delta q}{|q'|} > C w_{eff}, \quad \text{where } \Delta q = \sqrt{(q_{mn} - q_{m'n'})^2 + (q_{m'n'} - q_{m''n''})^2 + +(q_{m''n''} - q_{mn})^2}, \ w_{eff} = \sqrt{w_{mn}{}^2 + w_{m'n'}{}^2 + w_{m''n''}{}^2}$$

- Explicit time advance with midpoint method

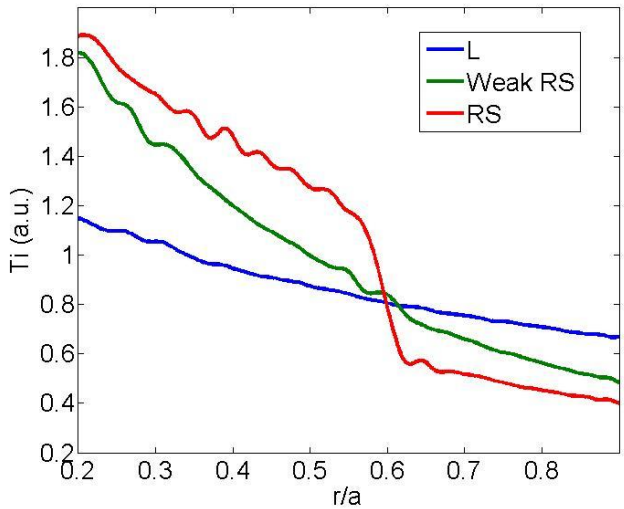
$$\frac{\partial F}{\partial t} = Q(F,t) \ \rightarrow \ F_{t+\Delta t} = F_t + \Delta t \cdot Q\left(F_{t+\Delta t/2}, t + \frac{\Delta t}{2}\right) \ \text{ where } \ F_{t+\Delta t/2} = F_t + \frac{\Delta t}{2} \cdot Q(F_t, t)$$

TRB simulations on ITB formation

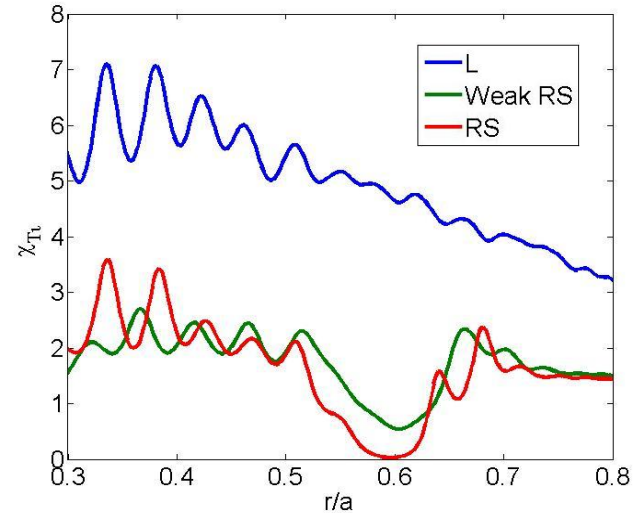
q profile



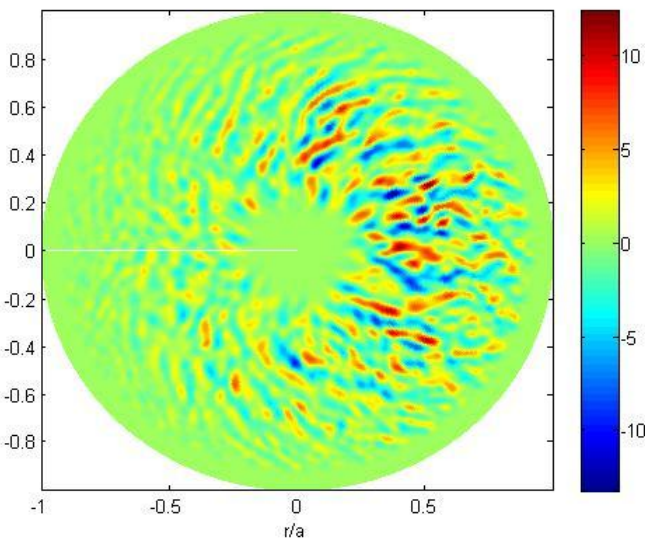
Ion temperature profile



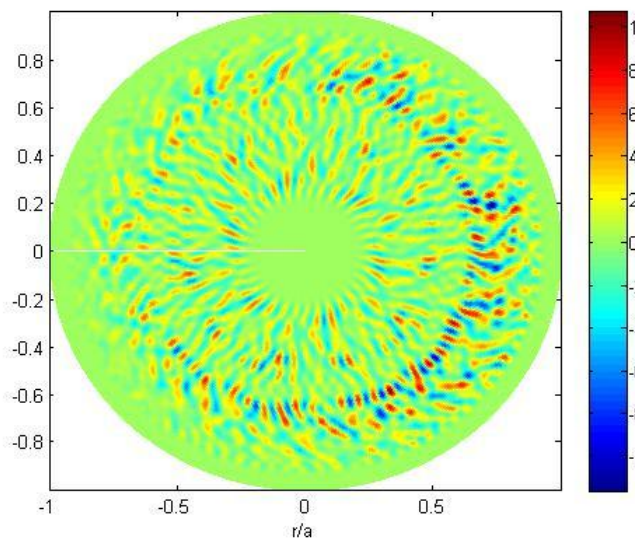
Ion thermal diffusivity



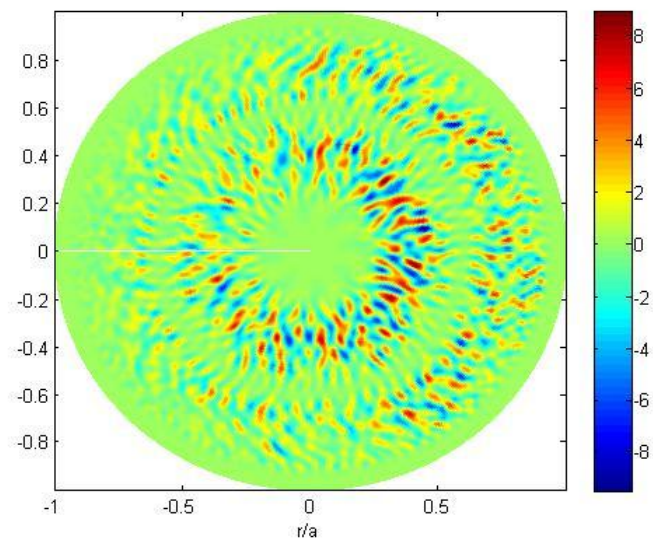
L-mode



Weak RS-mode

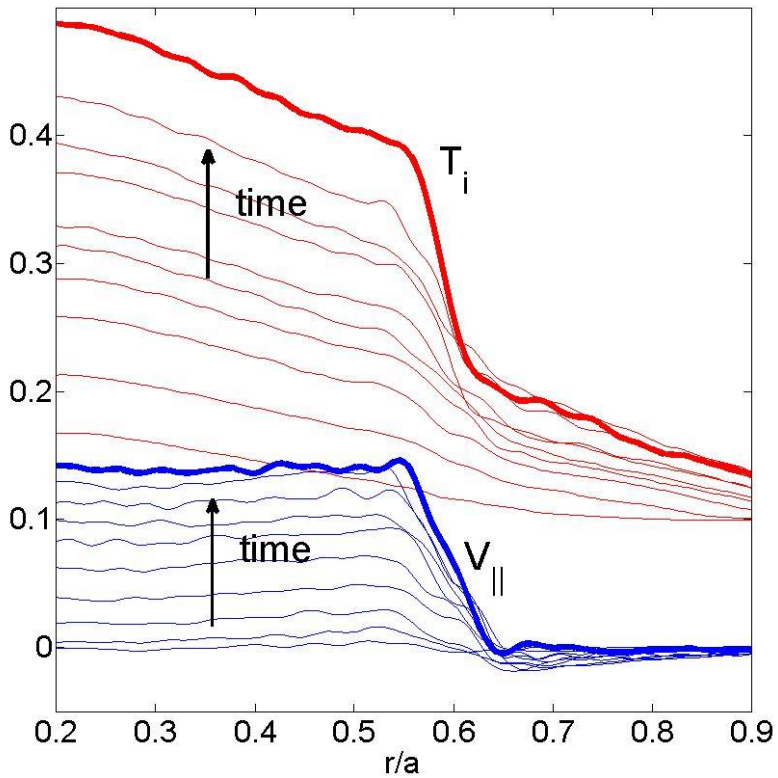


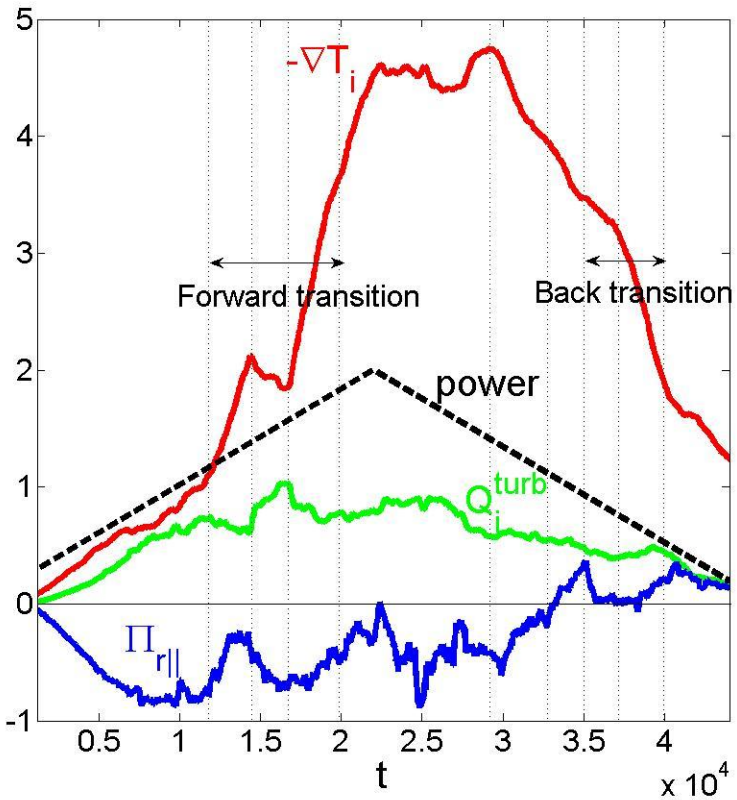
RS-mode



Dynamics of Internal Transport Barrier

- Long time power ramp simulations show (Kim et.al., NF 2011)
  - Both heat and momentum transport barriers are formed simultaneously
  - Reynolds stress or intrinsic rotation is crucial in ITB dynamics

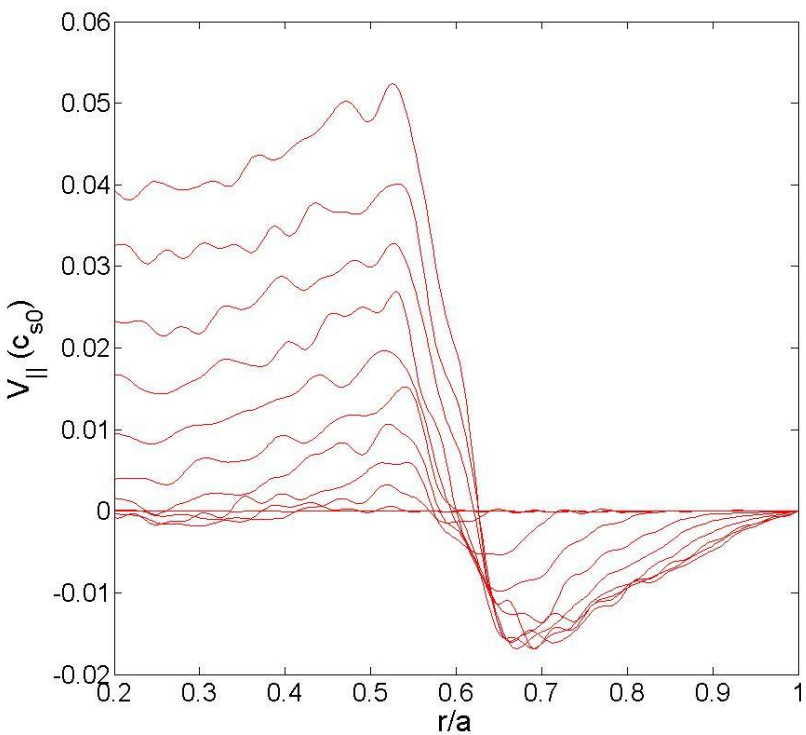


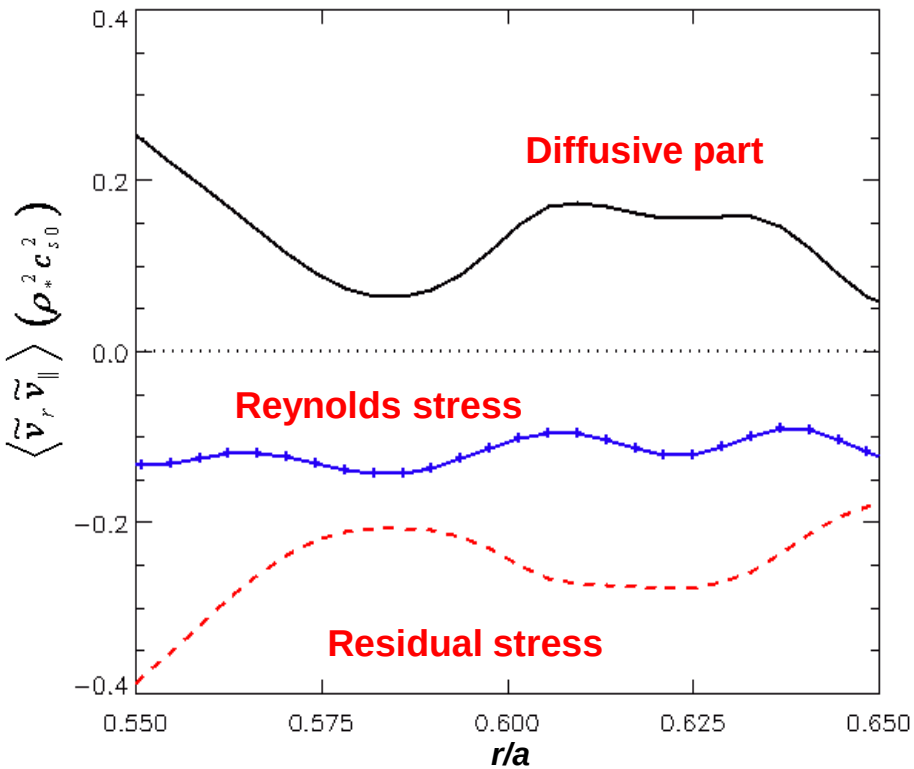


Intrinsic rotation is generated by non-diffusive part of Reynolds stress

- Intrinsic rotation is generated near **ITB head** and, initially, propagates into the core
- Reynolds stress $\Pi_{r\parallel} = -\chi_\phi \nabla V_\parallel + \Pi_{r\parallel}^{resid} < 0$, because of large **inward** residual stress

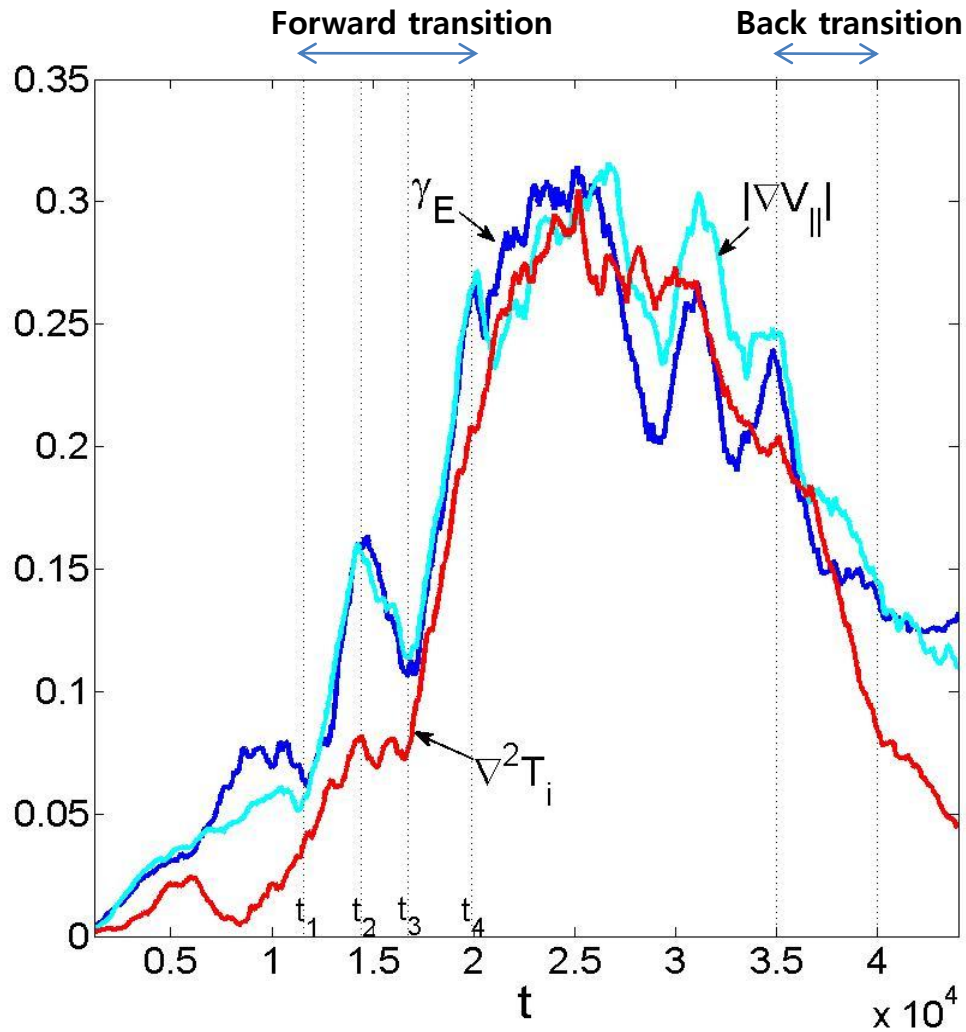
$V_{\parallel}$ evolution during initial phase



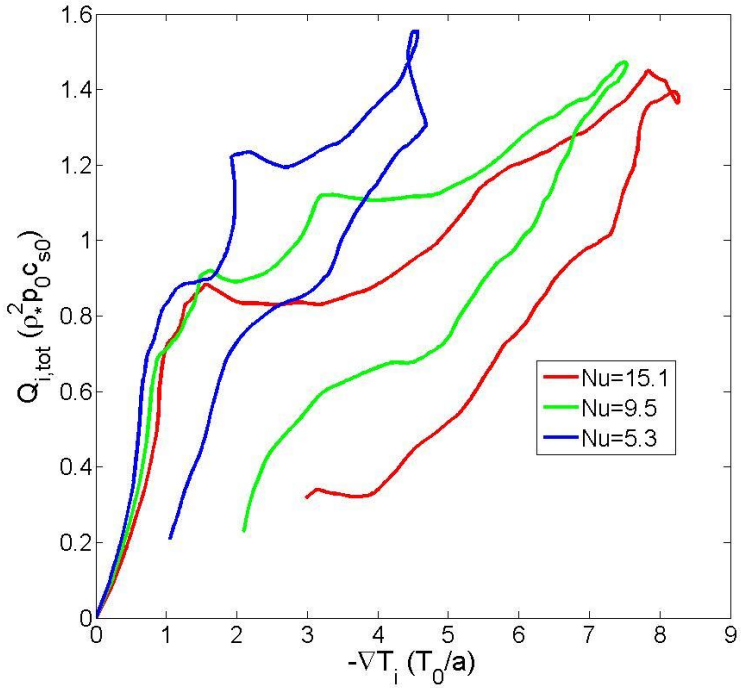


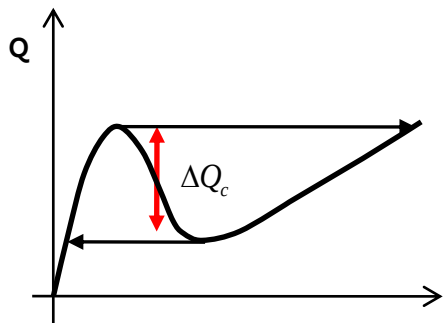
Intrinsic rotation is strongly coupled to ITB dynamics

- ExB shearing rate closely tracks $\nabla V_{||}$ at q_{min} position.
- Hence, the intrinsic rotation has a significant influence on ITB dynamics.
- A similar trend has been observed in ITB experiments using off-axis ICRH at Alcator C-Mod [Fiore et.al. APS'10].



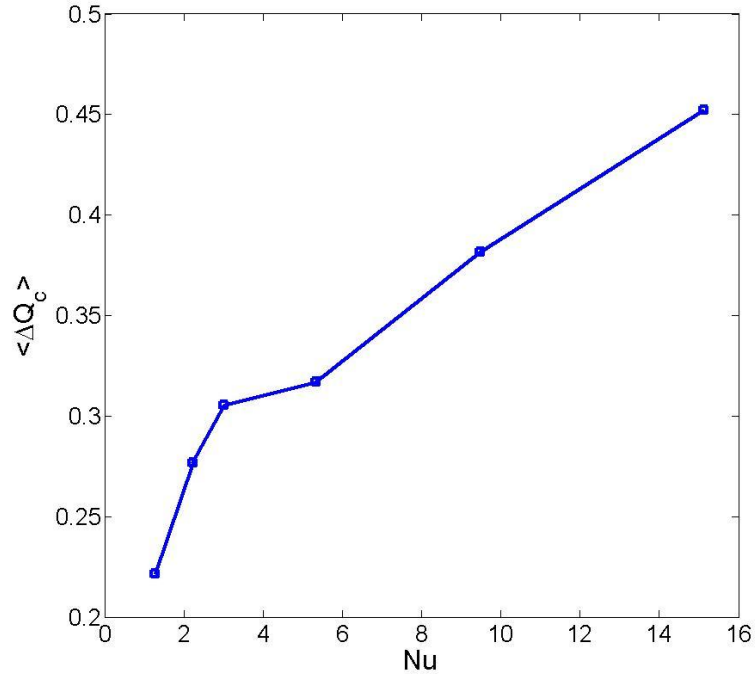
Hysteresis happens!





$$\langle \Delta Q_c \rangle = A / \Delta(\nabla T_i),$$

A: area spanned by hysteresis curve

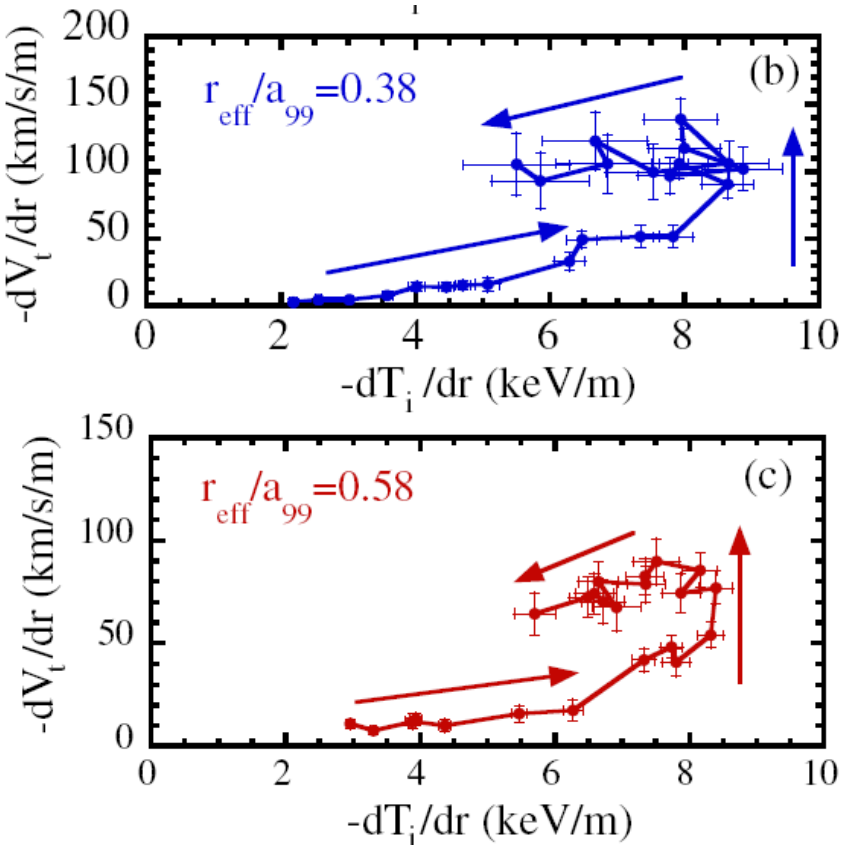


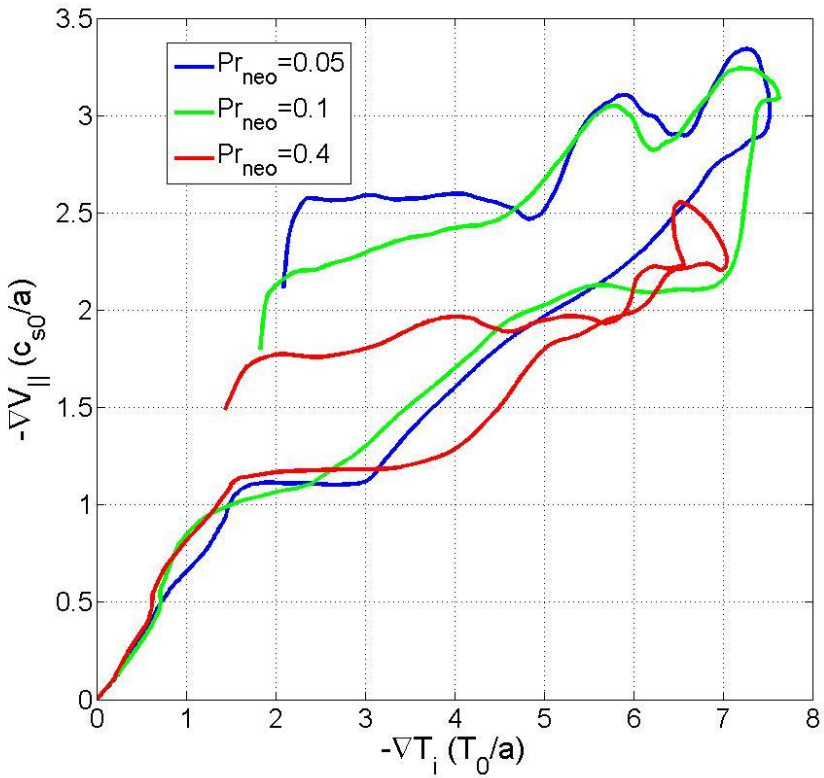
- Open loop hysteresis is seen in $Q_{i,tot}$ *vs.* $-\nabla T_i$ plot
  → Contrast to closed loop S-curve model

- Strength of hysteresis increases with Nusselt number ($Nu = \chi_{i,turb} / \chi_{i,neo}$) increases
  → agrees with prediction based on bifurcation theory
  [Malkov & Diamond, PoP'08]

NFRI National Fusion Research Institute

Relative hysteresis between ∇T_i & $\nabla V_{||}$ observed

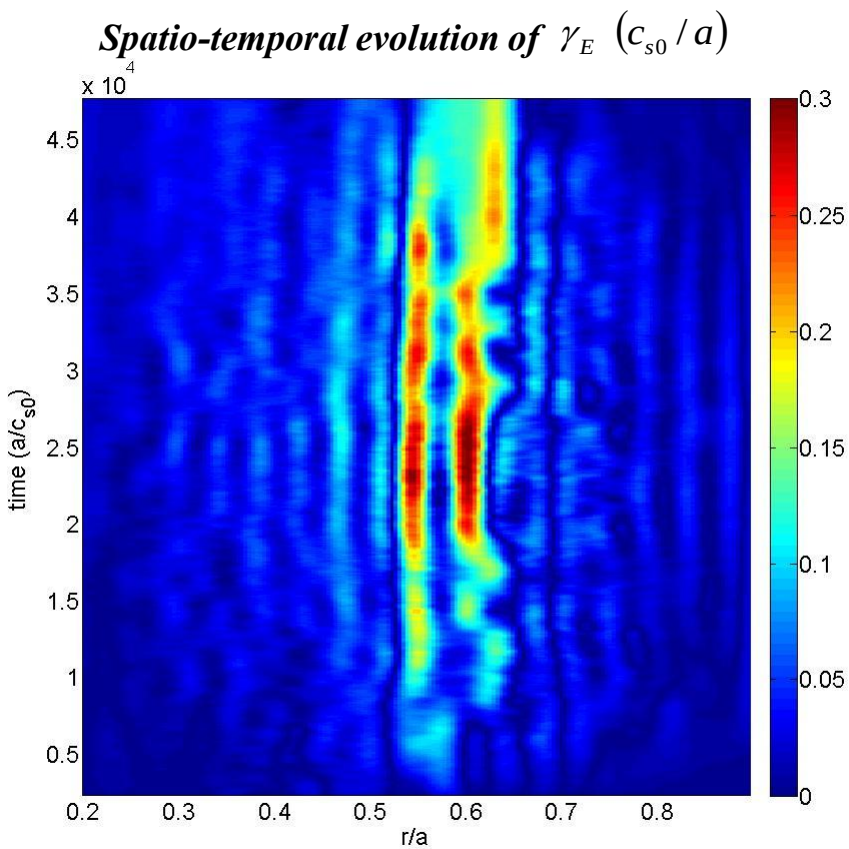


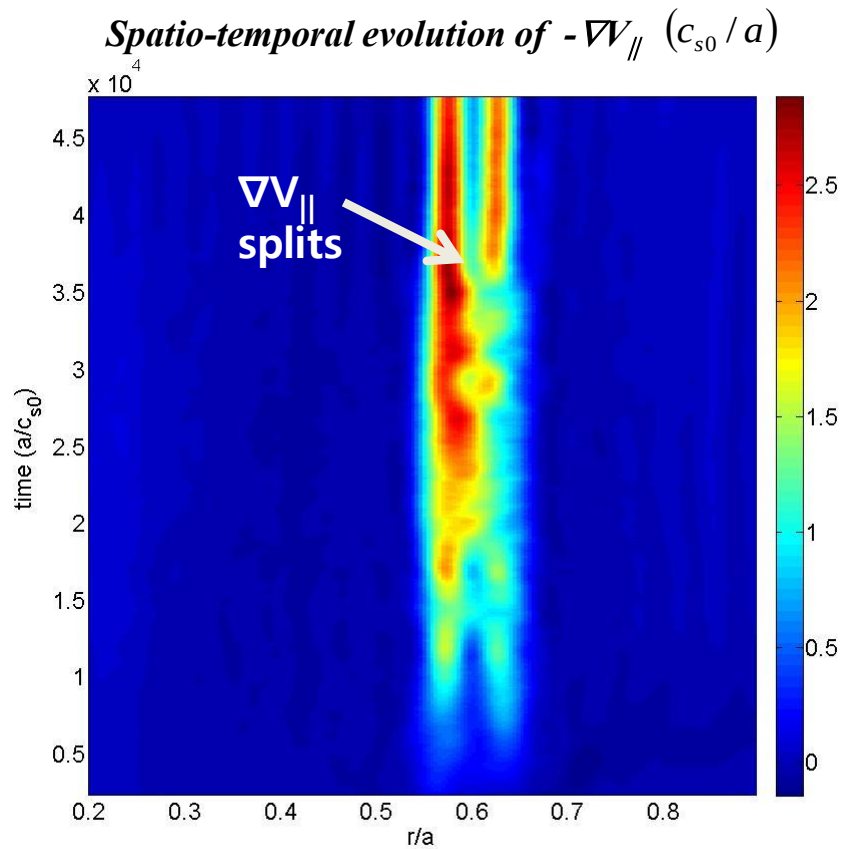


Relatively stronger hysteresis of intrinsic rotation over temperature gradient is observed ➡ **Recover** features of recent experimental observation in LHD [K. Ida et. al., NF 50 (2010) 064007]

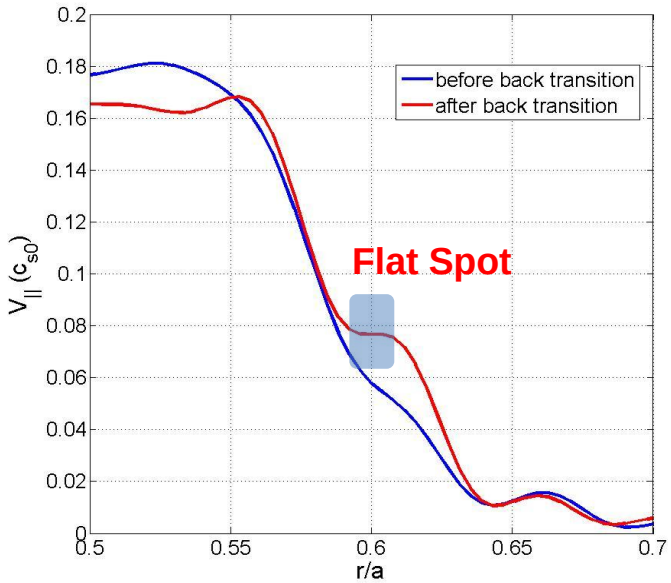
Splitting of $\nabla V_{||}$ observed in back transition

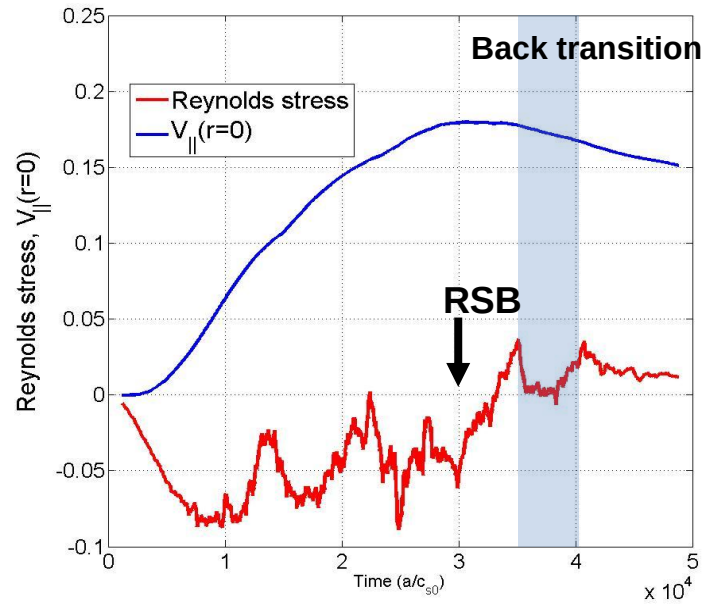
- V_{ExB} shear collapse starts from the ITB foot position and causes negative feedback.
- Flow evolution simultaneous with ExB shear. $\nabla V_{||}$ splits into two sections at back transition.

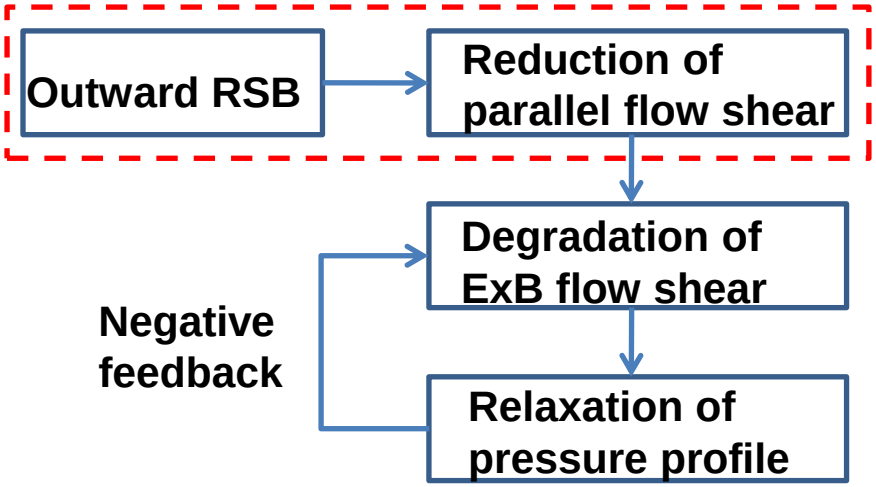




Back transition is triggered by Reynolds stress burst (RSB)







- Possible mechanisms of RSB are

  (1) parallel shear flow instability
  (2) nonlinear flip in sign of residual stress

- Similar momentum transfer events were observed in DIII-D (Osborne et. al., NF'95)

NFRI National Fusion Research Institute

Summary and future works

- Model and recent results of global flux-driven gyrofluid simulation code TRB are introduced.
- Robust **intrinsic rotation** with $0.1 < M_{th} < 0.2$ found in reversed shear ITB from gyrofluid simulations of ITG turbulence. Intrinsic rotation dynamics are strongly coupled to ITB evolution.
- **Open-loop hysteresis** in Q *vs.* ∇T_i discovered and correlated with Nusselt number. Relative hysteresis between ∇T_i and $\nabla V_{||}$ noted to correlate with Pr_{neo} .
- **Reynolds stress bursts** in regions of steep $\nabla V_{||}$ ***triggers*** back transition.

- **Ongoing and future works**
  - Detailed analysis of back transition: parallel shear flow instability
  - Density profile and q-curvature effects
  - Two field model (pressure + parallel flow) for transport bifurcation
  - Effects of external momentum source
  - Density dynamics and p-ITB formation