Impact of FLR effect and Landau damping on ELMs

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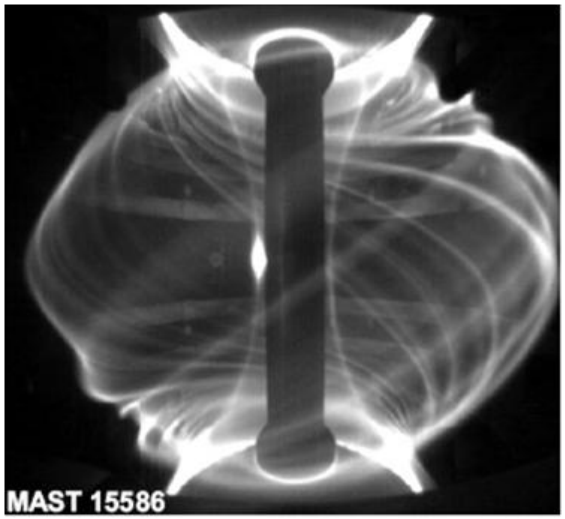
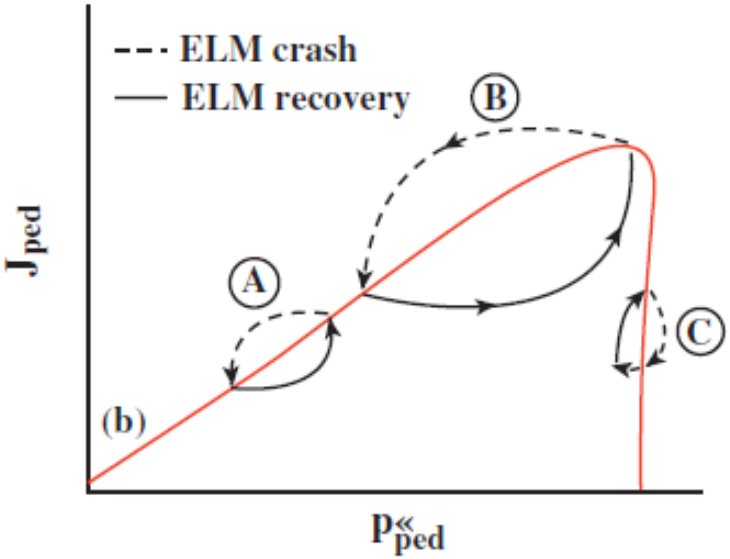
1. **Introduction**
2. **FLR effect in isothermal limit**
   - 2.1 Disappear of ion diamagnetic effect in twofluid model
   - 2.2 Full FLR stabilizing in gyrofluid model and gyroviscosity
   - 2.3 Nonlinear simulations
3. **The impact of Landau damping**
4. **Summary**

Background : Peeling-ballooning model for ELMs



A. Kirk, PRL **96**, 185001 (2006)



P.B. Snyder, et.al *Nucl. Fusion* **47** (2007) 961

- **Edge-Localized-Modes (ELMs)**
  - ❖ Erosion of PFC;
  - ❖ Impurity transport;
  - ❖ Confinement;

- **Peeling-ballooning model**
  - ✓ ideal peeling-ballooning instability;
  - ✓ Ion diamagnetic effect;
  - ✓ Resistivity/hyper-resistivity;

- **Kinetic effect?**
  - ➢ FLR effect;
  - ➢ Landau damping;
  - ➢ Toroidal resonance;

BOUT++: A framework for nonlinear twofluid and gyrofluid simulations

- **Different twofluid and gyrofluid models are developed under BOUT++ framework for ELM simulations**

	Twofluid	Gyrofluid (FLR effect)	Physics
	3-field $(\varpi, P, A_\parallel)$	1+0 $(n_{iG}, n_e, A_\parallel)$	Peeling-ballooning mode
Available	4-field $(\varpi, P, A_\parallel, V_\parallel)$	2+0 $(n_{iG}, n_e, A_\parallel, V_\parallel)$	+ Acoustic wave
	6-field $(\varpi, n_i, A_\parallel, V_\parallel, T_i, T_e)$	3+1 $(n_{iG}, n_e, A_\parallel, V_\parallel, T_{i\perp}, T_{i\parallel}, T_e)$	+ Thermal transport

Numerical method

- Landau damping
- Toroidal resonance
- Zonal flow closure

→ Nonlinear ITG/KBM simulations at pedestal

BOUT++: A framework for nonlinear twofluid and gyrofluid simulations

- **Different twofluid and gyrofluid models are developed under BOUT++ framework for ELM simulations**

Twofluid	Gyrofluid (FLR effect)	Physics
3-field $(\varpi, P, A_\parallel)$	**1+0** $(n_{iG}, n_e, A_\parallel)$	**Peeling-ballooning mode**
4-field $(\varpi, P, A_\parallel, V_\parallel)$	**2+0** $(n_{iG}, n_e, A_\parallel, V_\parallel)$	**+ acoustic wave**
6-field $(\varpi, n_i, A_\parallel, V_\parallel, T_i, T_e)$	**3+1** $(n_{iG}, n_e, A_\parallel, V_\parallel, T_{i\perp}, T_{i\parallel}, T_e)$	**+ Thermal transport**

+ Landau damping

FLR effect with large density gradient

Landau damping

Numerical method

- **Toroidal resonance**
- **Zonal flow closure**

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3-field twofluid model including the large density gradient at H-mode pedestal

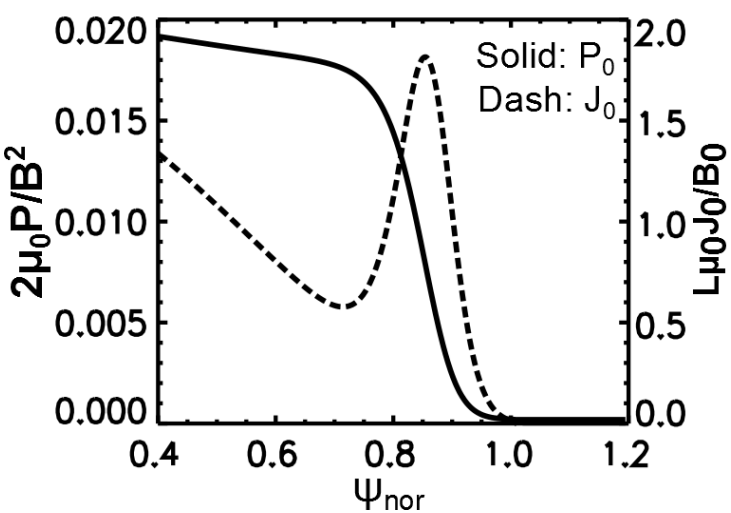
$$\frac{d\varpi}{dt} = B\nabla_{\parallel} J_{\parallel} + 2\mathbf{b}_0 \times \boldsymbol{\kappa} \cdot \nabla \tilde{P} + \mu_{i,\parallel} \partial_{\parallel}^2 \varpi$$

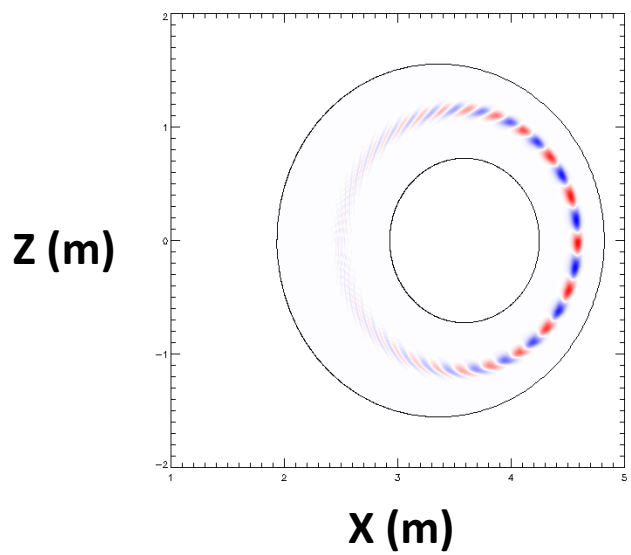
$$\frac{d\tilde{P}}{dt} + \mathbf{V}_E \cdot \nabla P_0 = 0$$

$$\frac{\partial A_{\parallel}}{\partial t} + \partial_{\parallel} \phi_T = \frac{\eta}{\mu_0} \nabla_{\perp}^2 A_{\parallel} - \frac{\eta_H}{\mu_0} \nabla_{\perp}^4 A_{\parallel}$$

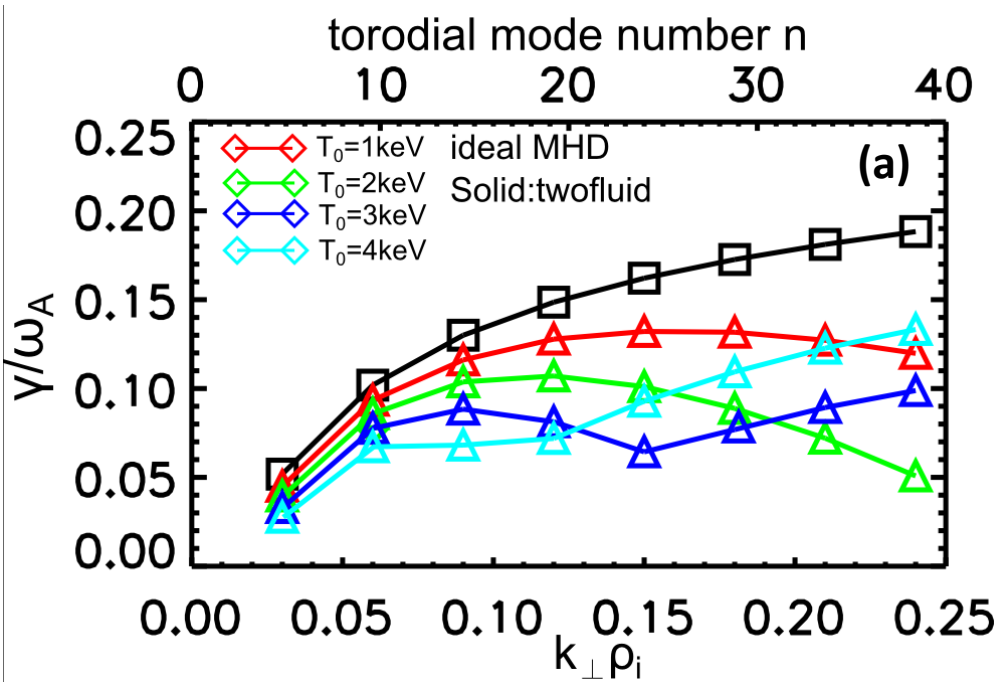
$$\varpi = \frac{m_i n_0}{B} \left(\nabla_{\perp}^2 \phi + \frac{1}{e n_0} \nabla_{\perp}^2 \tilde{P}_i + \frac{1}{n_0} \nabla n_0 \cdot \nabla \phi \right)$$

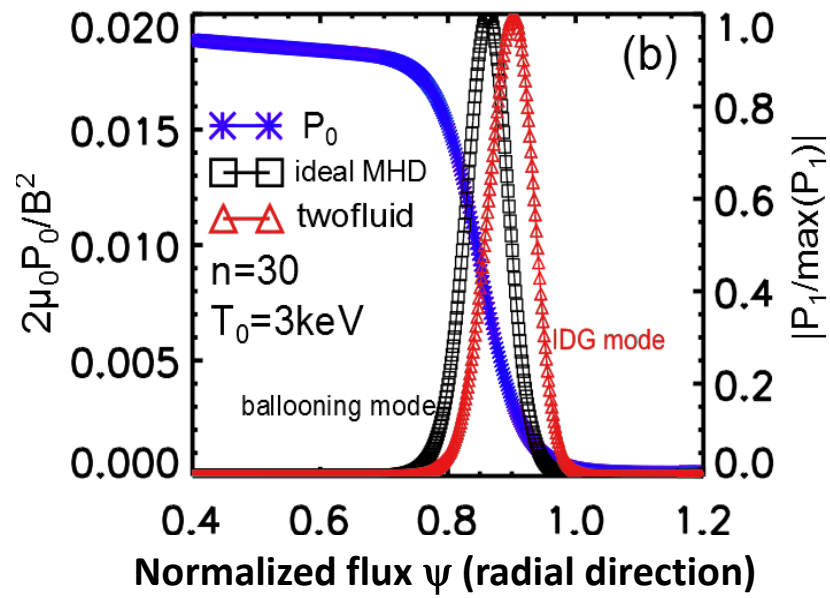
$$d/dt = \partial/\partial t + \mathbf{V}_{ET} \cdot \nabla,\ \mathbf{V}_{ET} = \frac{1}{B} \mathbf{b}_0 \times \nabla \phi_T,\ \phi_T = \phi_0 + \phi,\ \nabla_{\parallel} f = B \partial_{\parallel} \frac{f}{B},\ \partial_{\parallel} = \partial_{\parallel}^0 + \delta\mathbf{b} \cdot \nabla,\ \delta\mathbf{b} = \frac{1}{B} \nabla A_{\parallel} \times \mathbf{b}_0,\ J_{\parallel} = J_{\parallel 0} + \tilde{J}_{\parallel},\ \tilde{J}_{\parallel} = -\nabla_{\perp}^2 A_{\parallel} / \mu_0$$





In twofluid simulations, the ion diamagnetic stabilization on high-n modes disappears when temperature increases





→ The instability does not localize at peak pressure gradient region

→ Not pressure gradient driven ballooning mode, but other instability

→ Lowest order ballooning equation changes

$$\frac{1}{J}\frac{\partial}{\partial\chi}\left[\frac{k_\perp^2}{B^2 J}\frac{\partial}{\partial\chi}\hat{\phi}\right]+\left[\frac{k_\perp^2}{V_A^2}\omega(\omega+\omega_{*i})+\frac{2\omega_\kappa\omega_{*i}}{V_A^2\rho_i^2}+i\frac{q' g^{\psi\psi} n\chi}{V_A^2 L_n}\omega^2\right]\hat{\phi}=0$$

Local analysis: Ion-Density-Gradient mode in twofluid model

- **Local dispersion relation with density gradient**

$$\left(\omega+\omega_{*i}-\frac{ik_r}{k_\perp^2 L_n}\omega\right)\omega=-\gamma_I{}^2$$

- **Typically, the density gradient is weak, so we get the well-known ion diamagnetic stabilization on ballooning modes**

$$k_\perp L_n >> 1 \Rightarrow \qquad (\omega+\omega_{*i})\omega=-\gamma_I{}^2 \Rightarrow \omega=-\frac{\omega_{*i}}{2}+i\sqrt{\gamma_I{}^2-\frac{\omega_{*i}^2}{4}}$$

- **But at H-mode pedestal, density gradient is large, so we get**

$$k_\perp L_n \sim 1 \Rightarrow \qquad \gamma=\frac{1}{\sqrt{C}}\left(\sqrt{\gamma_I^2-\frac{\omega_{*i}^2}{4}}\cos\frac{\alpha}{2}+\frac{\omega_{*i}}{2}\sin\frac{\alpha}{2}\right)$$

$$C^2=1+\frac{k_r^2}{k_\perp^4 L_n^2}$$

$$\cos\alpha=\frac{1}{C}$$

$$\sin\alpha=\frac{1}{C}\frac{k_r}{k_\perp^2 L_n}$$

Ion diamagnetic stabilization on ballooning modes

Instability introduced by density gradient

When ω_{*i} increases:

- Ion diamagnetic effect stabilizes ballooning modes → First term decreases
- Ion density gradient introduces **Ion-Density-Gradient mode**: Second term becomes dominant

Is the IDG mode a kind of real physics instability?
Or it reveals certain disadvantage of twofluid model?
→ Gyrofluid model

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3-field isothermal gyrofluid model

- We derived an isothermal electromagnetic 3-field gyro-fluid model with vorticity formulation generalized from Snyder-Hammett gyro-fluid model [1] for edge plasmas.

$$\begin{pmatrix} \varpi_G \\ \tilde{P}_G \end{pmatrix} = \begin{bmatrix} -eB & eB \\ T_0 & T_0 \end{bmatrix} \begin{pmatrix} \tilde{n}_{iG} \\ \tilde{n}_e \end{pmatrix}$$

Gyro-averaged $\mathbf{V}_\Phi = \frac{1}{B}\mathbf{b} \times \nabla \Gamma_0^{1/2}(b)\phi$

Non Gyro-averaged $\mathbf{V}_E = \frac{1}{B}\mathbf{b}_0 \times \nabla \phi$

$$\frac{d\varpi_G}{dt} + \mathbf{V}_E \cdot \nabla \varpi_{G0} - eB(\mathbf{V}_{\Phi T} - \mathbf{V}_{ET}) \cdot \nabla n_{iG} = B\nabla_\parallel J_\parallel + 2\mathbf{b}_0 \times \boldsymbol{\kappa} \cdot \nabla \tilde{P}_G + \mu_{i,\parallel} \partial_\parallel^2 \varpi_G$$

$$\frac{d\tilde{P}_G}{dt} + \mathbf{V}_E \cdot \nabla P_{G0} + T_0(\mathbf{V}_{\Phi T} - \mathbf{V}_{ET}) \cdot \nabla n_{iG} = 0$$

EXB FLR correction

$$\frac{\partial A_\parallel}{\partial t} + \partial_\parallel \phi_T = \frac{\eta}{\mu_0} \nabla_\perp^2 A_\parallel - \frac{\eta_H}{\mu_0} \nabla_\perp^4 A_\parallel$$

$$\varpi_G = eB\left(\Gamma_0^{1/2} \tilde{n}_{iG} - n_0(1-\Gamma_0)\frac{e\phi}{T_0} + \frac{e\rho_i^2}{T_0} \nabla n_0 \cdot \nabla(\Gamma_0 - \Gamma_1)\phi - \tilde{n}_{iG} \right)$$

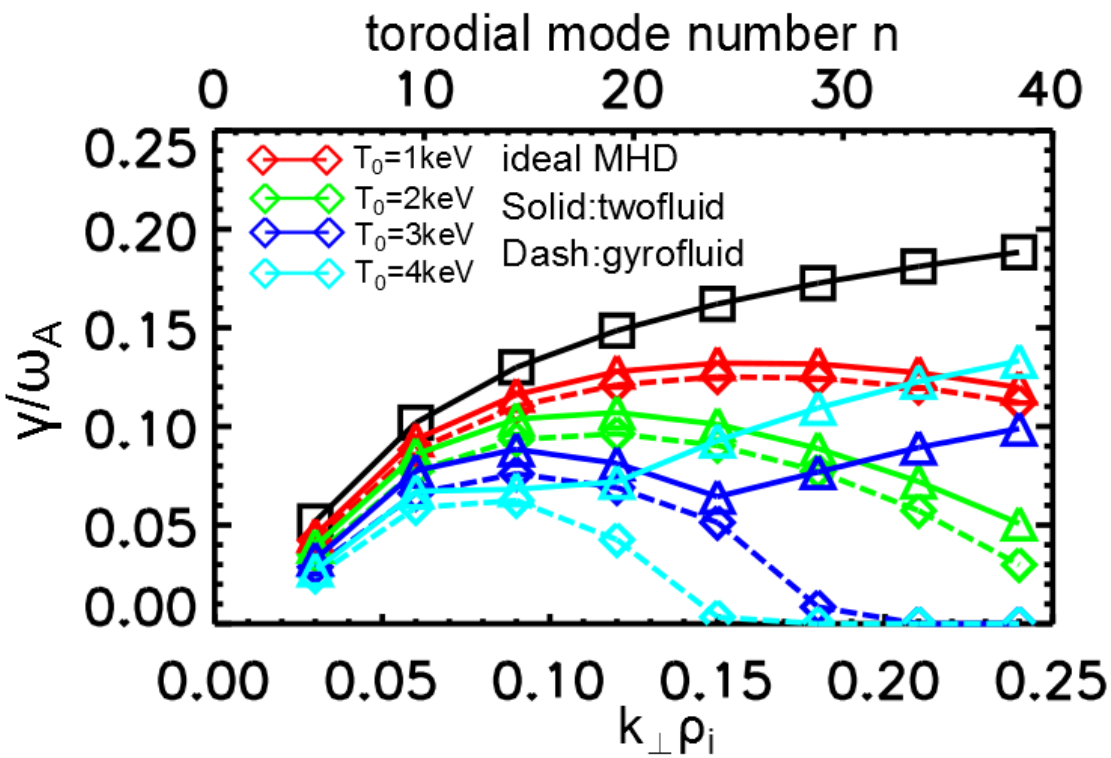
Density gradient

Padé approximation

$$\begin{cases} \Gamma_0^{1/2} \approx \dfrac{1}{1+b/2} \\ \Gamma_0 \approx \dfrac{1}{1+b} \quad , \quad b = -\rho_i^2 \nabla_\perp^2 \\ \Gamma_0 - \Gamma_1 \approx 1 \end{cases}$$

[1] P. B. Snyder and G. W. Hammett, Phys. Plasmas 8, 3199 (2001).

Gyrofluid model shows FLR stabilization on high-n modes and no IDG mode is found!



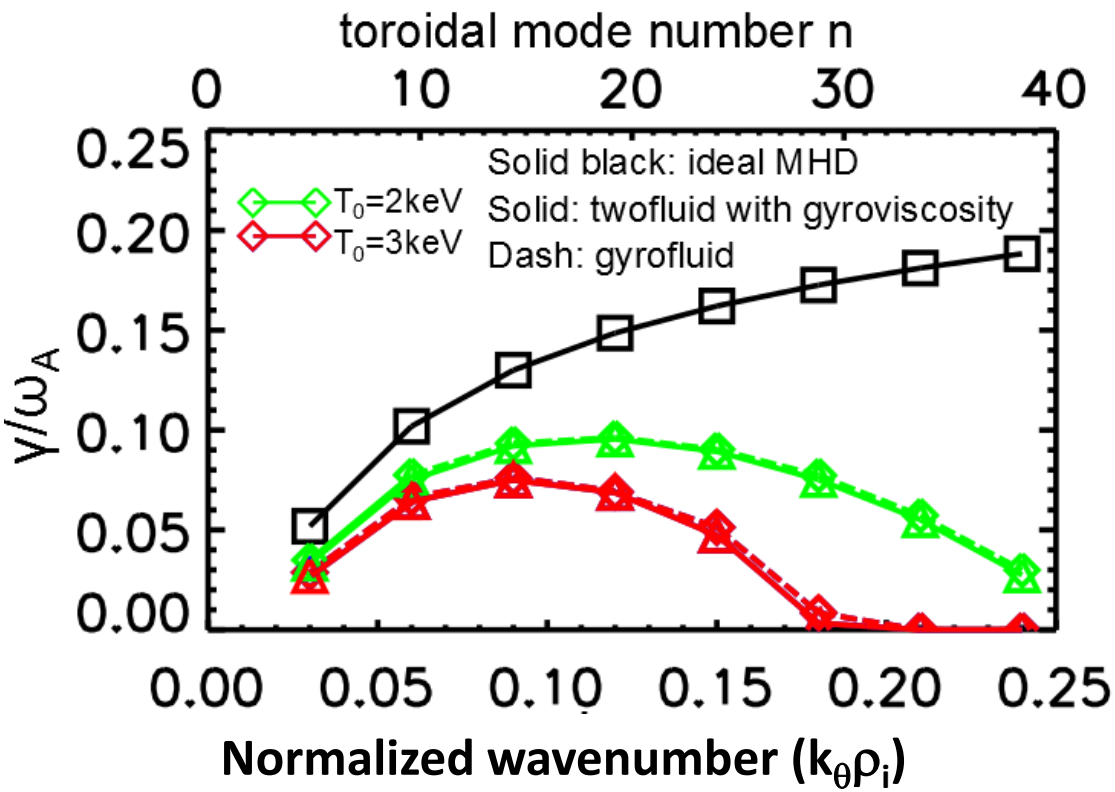
- ✓ **Higher temperature leads to more FLR stabilization on high-n modes;**
- ✓ **No IDG mode is found in gyrofluid simulations**

Ion diamagnetic effect alone in two-fluid model is not sufficient to represent FLR stabilization if density gradient is large!

Gyroviscous terms are necessary to stabilize Ion-Density-Gradient modes and should be kept in twofluid model

- **At long wavelength limit, gyro-fluid goes back to two-fluid but with additional gyroviscous terms**

$$\frac{d\varpi_G}{dt} + \mathbf{V}\cdot\nabla\varpi_{G0} - eB(\mathbf{V}_{\Phi T} - \mathbf{V}_{ET})\cdot\nabla n_{iG} = \frac{d\varpi}{dt} + \boxed{\frac{1}{2\omega_{ci}}\left\{\nabla_\perp^2[\phi, P_i] - [\nabla_\perp^2\phi, P_i] - [\phi, \nabla_\perp^2 P_i]\right\}}$$



$$\varpi \approx \varpi_G + \frac{1}{en_0}\nabla_\perp^2 \tilde{P}_{iG}, \quad \rho_i^2 k_\perp^2 << 1$$

- **Gyroviscous terms represent necessary FLR effect to stabilize the virtual IDG modes and should be kept in twofluid model.**

Gyroviscosity introduces more numerical noise in In nonlinear simulations

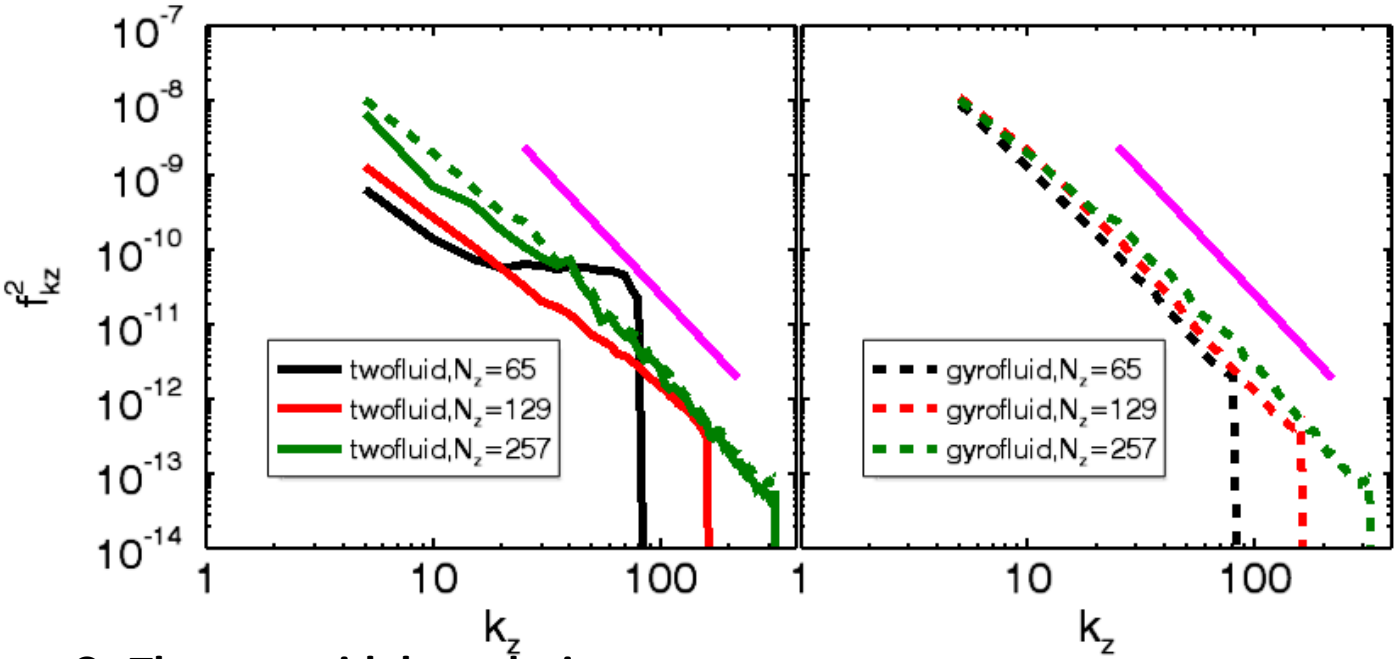


Fig. Time averaged power spectrum in k_z for twofluid and gyrofluid simulations. The additional line shows the $k^{-3.3}$ trend

- **Three toroidal resolutions:**
  $N_z = 65,129,257$
- **Gyrofluid:**
  - ✓ Three power spectrums have same index;
  - ✓ Good convergence;
- **Twofluid:**
  - ✓ $N_z = 65$ case shows a tail at high k_z region;
  - ✓ The tail disappears in $N_z = 129$ case;
  - ✓ Need $N_z = 257$ to recover gyrofluid spectrum;
  - ➢ Gyroviscosity needs 3rd finite difference → numerical noise

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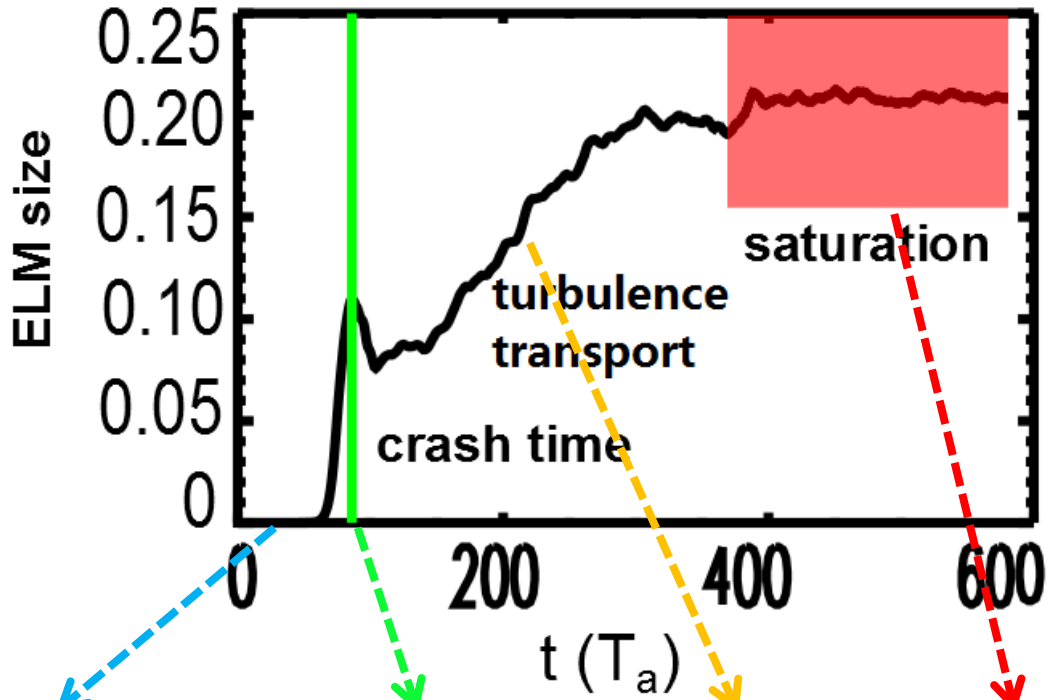
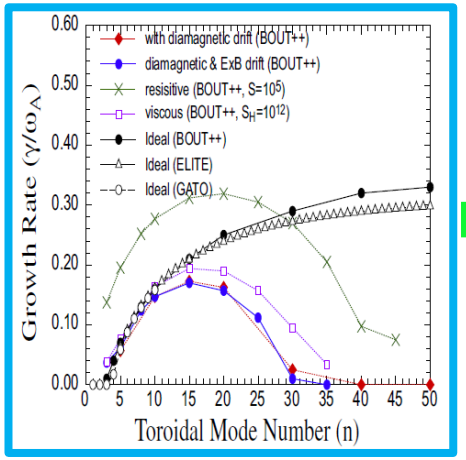
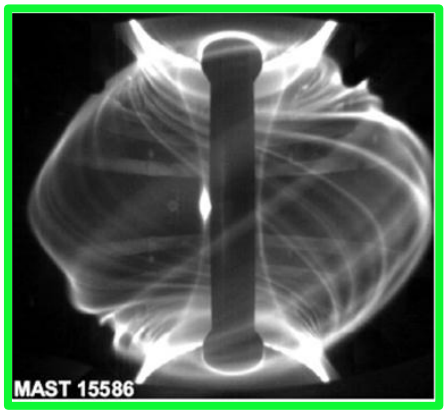
Energy losing process during one ELM event



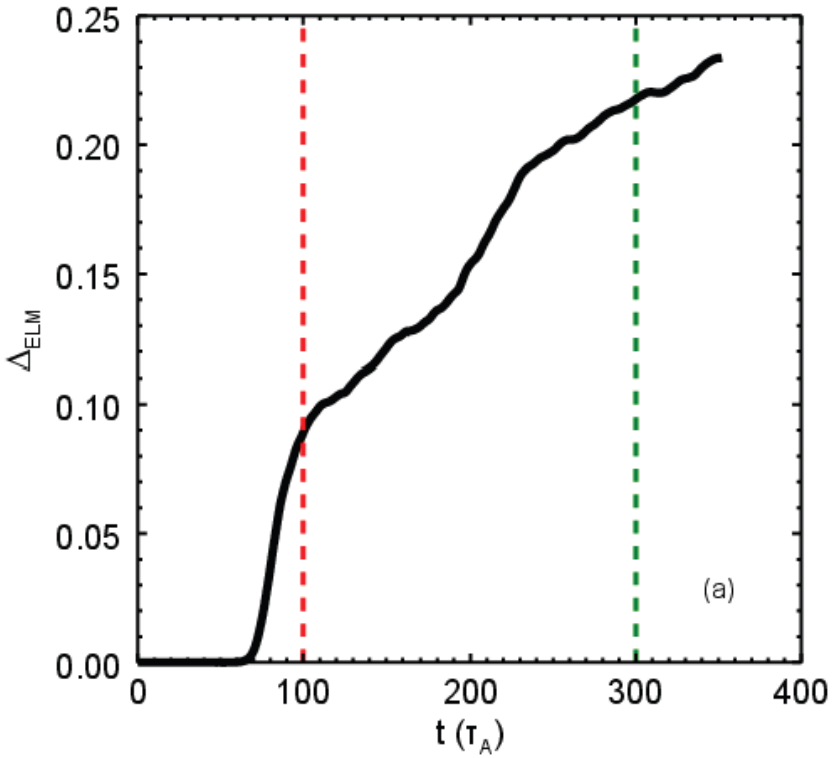
Linear phase → Initial crash | relaxation | Saturation

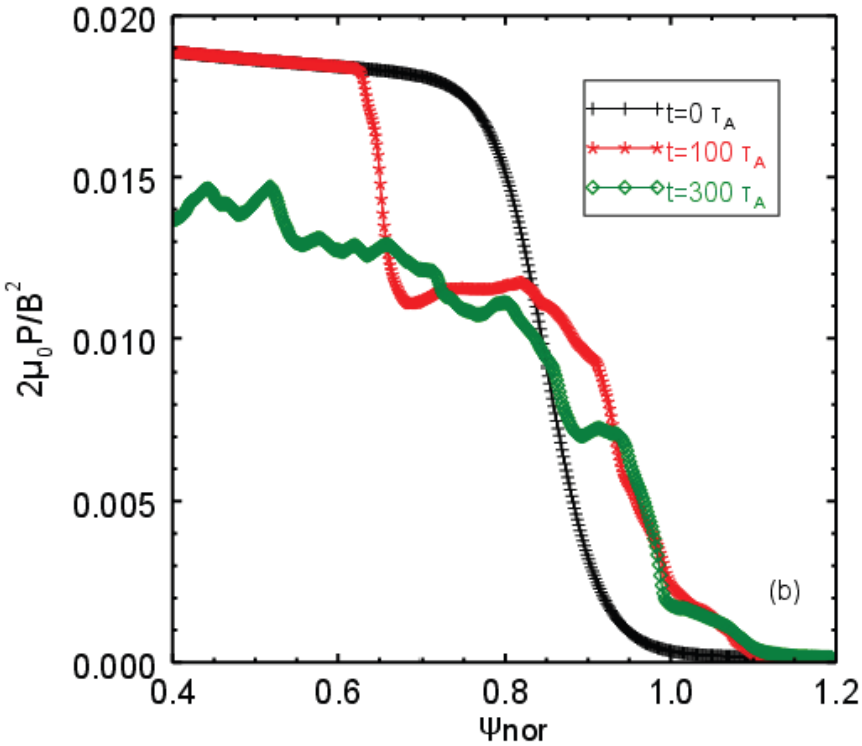




phase	characteristic
Linear phase	Growing of various linear instabilities
Initial crash	• Reconnection • Formation of filaments • Energy loss carried by filaments(used in BOUT++)
relaxation	Turbulence transport, mainly EXB convection
Saturation	Total energy loss(measured in experiments)

Nonlinear gyrofluid simulation with large density gradient shows ELM size keeps increasing and no saturation phase is found

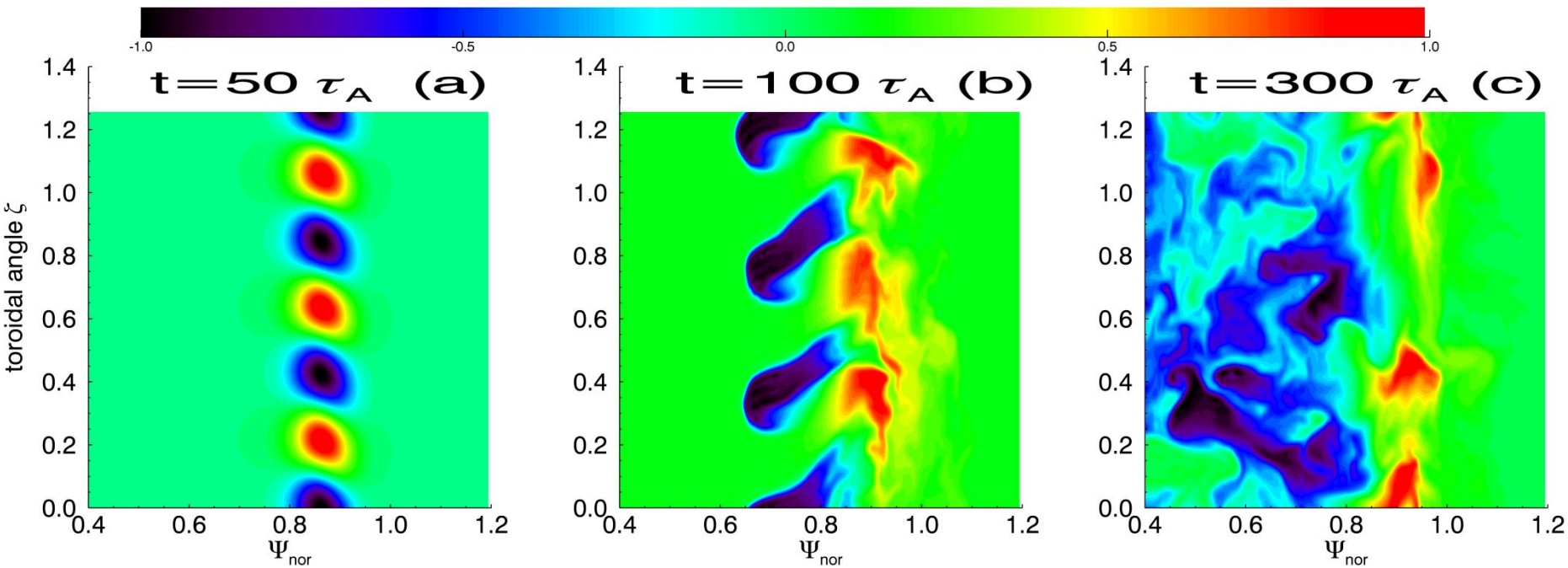




$$\Delta_{ELM}^{th} = \frac{\Delta W_{ped}}{W_{ped}} = \frac{\int_{R_{in}}^{R_{out}} \oint dR d\theta (P_0 - \langle P \rangle_\zeta)}{\int_{R_{in}}^{R_{out}} \oint dR d\theta P_0}$$

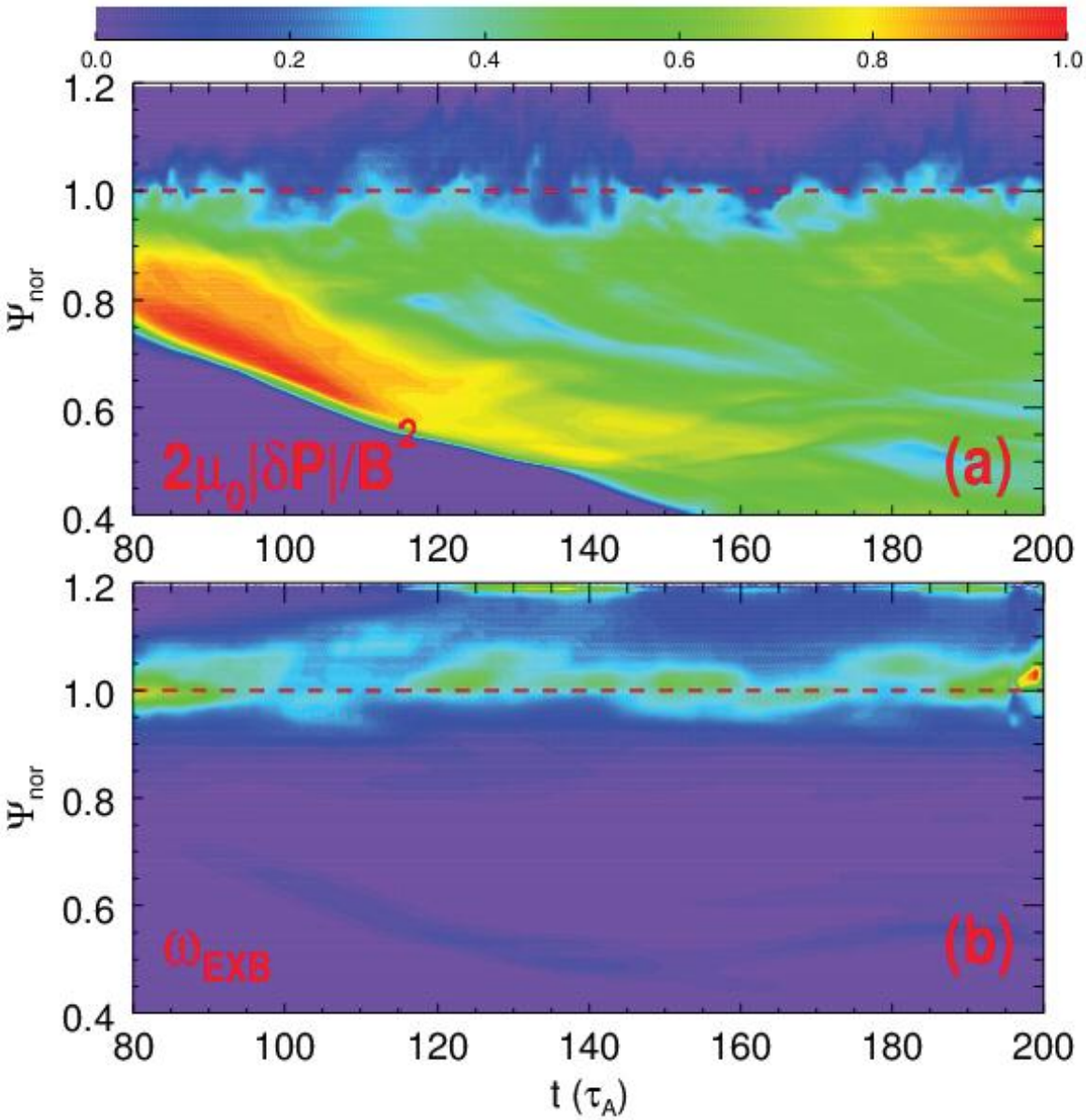
- After the initial crash, the pedestal keeps collapse until it is fully destroyed;
- Energy loss keeps increasing;
- No saturation.

Filamentary structure is generated at initial crash and the following turbulence transport further relaxes pressure profile



Linear phase **Initial crash** **Turbulence transport**

EXB shearing is too weak to suppress the local turbulence at the top of pedestal



- **ExB shearing rate**

$$\omega_{EXB} = \frac{(RB_p)^2}{B}\frac{\partial}{\partial\psi}\left(\frac{E_r}{RB_p}\right)$$

- **Local suppression of turbulence by EXB shearing**

Pedestal	top	bottom
ω_{EXB}	Weak	Strong
$\tilde{P}$	Large	Small

- **Density profile changes the distribution of ExB shearing**

$$\omega_{EXB} \propto \frac{1}{n_0}\frac{k_r^2}{-k_\perp^2 + ik_r/L_n}\left(\frac{\varpi}{m_i} - \frac{1}{Ze}\nabla_\perp^2 P_i\right)$$

Since the saturation phase determines the total energy loss of an ELM event, more physics are necessary to correctly predict the saturation

- Thermal transport
  - Parallel thermal conductivity is demonstrated to be important in our previous work; *(Xia, 2012 IAEA talk)*
  - How about nonlocal thermal transport?
    - ✓Landau damping

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6-field two fluid model in BOUT++ framework

Vorticity

$$\frac{\partial}{\partial t}\varpi = -\left(\frac{1}{B_0} b \times \nabla_\perp \phi + V_{\parallel i} b\right)\cdot \nabla \varpi + B^2 \nabla_\parallel \left(\frac{J_\parallel}{B}\right) + 2b \times \kappa \cdot \nabla P$$
$$-\frac{1}{2\Omega_i}\left[\frac{1}{B} b \times \nabla P_i \cdot \nabla(\nabla_\perp^2 \phi) - Z_i e B b \times \nabla n_i \cdot \nabla \left(\frac{\nabla_\perp \phi}{B}\right)^2\right]$$
$$+\frac{1}{2\Omega_i}\left[\frac{1}{B} b \times \nabla \phi \cdot \nabla(\nabla_\perp^2 P_i) - \nabla_\perp^2 \left(\frac{1}{B} b \times \nabla \phi \cdot \nabla P_i\right)\right] + \mu_{\parallel i} \nabla_{\parallel 0}^2 \varpi$$

Ion density

$$\frac{\partial}{\partial t} n_i = -\left(\frac{1}{B_0} b \times \nabla_\perp \phi + V_{\parallel i} b\right)\cdot \nabla n_i - \frac{2 n_i}{B} b \times \kappa \cdot \nabla \phi - \frac{2}{Z_i e B} b \times \kappa \cdot \nabla P - n_i B \nabla_\parallel \left(\frac{V_{\parallel i}}{B}\right)$$

Parallel motion

$$\frac{\partial}{\partial t} V_{\parallel i} = -\left(\frac{1}{B_0 b} \times \nabla_\perp \phi + V_{\parallel i} b\right)\cdot \nabla V_{\parallel i} - \frac{1}{m_i n_i} b \cdot \nabla P$$

Ohm's law

$$\frac{\partial}{\partial t}\psi = -\frac{1}{B}\nabla_\parallel \phi + \frac{\eta}{\mu_0 \nabla_\perp^2 \psi} + \frac{1}{e n_e B}\nabla_\parallel P_e \frac{\eta_H}{\mu_0}\nabla_\perp^4 \psi$$

Ion temperature

$$\frac{\partial}{\partial t} T_i = -\frac{2}{3} T_i \left[\left(\frac{2}{B} b \times \kappa\right)\cdot\left(\nabla \phi + \frac{1}{Z_i e n_i}\nabla P_i + \frac{5 k_B}{2 Z_i e}\nabla T_i\right) + B \nabla_\parallel \left(\frac{V_{\parallel i}}{B}\right)\right]$$
$$-\left(\frac{1}{B_0} b \times \nabla_\perp \phi + V_{\parallel i} b\right)\cdot \nabla T_i - \frac{2}{3 n_i k_B}\nabla_\parallel q_{\parallel i} + \frac{2 m_e}{m_i}\frac{Z_i}{\tau_e}(T_e - T_i)$$

Electron temperature

$$\frac{\partial}{\partial t} T_e = -\frac{2}{3} T_e \left[\left(\frac{2}{B} b \times \kappa\right)\cdot\left(\nabla \phi + \frac{1}{Z_i e n_i}\nabla P_i + \frac{5 k_B}{2 Z_i e}\nabla T_e\right) + B \nabla_\parallel \left(\frac{V_{\parallel e}}{B}\right)\right]$$
$$-\left(\frac{1}{B_0} b \times \nabla_\perp \phi + V_{\parallel i} b\right)\cdot \nabla T_e - \frac{2}{3 n_e k_B}\nabla_\parallel q_{\parallel e} - \frac{2 m_e}{m_i}\frac{Z_i}{\tau_e}(T_e - T_i)$$

Landau damping has larger stabilizing effect on P-B modes than flux limited expression

- **Flux limited thermal conductivity**

$$q_{\parallel i} = -\kappa_{\parallel i} \nabla_{\parallel} k_B T_i$$
$$q_{\parallel e} = -\kappa_{\parallel e} \nabla_{\parallel} k_B T_e$$

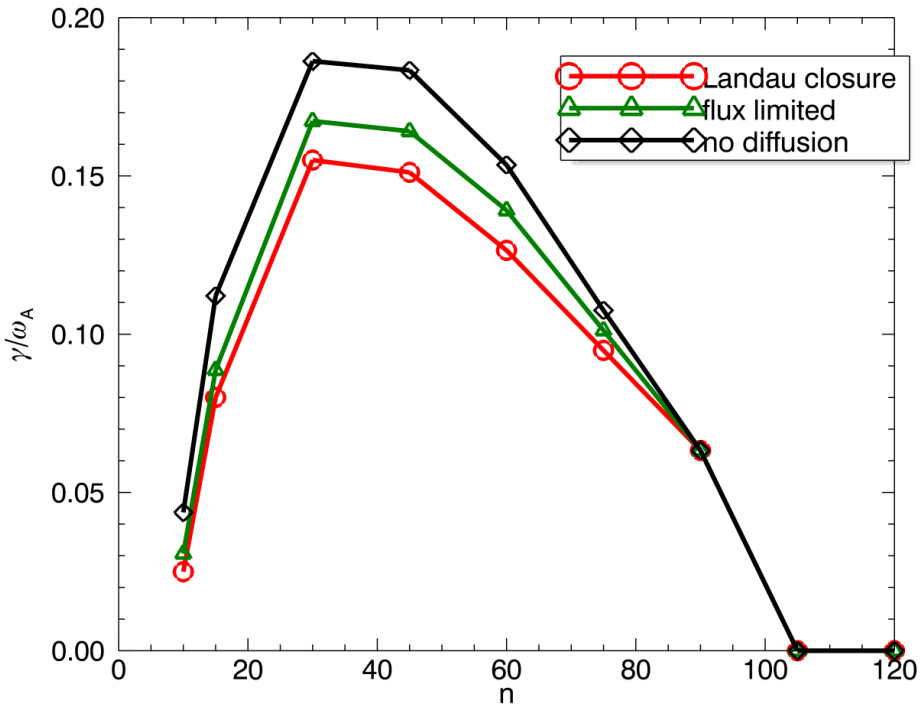
Where

$$\kappa_{\parallel i} = \left(\kappa_{\parallel i}^{SH\,-1} + \kappa_{\parallel i}^{FS\,-1}\right)^{-1}$$
$$\kappa_{\parallel e} = \left(\kappa_{\parallel e}^{SH\,-1} + \kappa_{\parallel e}^{FS\,-1}\right)^{-1}$$
$$\kappa_{\parallel i}^{SH} = 3.9 n_i v_{th,i}^2 / \nu_i$$
$$\kappa_{\parallel e}^{SH} = 3.2 n_e v_{th,e}^2 / \nu_e$$
$$\kappa_{\parallel j}^{FS} = n_j v_{th,j} q R_0$$

- **Landau damping closure ***

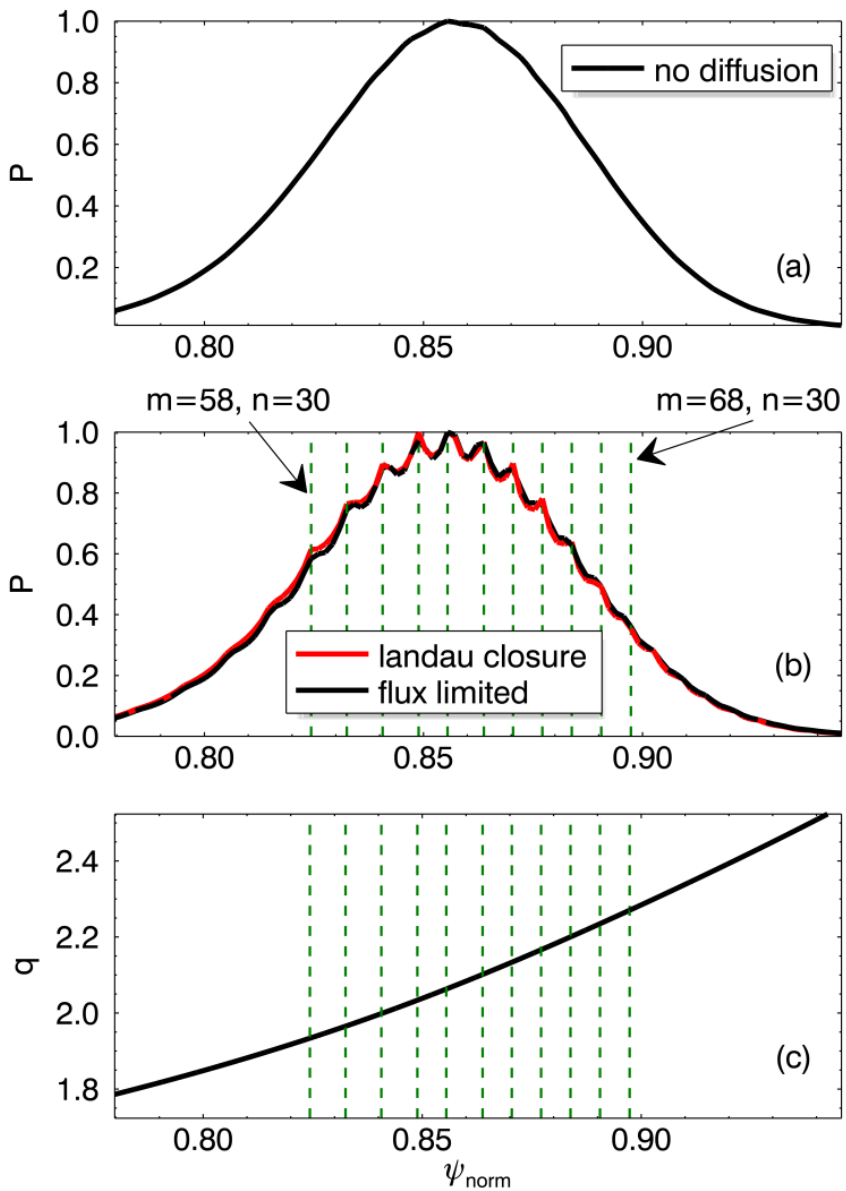
$$\tilde{q}_{\parallel,\alpha} = -n_0 \sqrt{\frac{8}{\pi}} v_{t\parallel} \frac{i k_{\parallel} k_B \tilde{T}_{\alpha}}{|k_{\parallel}|}, \alpha = i, e$$

*



- ✓ Parallel thermal conductivity has stabilizing effect on peeling-ballooning modes;
- ✓ Landau damping shows stronger stabilizing effect;

Landau damping and flux limited heat flux has no damping effect on rational surface due to $k_{\parallel} = 0$



- Radial mode structure:
  - Without parallel diffusion: smooth;
  - With Landau damping or flux limited heat flux: peaked at rational surfaces.

	Rational surface	Non-rational surface
Instability	Strong	Weak
Parallel damping	Weak	Strong

✓ The mismatch between instability and parallel damping reduces the efficiency of parallel damping stabilization on peeling-ballooning modes.

Summary

- In the presence of large density gradient, ion diamagnetic effect alone is not sufficient to represent FLR stabilizing on ballooning mode;
- With gyroviscosity, gyrofluid and twofluid show good consistence on linear instability;
- No saturation phase in isothermal limit with large density gradient;
- Landau damping can reduce linear growth rate and change radial mode structure;

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https://bout2013.llnl.gov/