Six-field two-fluid ELM simulations with Landau-Fluid closures

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Outline

1. Introduction
2. Physics Model
3. Simulation Result
   1) Effect of different parallel heat flux closures
   2) $L_n/L_t$ scan
   3) Effect of thermal force
4. Summary
Background

• H-mode
  ➢ Better power confinement for plasmas
  ➢ Edge transport barrier and pedestal region

• ELMs
  ➢ Periodic MHD events at H-mode pedestal;
  ➢ Damage to PFC;
  ➢ Affect confinement;

• Peeling-ballooning model
  ➢ Driven by combination of high pressure gradient and current
  ➢ Different linear instabilities, different types of ELMs.

P.B. Snyder, et.al Nucl. Fusion 47 (2007) 961
Landau fluid model can fill the gap between hot and cold boundary plasma.

**Core region**
- Temperature: high
- Collision: weak
- Model: gyro-kinetic

**Boundary region**
- Temperature: low
- Collision: strong
- Model: two fluid

**Landau fluid model**
- Two fluid model for ideal peeling-balloonning mode
- Using non-local transport closure to simulate kinetic effect on hot, collisionless region

**Landau closure**

**Two fluid model**
6-field includes the effect of thermal conductivity and temperature profile

<table>
<thead>
<tr>
<th>Model</th>
<th>Variables</th>
<th>Physics</th>
</tr>
</thead>
<tbody>
<tr>
<td>3-field</td>
<td>$\omega, A_\parallel, P$</td>
<td>Peeling-ballooning model</td>
</tr>
<tr>
<td>6-field</td>
<td>$\omega, A_\parallel, n_i, V_{\parallel i}, T_i, T_e$</td>
<td>+Thermal conductivity</td>
</tr>
</tbody>
</table>

- **3-field:**
  - Only peeling-ballooning model

- **6-field:**
  - **Thermal conductivity**
    - Landau closure: collisionless wave-particle resonances
    - Flux limited heat flux: collisional transport and flux streaming
  - **Effect of temperature profile** $\eta_i = L_n/L_T$ scan
6-field two fluid model with parallel heat flux

**Vorticity:**

\[
\frac{\partial \mathbf{\omega}}{\partial t} = -\left( \frac{1}{B_0} b \times \nabla \phi + V_{\parallel} b \right) \cdot \nabla \mathbf{\omega} + B^2 \nabla \left( \frac{J_{\parallel}}{B} \right) + 2b \times \kappa \cdot \nabla P
\]

\[
- \frac{1}{2\Omega_i} \left[ \frac{1}{B} b \times \nabla P \cdot \nabla \left( \nabla^2 \phi \right) - Z_i e B b \times \nabla n_i \cdot \nabla \left( \nabla^2 \phi \right)^2 \right]
\]

\[
+ \frac{1}{2\Omega_i} \left[ \frac{1}{B} b \times \nabla \phi \cdot \nabla \left( \nabla^2 P \right) - \nabla^2 \left( \frac{1}{B} b \times \nabla \phi \cdot \nabla P \right) \right] + \mu_i \nabla^2 \mathbf{\omega}
\]

**Ion density:**

\[
\frac{\partial n_i}{\partial t} = -\left( \frac{1}{B_0} b \times \nabla \phi + V_{\parallel} b \right) \cdot \nabla n_i - \frac{2n_i}{B} b \times \kappa \cdot \nabla \phi - \frac{2}{Z_i e B} b \times \kappa \cdot \nabla P - n_i B \nabla \left( \frac{V_{\parallel}}{B} \right)
\]

**Ion parallel velocity:**

\[
\frac{\partial V_{\parallel}}{\partial t} = -\left( \frac{1}{B_0} b \times \nabla \phi \right) \cdot \nabla n_i - \frac{1}{m_i n_i} b \cdot \nabla P
\]

**Ohm’s law:**

\[
\frac{\partial A_{\parallel}}{\partial t} = -\nabla \phi \cdot \nabla_{\perp} A_{\parallel} + \frac{\mu_0}{en_e} \nabla_{\parallel} P_e + \frac{\alpha e k_B}{\mu_0} \nabla_{\parallel} T_e - \frac{\eta_H}{\mu_0} \nabla^4 A_{\parallel}
\]

**Ion temperature:**

\[
\frac{\partial T_i}{\partial t} = -\frac{2}{3} T_i \left[ \left( \frac{2}{B} b \times \kappa \right) \cdot \nabla \phi + \nabla \cdot \frac{1}{Z_i e n_i} \nabla P_i + \frac{5k_B}{2Z_i e} \nabla T_i \right] + B \nabla \left( \frac{V_{\parallel}}{B} \right)
\]

\[
-\left( \frac{1}{B_0} b \times \nabla \phi + V_{\parallel} b \right) \cdot \nabla T_i - \frac{2}{3n_i k_B} \nabla_{\parallel} q_i + \frac{2m_e Z_i}{m_i \tau_e} (T_e - T_i)
\]

**Electron temperature:**

\[
\frac{\partial T_e}{\partial t} = -\frac{2}{3} T_e \left[ \left( \frac{2}{B} b \times \kappa \right) \cdot \nabla \phi + \nabla \cdot \frac{1}{Z_i e n_i} \nabla P_e + \frac{5k_B}{2Z_i e} \nabla T_e \right] + B \nabla \left( \frac{V_{\parallel}}{B} \right)
\]

\[
-\left( \frac{1}{B_0} b \times \nabla \phi + V_{\parallel} b \right) \cdot \nabla T_e - \frac{2}{3n_i k_B} \nabla_{\parallel} q_e - \frac{2m_e Z_i}{m_i \tau_e} (T_e - T_i)
\]

**Thermal force:**

\[

**Parallel heat flux:**
Different balance pressure profile

- Simulations are based on the shifted circular cross-section toroidal equilibria (cbm18_den6) generated by the TOQ code*. 

Classical thermal conductivities and Landau damping closure

Landau damping closure

\[ q_{\parallel i} = -n_0 \sqrt{\frac{8}{\pi}} \frac{\nu_{T_{\parallel i}}}{k_{\parallel}} \frac{ik_{\parallel} k_B T_i}{|k_{\parallel}|} \]

\[ q_{\parallel e} = -n_0 \sqrt{\frac{8}{\pi}} \frac{\nu_{T_{\parallel e}}}{k_{\parallel}} \frac{ik_{\parallel} k_B T_e}{|k_{\parallel}|} \]

✓ Non-local thermal transport

Classical thermal conductivities

\[ q_{\parallel i} = -\kappa_{\parallel i} \nabla_{\parallel} k_B T_i \]

\[ q_{\parallel e} = -\kappa_{\parallel e} \nabla_{\parallel} k_B T_e \]

Where

\[ \kappa_{\parallel i} = \left( \kappa_{\parallel i}^{SH-1} + \kappa_{\parallel i}^{FS-1} \right)^{-1} \]

\[ \kappa_{\parallel e} = \left( \kappa_{\parallel e}^{SH-1} + \kappa_{\parallel e}^{FS-1} \right)^{-1} \]

\[ \kappa_{\parallel i} = 3.9n_i \nu_{th,i}^2 / \nu_i \]

\[ \kappa_{\parallel e} = 3.2n_e \nu_{th,e}^2 / \nu_e \]

\[ \kappa_{FS} = n_j \nu_{th,j} q R_0 \]

✓ Classical heat flux

Spitzer-Harm

Flux streaming
Landau closure has more damping effect on linear growth rate, but not very strong

- Both Landau closure and flux limited thermal conductivity has stabilizing effect on peeling-ballooning modes;
- Landau closure has stronger stabilizing effect;
- Thermal conductivity doesn’t change the unstable island of modes.

Why the stabilizing effect from local/nonlocal parallel thermal conductive is not that strong?
For ideal ballooning mode, dispersion relation is
\[
\omega \left( \omega - i \chi_{||} k_{||}^2 \right) + \gamma_i^2 = 0
\]
We get growth rate
\[
\gamma = \frac{1}{2} \left( -\chi_{||} k_{||}^2 + \sqrt{\chi_{||}^2 k_{||}^4 + 4 \gamma_i^2} \right)
\]
Parallel conductivity has a stabilizing effect on peeling ballooning mode.
Parallel conductivity should have no effect on rational surface which \( k_{||} = 0 \) \( \Rightarrow \) Radial structure
Our simulations show consistent radial structure with theoretic expectation!

- **Radial mode structure**
  - Without parallel diffusion: smooth;
  - With Landau damping for flux limited thermal conductivity: peaked at rational surface.

<table>
<thead>
<tr>
<th></th>
<th>Rational surface</th>
<th>Irrational surface</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Instability</strong></td>
<td>Strong</td>
<td>Weak</td>
</tr>
<tr>
<td><strong>Parallel damping</strong></td>
<td>Weak</td>
<td>Strong</td>
</tr>
</tbody>
</table>

- The mismatch between instability and parallel diffusion reduces the damping effect on peeling ballooning modes.
Landau damping leads to smaller ELM size in nonlinear simulations than flux limited expression.

- Nonlinear result is similar as linear result;
- Landau damping closure has more damping effect on the turbulence transport phase of ELM crash;
- ELM size with Landau closure is smaller than ELM size with flux limited heat flux;
For larger $L_n/L_t=4$, ELMs become saturated faster.

- Elm size soon becomes saturated for larger $L_n/L_t$;
- With Landau closure, we get smaller ELM size.
Landau closure has more damping effect on temperature disturbance than pressure.

- Pressure disturbance is similar.
- Temperature disturbance with Landau closure is small than the case with flux limited heat flux.
Effect on rational surface during linear phase

- Electron temperature contour of nonlinear run
- Parallel heat flux has no effect on rational surface
- Trace of rational surface disappear in the turbulence state
Poloidal temperature oscillating during turbulence transport phase

Landau closure makes poloidal temperature profile more flat
Mode structure on Cylindrical coordinates

Before crash

T=60

After crash

T=70

Turbulence state

T=500

Turbulence state

T=1000
Different $L_n/L_t$

- Keep the same pressure profile, change density and temperature;
- $L_n/L_t$ scan:

<table>
<thead>
<tr>
<th>$L_n/L_t$</th>
<th>Height</th>
<th>tail</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>0.800</td>
<td>0.000</td>
</tr>
<tr>
<td>2</td>
<td>0.400</td>
<td>0.200</td>
</tr>
<tr>
<td>4</td>
<td>0.240</td>
<td>0.280</td>
</tr>
<tr>
<td>6</td>
<td>0.171</td>
<td>0.316</td>
</tr>
<tr>
<td>8</td>
<td>0.133</td>
<td>0.333</td>
</tr>
<tr>
<td>10</td>
<td>0.109</td>
<td>0.345</td>
</tr>
</tbody>
</table>
Damping effect is strong when $\frac{L_n}{L_T}$ is large

- Keep pressure profile and local density and temperature profile fixed, change $\frac{L_n}{L_T}$
- $\frac{L_n}{L_T}$ has small effect on peeling ballooning mode
- Parallel conductivity has small effect when $\frac{L_n}{L_T}$ is small
$L_n/L_T$ has little effect on growth rate spectrum

- $L_n/L_T$ has small effect on the growth rate and spectrum;
- Small effect from diamagnetic term because of different density gradient profile.
Thermal force, when coupled with parallel heat flux, can destabilizes modes

- With Landau closure or flux limited diffusion: Thermal force has an unstable effect on modes;
- Without parallel diffusion: Thermal force has small effect on the linear growth rate of modes.
Summary

• Parallel conductivity term has stabilize effect on peeling-ballooning mode and can reduce elm size;
• Landau closure has more damping effect on the linear growth rate of peeling-ballooning mode;
• With Landau closure, temperature disturbance is small on the turbulence transport phase of elm crash.