Multi-field two-fluid and gyro-fluid simulations of Edge-Localized-Modes

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Principal Results

Part one: two-fluid model
(1) Fundamental model: 3-field 2-fluid model is a good enough model for P-B instability and ELM crashes.

(2) Multi-field 2-fluid models are necessary to describe heat transport.

(3) BOUT++ simulations show that bright stripes from visible camera on EAST match ELM filamentary structures.

Part two: gyro-fluid model
(1) First order FLR corrections from “gyro-viscous cancellation” in two-fluid model are necessary to agree with gyro-fluid results for high ion temperature.

(2) Higher ion temperature introduces more FLR stabilizing effects, thus reduces ELM size.
BOUT++ code for modeling tokamak edge ELMs and turbulence*

- Framework for writing fluid / plasma simulations in complex tokamak geometry
  - Proximity of open+closed flux surface
  - Presence of X-point
- Written in C++, started from BOUT code, jointly developed by LLNL, Univ. York, other U.S. and international partners
- Well benchmarked with ELITE, GATO and other codes

• Dudson, Umansky, Xu et al., Comp. Phys. Comm. V.180 (2009) 1467.
Multi-field two-fluid Peeling-Ballooning modes simulation with BOUT++

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Three-field ($\mathcal{W}$, $P$, $A_{||}$): peeling-ballooning model.

Four-field ($\mathcal{W}$, $P$, $A_{||}$, $V_{||}$): include sound waves.

Five-field ($\mathcal{W}$, $n_i$, $T_i$, $T_e$, $A_{||}$): parallel thermal conductivities

Six-field ($\mathcal{W}$, $n_i$, $T_i$, $T_e$, $A_{||}$, $V_{||}$): combine all the models together, based on Braginskii equations, the density, momentum and energy of ions and electrons are described in drift ordering[1].

ELM crash in BOUT++ simulations

- ELM size
- Density $n_i (10^{19} \text{ m}^{-3})$
- Normalized Time ($\tau_A$)
- Major radius $R (\text{ m})$
- Pressure Perturbation $\tilde{P}$
- Vertical $Z (\text{ m})$
- Electron Temperature perturbation $\tilde{T}_e (\text{ keV})$
Five-field model: low density leads to larger stabilizing effects by diamagnetic drifts, and density gradient drives larger ELM size.

3-field:

\[ \frac{\partial P}{\partial t} + \mathbf{V}_E \cdot \nabla P = 0 \] (previous model)

5-field

\[ \frac{\partial n_i}{\partial t} + \mathbf{V}_E \cdot \nabla n_i = 0, \]

\[ \frac{\partial T_j}{\partial t} + \mathbf{V}_E \cdot \nabla T_j = \nabla \left( \kappa \| \nabla \| T_j \right) \]

Density quantity affects linear growth rate through \( \omega_* \propto 1/n_i \)

With fixed pressure profile, ion density gradient leads to larger ELM size.
Thermal conductivities suppress the energy transport at inner boundary

- Thermal conductivities with flux limited expressions suppress the increase of ELM:

\[ \kappa_{i} = 3.9 \frac{v_{th,i}^2}{v_i}, \quad \kappa_{e} = 3.2 \frac{v_{th,e}^2}{v_e}, \quad \kappa_{fl,j} = v_{th,j} q_{95} R_0 \]

Flux limited expression:

\[ \kappa_{ij}^e = \left( \frac{1}{\kappa_{ij}} + \frac{1}{\kappa_{fl,j}} \right)^{-1} \]

Electron temperature is most sensitive to \( \kappa_{ij} \).
3-field 2-fluid model is good enough to simulate P-B stability and ELM crashes, additional physics from multi-field contributes less than 25% corrections.

- **Fundamental physics in ELMs:**
  - Peeling-Ballooning instability
  - Ion diamagnetic stabilization → kinetic effect
  - Resistivity and hyper-resistivity → reconnection

- **Additional physics:**
  - Ion acoustic waves
  - Thermal conductivities
  - Hall effect
  - Compressibility
  - Electron-ion friction

Change the linear growth rate less than 25%

Power depositions on PFCs. Turbulence and transport
BOUT++ simulations show that the stripes from visible camera match ELM filamentary structures.

EAST#41019@3034ms
Visible camera shows bright ELM structure

BOUT++ simulation shows that the ELM stripe are filamentary structures

$\text{BOUT++ simulations show that the stripes from visible camera match ELM filamentary structures.}$

$\text{EAST#41019@3034ms}$
$\text{Visible camera shows bright ELM structure}$$^5$

$\text{BOUT++ simulation shows that the ELM stripe are filamentary structures}^*$$

$\text{Pitch match!}$
$\text{Mode number match!}$

$\text{Taken by J. H. Yang}$
$\text{Figure by W.H. Meyer}$
EAST experiments verified BOUT++ predictions that low-n modes become dominant at high $I_p$.

**BOUT++:**
- Higher current has lower n mode

**Experiment:**
- 400kA $\rightarrow$ n=25
- 300kA $\rightarrow$ n=30

Qualitatively consist

Z.X. Liu et al, EX/p7-11, “Study of ELMy H-mode plasmas and BOUT++ simulation on EAST”
Theory and Gyro-fluid Simulations of Edge-Localized-Modes

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We derived an isothermal electromagnetic 3-field gyro-fluid model with vorticity formulation generalized from Snyder-Hammett gyro-fluid model [1] for edge plasmas.

Utilizing the Padé approximation for the modified Bessel functions, this set of gyro-fluid equations is implemented in the BOUT++ framework with full ion FLR effects,

In long-wavelength limit, this set of gyro-fluid equations is reduced to previous 3-field two-fluid model with additional gyro-viscous terms resulting from the incomplete “gyro-viscous cancellation” in two-fluid model given by Xu et al [2].

Higher ion temperature introduces more FLR stabilizing effects, thus reduces ELM size.

In the presence of large density gradient, gyro-fluid and two-fluid model show qualitative difference when $k_\perp \rho_i$ is large.

$$\frac{d\omega_G}{dt} + \mathbf{V}_E \cdot \nabla \omega_{G0} - eB(\mathbf{V}_{\Phi T} - \mathbf{V}_{ET}) \cdot \nabla n_{iG} = B \nabla || J || + 2b_0 \times \kappa \cdot \nabla \tilde{P}_G + \mu_{i,\parallel} \partial_{||0}^2 \omega_G$$

$$\frac{d\tilde{P}_G}{dt} + \mathbf{V}_E \cdot \nabla P_{G0} + T_0(\mathbf{V}_{\Phi T} - \mathbf{V}_{ET}) \cdot \nabla n_{iG} = 0$$

$$\frac{\partial A||}{\partial t} + \partial_{\|\Phi T} = \frac{\eta}{\mu_0} \nabla^2 A|| - \frac{\eta_H}{\mu_0} \nabla^4 A||$$

$$\omega_G = eB(\Gamma_0^{1/2} \tilde{n}_{iG} - n_0(1 - \Gamma_0) \frac{e\phi}{T_0} + \frac{e\rho_i^2}{T_0} \nabla n_0 \cdot \nabla ((\Gamma_0 - \Gamma_1)\phi) - \tilde{n}_{iG})$$

Consider the large density gradient at H-mode pedestal:

- **Two-fluid model:** no stabilizing on high-$n$ modes,
- **Gyro-fluid model:** strong FLR stabilizing on high-$n$ modes.

Simple ion diamagnetic effect in two-fluid model is not sufficient to represent FLR stabilizing if density gradient is large!
Gyroviscous terms are necessary to stabilize Ion-Density-Gradient modes, which appear in two-fluid model

- Two-fluid dispersion relation:

\[
\gamma = \frac{1}{\sqrt{C}} \left( \sqrt{\gamma^2 - \frac{\omega_{*i}^2}{4}} \cos \frac{\alpha}{2} + \frac{\omega_{*i}}{2} \sin \frac{\alpha}{2} \right)
\]

\[ C^2 = 1 + \frac{k_x^2}{k_{*i}^2 L_n^2}, \quad \cos \alpha = \frac{1}{C}, \quad \sin \alpha = \frac{1}{C} \frac{k_x}{k_{*i}^2 L_n} \]

- At long wavelength limit, gyro-fluid goes back to two-fluid but with additional gyroviscous terms

\[
\frac{d\vec{\omega}_G}{dt} + \vec{V}_E \cdot \nabla \vec{\omega}_G - eB(\vec{V}_{\phi T} - \vec{V}_{ET}) \cdot \nabla n_{iG} = \]

\[
\frac{d\vec{\omega}}{dt} + \frac{1}{2\omega_{ci}} \left\{ \nabla_{\perp}^2 [\phi, P] - [\nabla_{\perp}^2 \phi, P] - [\phi, \nabla_{\perp}^2 P] \right\}
\]
Higher ion temperature introduces more FLR stabilizing effects, thus reduces ELM size.

- Hyper-resistivity is necessary to ELM crash, but ELM size is weakly sensitive to hyper-resistivity;
- With fixed pressure profile, high ion temperature introduce stronger FLR effect and thus leads to smaller ELM size.

**Definition of ELM size:**

\[
\Delta_{\text{ELM}}^H = \frac{\Delta W_{\text{ped}}}{W_{\text{ped}}} = \frac{\langle \int_{R_{\text{in}}}^{R_{\text{out}}} dR \sigma (P_0 - \langle P \rangle \xi) \rangle_t}{\int_{R_{\text{in}}}^{R_{\text{out}}} dR \sigma P_0}.
\]

![Graph showing the relationship between ELM size and ion temperature](image)
Accurate non-Fourier methods for Landau-fluid operators

Tokamak edge:

- kinetic effects important -> need Landau-fluid (LF) operators

\[ \gamma \propto -v_{\text{char}} |k| \]

- Large spatial inhomogeneities & complicated boundary
  - need non-Fourier implementation
  - Useful accurate approximation:

\[ \left| \frac{1}{k} \right| \approx \sum_{n=0}^{N} \frac{\alpha^n k_0}{k^2 + (\alpha^n k_0)^2} \]

- The new method has favorable Fourier-like computational scaling

\( \checkmark \) The error is less than 1.5%.
Principal Results

**Part one: two-fluid model**

(1) Fundamental model: 3-field 2-fluid model is a good enough model for P-B stability and ELM crashes.

(2) High-n P-B mode is strongly stabilized at low density by diamagnetic drifts at low temperature.

(3) BOUT++ simulations show that bright stripes from visible camera on EAST match ELM filamentary structures.

**Part two: gyro-fluid model**

(1) First order FLR corrections from “gyro-viscous cancellation” in two-fluid model are necessary to agree with gyro-fluid results for high ion temperature.

(2) Higher ion temperature introduces more FLR stabilizing effects, thus reduces ELM size.