Numerical Simulation of Profile Evolution during SMBI at HL-2A with BOUT++

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Outline

• Motivation
• Physical Model
• Numerical Results
  → Transport During SMBI
  → SMBI at Low and High Plasmas
  → Comparisons Between SMBI and GP
  → Comparisons Between Simulation and Experiment
• Conclusions and Future Plans
SMBI Fuelling for Fusion

- SMBI (Supersonic molecular beam injection) is a simple and efficient fuelling source by comparing with GP (gas puffing) and Pellet injection fueling methods.
  - Results in a high density, directed particle source, with fuelling efficiencies often in excess of 30%.
- SMBI is an useful tool for density control to
  - maintain plasmas within stable operational boundaries
  - actively manage deleterious ELMs
- SMBI is also an useful tool for:
  - Confinement improvement, such as L-H transition
  - Perturbative transport study, i.e., nonlocal transport
BOUT++ has been originally developed for modeling tokamak edge turbulence* and is extending to include neutrals for 3D plasma transport

- BOUT++ is an unique code to simulate boundary plasma turbulence in a complex geometry
  - Proximity of open+closed flux surface
  - Presence of X-point
- BOUT++ is being to include neutrals for 3D plasma transport in a new trans-neut module
  - B2, SOLPS and UEDGE are 2D plasma transport codes
  - Added BOUT++ capabilities can handle the localized 3D sources for neutrals, and late on for impurities.

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Process of Molecule Reaction

Franck-Condon
Molecules $\rightarrow$ Atoms $\rightarrow$ Ionization $\rightarrow$ Plasmas

Supersonic Molecular Beam

$H_2 \rightarrow 2H^0 \rightarrow 2H^+ + 2e$

e + $H_2 \rightarrow 2H^0 + e$

Dissociation

Charge Exchange (CX)

$H^+ + H^0 \rightarrow H^0 + H^+$

e + $H^0 \rightarrow H^+ + 2e$

At Edge plasma

\[
\langle \sigma V \rangle_{\text{diss}} \approx \langle \sigma V \rangle_{\text{I}} < \langle \sigma V \rangle_{\text{CX}}
\]

\[
\langle \hat{\sigma}_I \hat{V}_{\text{th,e}} \rangle \approx \langle \hat{\sigma}_\text{diss} \hat{V}_{\text{th,e}} \rangle = 3 \times 10^{-8} \left(0.1 T_e \text{eV}\right)^2 \left[3 + \left(0.1 T_e \text{eV}\right)^2\right] \text{cm}^3 / \text{s}
\]

\[
\langle \sigma_{\text{CX}} V_{\text{th,i}} \rangle = 1.7 \times 10^{-8} + 1.9 \times 10^{-8} \left(1.5 T_i \text{eV}\right)^{1/3} - (15 \text{eV})^{1/3} \left(150 T_i \text{eV}\right)^{1/3} - (15 \text{eV})^{1/3} \text{cm}^3 / \text{s}
\]

Figure 1.25. The rate coefficients for atomic and molecular hydrogen [1,23]. The numbered reactions are (1): $e + H_2 \rightarrow H^+_2 + 2e$, (2): $e + H_2 \rightarrow 2H^0 + e$, (3): $e + H_2 \rightarrow H^0 + H^+ + 2e$, (4): $e + H^+_2 \rightarrow 2H^0$, (5): $e + H^+_2 \rightarrow H^0 + H^+ + e$, (6): $e + H^0 \rightarrow H^+ + 2e$, and charge exchange (7): $H^0 + H^+ \rightarrow H^+ + H^0$. 

Peter C. Stangeby *The Plasma Boundary of Magnetic Fusion Devices*, Institute of Physics publishing, 2000
Physical Model (1) - Plasmas

\[ \frac{\partial \hat{N}_i}{\partial t} + \nabla_{\parallel} \left( \hat{V}_{\parallel} \hat{N}_i \right) = D^c_{\perp i} \nabla^2_{\perp} \hat{N}_i + \hat{S}_I^p \]

Quasi-neutral \( \hat{N}_e = \hat{N}_i \)

\[ \frac{\partial \hat{T}_e}{\partial t} = \frac{2}{3 \hat{N}_i} \nabla_{\parallel} \left( \hat{k}_{\parallel e} \nabla_{\parallel} \hat{T}_e \right) + \frac{2}{3} \hat{\chi}_{\perp e} \nabla^2_{\perp} \hat{T}_e - \hat{V}_I \left( \hat{T}_e + \frac{2}{3} \hat{W}_I \right) - \frac{2}{3} \hat{v}_{\text{diss}} \left( \hat{W}_{\text{diss}} + \hat{W}_{\text{bind}} \right) \left( \frac{2m_e}{M_i} \right) \frac{\hat{T}_e - \hat{T}_i}{\hat{t}_e} \]

\[ \frac{\partial \hat{T}_i}{\partial t} + \hat{V}_{\parallel} \nabla_{\parallel} \hat{\hat{T}}_i = \frac{2}{3 \hat{N}_i} \nabla_{\parallel} \left( \hat{k}_{\parallel i} \nabla_{\parallel} \hat{T}_i \right) - \frac{2}{3} \hat{T}_i \nabla_{\parallel} \hat{V}_i + \frac{2}{3} \hat{\chi}_{\perp i} \nabla^2_{\perp} \hat{T}_i - \hat{V}_I \hat{T}_i + \left( \frac{2m_e}{M_i} \right) \frac{\hat{T}_e - \hat{T}_i}{\hat{t}_e} \]

\[ \frac{\partial \hat{V}_{\parallel i}}{\partial t} + \hat{V}_{\parallel} \nabla_{\parallel} \hat{V}_{\parallel i} = \frac{4}{3} \frac{1}{\hat{N}_i \hat{M}_i} \nabla_{\parallel} \left( \hat{\eta}^0_{\parallel} \nabla_{\parallel} \hat{V}_{\parallel i} \right) - \frac{\nabla_{\parallel} \hat{P}}{\hat{N}_i \hat{M}_i} - \left( \hat{v}_{\text{CX}} + \hat{V}_I \right) \left( \hat{V}_{\parallel i} - \hat{V}_{\parallel a} \right) \]

\[ \hat{V}_I = \hat{N}_a \left\langle \hat{\sigma}_I \hat{V}_{th,e} \right\rangle \quad \hat{S}_I^p = \hat{N}_e \hat{V}_I \quad \hat{k}_{\parallel i} = 3.9 \hat{T}_i^{5/2} / t_0 \nu_{ii0} \]

\[ \hat{V}_{\text{CX}} = \hat{N}_a \left\langle \hat{\sigma}_{\text{CX}} \hat{V}_{th,i} \right\rangle \quad \hat{S}_{\text{CX}}^p = \hat{N}_i \hat{V}_{\text{CX}} \quad \hat{k}_{\parallel e} = 3.2 (M_i / M_e) \hat{T}_e^{5/2} / t_0 \nu_{ei0} \]

\[ \hat{V}_{\text{diss}} = \hat{N}_m \left\langle \hat{\sigma}_{\text{diss}} \hat{V}_{th,e} \right\rangle \quad \hat{\eta}^0_{\parallel i,a} = 0.96 \left( \hat{N}_{i,a} \hat{T}_{i,a} \right) / t_0 \nu_{ii0} \quad t_0 = L_0 / \nu_{th,i} \]
Physical Model (2) - Atoms

Due to ion CX, assumption: \( \hat{T}_a = \hat{T}_i \)

\[
\frac{\partial \hat{N}_a}{\partial \hat{t}} + \nabla \left( \hat{V}_a \hat{N}_a \right) = \hat{D}_\perp^c \nabla^2 \hat{N}_a - \hat{S}_I^p + 2\hat{S}_{\text{diss}}
\]

\[
\frac{\partial \hat{V}_a}{\partial \hat{t}} + \hat{V}_a \nabla \hat{V}_a = 4 \frac{1}{3 \hat{N}_a \hat{M}_a} \nabla \left( \hat{N}_a \nabla \hat{V}_a \right) - \nabla \left( \frac{\hat{P}_a}{\hat{N}_a \hat{M}_a} \right) + \hat{V}_{CX}^a (\hat{V}_i - \hat{V}_a) - \frac{2\hat{S}_{\text{diss}}}{\hat{N}_a} \hat{V}_a
\]

Where,

\[
\hat{P}_a = \hat{N}_a \hat{T}_i \\
\hat{S}_{\text{diss}} = \hat{N}_e \hat{v}_{\text{diss}} \\
\hat{v}_I^a = \hat{N}_e \left\langle \hat{\sigma}_I \hat{V}_{\text{th,e}} \right\rangle
\]

\[
\hat{D}_\perp^c = \hat{T}_i / \left( \hat{M}_a \hat{v}_{CX}^a \right) \\
\hat{v}_{\text{diss}} = \hat{N}_m \left\langle \hat{\sigma}_{\text{diss}} \hat{V}_{\text{th,e}} \right\rangle \\
\hat{v}_{CX}^a = \hat{N}_i \left\langle \hat{\sigma}_{CX} \hat{V}_{\text{th,i}} \right\rangle
\]

Flux limited conditions:

\[
\hat{D}_{a}^{\text{fl}} = \hat{V}_{\text{th,a}} \hat{L}_{a}^\text{min} \\
\hat{D}_{a}^{e} = \left( \frac{1}{\hat{D}_{a}^{\text{fl}}} + \frac{1}{\hat{D}_{a}^{c}} \right)^{-1}
\]

Strong numerical instability
Physical Model (3) - Molecules

Constant room temperature

\[ \hat{T}_m = 300[K] \approx 0.0258[eV] \]

\[ \frac{\partial \hat{N}_m}{\partial \hat{t}} + \nabla_x \left( \hat{V}_{xm} \hat{N}_m \right) = -\hat{S}_{\text{diss}} \]

\[ \frac{\partial \hat{V}_{xm}}{\partial \hat{t}} + \hat{V}_{xm} \nabla_x \hat{V}_{xm} = -\frac{\nabla_x \hat{P}_m}{\hat{N}_m \hat{M}_m} \]

Where,

\[ \hat{P}_m = \hat{N}_m \hat{T}_m \quad \hat{S}_{\text{diss}} = \hat{N}_e \hat{V}_{\text{diss}} \]

Flux limited thermal conductivities are also applied:

\[ \hat{K}^{fl}_{||j} = \hat{V}_{th, j} q_{95} \hat{L}_0 \quad \hat{K}^{e}_{||j} = \left( \frac{1}{\hat{K}^{fl}_{||j}} + \frac{1}{\hat{K}^{e}_{||j}} \right)^{-1} \quad j = e, i \]
Circular HL-2A Geometry w/ X point

**Plasma Radial B.C.**

\[ \frac{\partial \hat{N}_i}{\partial \hat{\psi}} \bigg|_{R_1} = 260 ; \quad \frac{\partial \hat{T}_{e,i}}{\partial \hat{\psi}} \bigg|_{R_1} = -14000 \]

**Private flux region B.C.**

\[ R_1 \text{ and } 0 - \theta_1, \theta_2 - 2\pi \text{ same as:} \]

\[ N_i \big|_{R_2} = 0.1N_0; \quad T_{e,i} \big|_{R_2} = 10eV \]

**Parallel Sheath B.C.**

\[ \partial_{||} N_i = 0 \]

\[ V_{||i} = c_{se} = \sqrt{(T_e + T_i)/M_i} \]

\[ q_{||se} = -\kappa_{||e} \partial_{||} T_e = \gamma_e N_i T_e c_{se} \]

\[ q_{||si} = -\kappa_{||i} \partial_{||} T_i = \gamma_i N_i T_i c_{se} \]

\[ \gamma_i \approx 2.5 \quad \gamma_e \approx 7 \]

- \( R_0 = 1.65m \)
- \( a = 0.42m \)
- \( \partial \hat{N}_i \bigg|_{R_1} = 260 \)
- \( \partial \hat{T}_{e,i} \bigg|_{R_1} = -14000 \)
- \( N_0 = 1 \times 10^{19} / m^3 \)
2D Plasmas Profiles at Steady State Before SMBI

\[ N_i[N_0] \quad N_0 = 1 \times 10^{19} / m^3 \]

\[ T_e[eV] \]

\[ T_i[eV] \]

\[ V_{||}[m/s] \]
Neutrals Propagate First Inwards then Outwards due to Competition between Fueling Rate and Ionization/Dissociation Rate during SMBI Plotted at Outside Mid Plane
Plasma Profiles Variation During Neutrals Inwards and Outwards Propagation Plotted at Outside Mid Plane

During SMBI, density increases while temperature decreases.
Parallel Plasma Transports Due to Parallel Ion Velocity and Thermal Conductivities Plotted at R=2.04m

\[ N_i [N_0] \quad N_0 = 1 \times 10^{19} / m^3 \]

\[ T_e [eV] \]

\[ T_i [eV] \]

\[ V_{||i} [m/s] \]

\[ \kappa_{||e} [N_0 m^2 / s] \]

\[ \kappa_{||i} [N_0 m^2 / s] \]

SMBI creates poloidal locality
Poloidal Propagation of Plasma Density $N_i$ During SMBI

$N_i [N_0]$ $N_0 = 1 \times 10^{19} / m^3$ $N_i [N_0]$

SMBI creates poloidal density blobs
2D Profiles of Plasmas and Neutrals Right after SMBI

\[ N_i[N_0] \]
\[ N_0 = 1 \times 10^{19} / m^3 \]

\[ T_e[eV] \]

\[ T_i[eV] \]

\[ R = 2.03m \]
separatrix

\[ R = 2.03m \]
separatrix

\[ V_{li}[m / s] \]

\[ R[m] \]
\[ \theta[rad] \]
Plasmas Profiles Relaxing after SMBI to Reach Steady States Determined by Boundary Conditions Plotted about 10ms after SMBI

Ni needs much more time to get steady state

After SMBI plasma density increases by about 100%
Neutrals Penetration Depths Influenced by Low and High Plasma Density and Temperature Profiles Plotted at Outside Mid Plane

\[ N_m [N_0] \quad N_0 = 1 \times 10^{19} / m^3 \quad N_a [N_0] \]

- Low Ni Low Tei
- High Ni Low Tei
- Low Ni High Tei
- Low Ni Low Tei
- High Ni Low Tei
- Low Ni High Tei

SMBI penetrates deeper in colder plasmas
Plasma Profiles Evolution during SMBI at Different Low and High Initial Plasma Density and Temperature Plotted at Outside Mid Plane

\[ N_i [N_0] \quad N_0 = 1 \times 10^{19} / m^3 \]

\[ T_e [eV] \]

<table>
<thead>
<tr>
<th>Low Ni</th>
<th>Low Tei</th>
</tr>
</thead>
<tbody>
<tr>
<td>High Ni</td>
<td>High Tei</td>
</tr>
</tbody>
</table>

Separatrix
Neutrals Penetrate Deeper Using SMBI than GP

\[ N_m [N_0] \quad N_0 = 1 \times 10^{19} / m^3 \quad N_a [N_0] \]

\[ \begin{align*}
0.0 & \quad 0.2 & \quad 0.4 & \quad 0.6 & \quad 0.8 & \quad 1.0 \\
0.0 & \quad 0.5 & \quad 1.0 & \quad 1.5 & \quad 2.0
\end{align*} \]

\[ \begin{align*}
t[ms] \quad 1.94 & \quad 1.96 & \quad 2.00 & \quad 2.02 & \quad 2.04 & \quad 2.06 \\
1.94 & \quad 1.96 & \quad 2.00 & \quad 2.02 & \quad 2.04 & \quad 2.06
\end{align*} \]

SMBI

GP

edge localized

separatrix
Plasma Density Increases and Temperature Decreases More during SMBI than GP Plotted at Outside Mid Plane

$N_i [N_0] \quad N_0 = 1 \times 10^{19} / m^3$

$T_e [eV]$

$T_i [eV]$

$R [m]$

SMBI provides more efficient fueling source
Total Injected Particles within 2ms SMBI in Simulation is about the Same Order as that in Experiment

Simulation

\[ V_{m0} = 500m/s \quad N_{m0} = 1 \times 10^{19} / m^3 \]

\[ a = 0.42m \quad w_\theta \approx 0.16m \]

\[ S_{injc} = 2\pi Rw_\theta = 2.08m^2 \]

\[ N_{total} = V_{m0} N_{m0} S_{injc} t_{injc} \]

\[ \approx 2.08 \times 10^{19} \]

Experiment

Yu DL et al NF 2012 52 082001
Line Averaged Plasmas Density and Temperatures Qualitatively Consist with Experiments

\[ R = 195 \text{cm} \quad N_0 = 1 \times 10^{19} / \text{m}^3 \quad Z = -3.5 \text{cm} \]

Yao L H et al NF, 2001, 41 817

Yu DL et al NF 2012, 52 082001
Conclusions and Future Plans

• A **eight-field** physical model of fueling has been **developed** to study 3D neutrals and plasmas transport and during SMBI (or GP);

• A new BOUT++ **module trans_neut** code has also been developed and the simulations of SMBI in a **real HL-2A geometry** with X point has been done;

• In colder plasmas, the neutrals **penetrate** into the separatrix during SMBI while neutrals are **difficult** to penetrate into the separatrix during GP;

• However, in hotter plasmas, neutral penetration depth decreases during SMBI due to the increase of dissociation and ionization rates;

• Line averaged plasma **density increases** and **temperatures decrease** have been observed during SMBI which are consistent with the experiments;

• Easily turn on the toroidal direction simulation in trans_neut code to **study 3D neutrals and plasmas transport**;

• More **Physics of SMBI** fueling will be studied, such as **fueling efficiency, penetration depth, ELM mitigation** etc.