LLNL-PRES-597235

ETG turbulence simulation of tokamak edge plasmas via 3+1 gyrofluid code

BOUT++
Boundary Plasma Turbulence Code

P.W.Xi[1,2], X.Q.Xu[2] , A.Dimits[2], M.Umansky[2], I.Joseph[2], S.S.Kim[3], P. H.Diamond[3]
[1]Fusion Simulation Center, School of Physics, Peking University, Beijing, China
[2]Lawrence Livermore National Laboratory, Livermore, CA 94550, USA
[3]WCI Center for Fusion Theory, NFRI, Korea

54th Annual Meeting of the APS-DPP

29/Oct-2/Nov. 2012, Providence, Rhode Island

This work was performed under the auspices of the U.S. DoE by LLNL under Contract DE-AC52-07NA27344 and is supported by the China NSF under Contract No.10721505, the National Magnetic Confinement Fusion Science Program of China under Contracts No. 2011GB107001, and also supported by the China Scholarship Committee under contract N0.2011601099.

Motivation: self-consistent model for hyper-resistivity based on MHD-ETG interaction

- Hyper-resistivity is demonstrated to play a crucial role in the ELM dynamics

$$\frac{\partial A_{\parallel}}{\partial t} + \partial_{\parallel}\phi_T = \frac{\eta}{\mu_0}\nabla_{\perp}^2 A_{\parallel} \boxed{- \frac{\eta_H}{\mu_0}\nabla_{\perp}^4 A_{\parallel}} \rightarrow$$

Numerical: **Remove current sheet**
Physics: **Hyper-resistive ballooning mode**
Reconnection

  - Simple estimation for hyper-resistivity is used in present BOUT++ ELM simulations → **not sufficient**
  - Classical: electron viscosity $\eta_H \propto \mu_e \sim \rho_e^2 \nu_{ei}$ → too small
  - A Self-consistent model for hyper-resistivity is necessary to determine the linear instability at H-mode pedestal and ELM size

- A hyper-resistivity model is proposed based on the MHD-ETG interaction theory*

*C. J. McDevitt and P. H. Diamond, *Phys. Plasmas* **13**, 032302(2006)

Reduced MHD equation included ETG turbulence

- **Ohm's law with electron inertial**

$$\frac{m_e}{e}\frac{d\tilde{u}_{\|e}}{dt}+\frac{m_e}{e}\tilde{\mathbf{V}}\cdot\nabla u_{\|e0}-\frac{\partial\tilde{A}_{\|}}{\partial t}=\partial_{\|}\tilde{\phi}-\frac{1}{n_0 e}\partial_{\|}\tilde{P}_e-\frac{\eta}{\mu_0}\nabla_{\perp}^2\tilde{A}_{\|}$$

$$\tilde{f}=f_1+\delta f$$

MHD perturbation
Slow time variation

ETG perturbation
Fast time variation

- **Time average on MHD time scale**

$$\frac{\partial A_{\|1}}{\partial t}+\partial_{\|}\phi_1-\frac{1}{n_0 e}\partial_{\|}\tilde{P}_{e1}=\frac{\eta}{\mu_0}\nabla_{\perp}^2 A_{\|1}-\frac{m_e}{e^2 n_0}\mathbf{V}_1\cdot\nabla J_{\|0}+\frac{m_e}{e}\frac{\partial\tilde{u}_{\|e1}}{\partial t}+\frac{m_e}{e}\left[\phi_1,\tilde{u}_{\|e1}\right]$$

$$+\left\langle\left[\delta A_{\|},\delta\phi\right]\right\rangle+\frac{1}{n_0 e}\left\langle\left[\delta A_{\|},\delta P_e\right]\right\rangle+\boxed{\frac{m_e}{e}\left\langle\left[\delta\phi,\delta\tilde{u}_{\|e}\right]\right\rangle}$$

Anomalous electron parallel momentum transport due to ETG turbulence

- **Modulation of MHD perturbation on ETG turbulence**
  - ➢ Profile modification
  - ➢ EXB shearing
  - ➢ Field line bending (EM ETG)

$$T_1, n_1 \Rightarrow |\delta\phi|^2$$
$$\phi_1 \Rightarrow |\delta\phi|^2$$
$$A_{\|1} \Rightarrow |\delta\phi|^2$$

Hyper-resistivity

$\boldsymbol{\eta_H}$ →Relation between $A_{\|1}$ and anomalous electron parallel momentum transport

A. Sen, R. Singh, D. Chandra, P. Kaw and D. Raju, *Nucl. Fusion* **49** (2009) 115012

3+1 gyrofluid model for ITG/ETG

$$\frac{d\tilde{n}}{dt}+\nabla_{\|}n_0\tilde{u}_{\|}+\frac{n_0}{2T_0}\left[\hat{\nabla}_{\perp}^2\mathbf{V}_{\Phi}\right]\cdot\nabla\tilde{T}_{\perp}-n_0\left(1+\frac{\eta_i}{2}\hat{\nabla}_{\perp}^2\right)i\omega_{*i}\frac{e\Phi}{T_0}+n_0\left(2+\frac{1}{2}\eta_i\hat{\nabla}_{\perp}^2\right)i\omega_d\frac{e\Phi}{T_0}+\frac{1}{T_0}i\omega_d(\tilde{p}_{\|}+\tilde{p}_{\perp})=0$$

$$\frac{d\tilde{u}_{\|}}{dt}+\frac{e}{m_i}\partial_{\|}\Phi+\frac{\nabla_{\|}\tilde{p}_{\|}}{m_in_0}-\frac{1}{2m_i}\left[\hat{\nabla}_{\perp}^2\mathbf{V}_{\bar{A}}\right]\cdot\nabla\tilde{T}_{\perp}+\frac{e}{m_i}\frac{\partial\bar{A}_{\|}}{\partial t}+v_t^2\left(1+\eta_i+\frac{\eta_i}{2}\hat{\nabla}_{\perp}^2\right)i\omega_{*i}\frac{e\bar{A}_{\|}}{T_0}+\left(\frac{\tilde{p}_{\perp}}{m_in_0}+\frac{e}{2m_i}\hat{\nabla}_{\perp}^2\Phi\right)\nabla_{\|}\ln B+8i\omega_d\tilde{u}_{\|}=0$$

$$\frac{d\tilde{p}_{\|}}{dt}+\nabla_{\|}(\tilde{q}_{\|}+3P_0\tilde{u}_{\|})+\frac{n_0}{2}\left[\hat{\nabla}_{\perp}^2\mathbf{V}_{\Phi}\right]\cdot\nabla\tilde{T}_{\perp}+2P_0\tilde{u}_{\|}\nabla_{\|}\ln B$$

$$-n_0T_0\left(1+2\eta_i+\frac{\eta_i}{2}\hat{\nabla}_{\perp}^2\right)i\omega_{*i}\frac{e\Phi}{T_0}+n_0T_0\left(4+\frac{1}{2}\hat{\nabla}_{\perp}^2\right)i\omega_d\frac{e\Phi}{T_0}+\frac{m_i}{T_0}i\omega_d\left(\tilde{r}_{\|,\|}+\tilde{r}_{\|,\perp}\right)=0$$

$$\frac{d\tilde{p}_{\perp}}{dt}+B\nabla_{\|}\left(\frac{\tilde{q}_{\perp}+3P_0\tilde{u}_{\|}}{B}\right)+\frac{1}{2}\left[\hat{\nabla}_{\perp}^2\mathbf{V}_{\Phi}\right]\cdot\nabla\tilde{p}_{\perp}-\frac{P_0}{2}\left[\hat{\nabla}_{\perp}^2\mathbf{V}_{\bar{A}}\right]\cdot\nabla\tilde{u}_{\|}+2n_0T_0\left(4+2\hat{\nabla}_{\perp}^2+\hat{\hat{\nabla}}_{\perp}^2\right)i\omega_d\frac{e\Phi}{T_0}$$

$$-n_0T_0\left[1+\frac{1}{2}\hat{\nabla}_{\perp}^2+\eta_i\left(2+\hat{\nabla}_{\perp}^2+\hat{\hat{\nabla}}_{\perp}^2\right)\right]i\omega_{*i}\frac{e\Phi}{T_0}+\frac{m_i}{T_0}i\omega_d\left(\tilde{r}_{\|,\perp}+\tilde{r}_{\perp,\perp}\right)=0$$

- **Definitions**

$$\frac{d}{dt}=\frac{\partial}{\partial t}+\mathbf{V}_{\Phi}\cdot\nabla,\ \mathbf{V}_{\Phi}=\frac{\mathbf{b}_0\times\nabla\Phi}{B},\ \mathbf{V}_{\bar{A}}=\frac{\mathbf{b}_0\times\nabla\bar{A}}{B}=-\tilde{\mathbf{b}},\ \nabla_{\|}f=B\partial_{\|}\frac{f}{B},\ \partial_{\|}=\mathbf{b}_0\cdot\nabla+\tilde{\mathbf{b}}\cdot\nabla,$$

$$\mathrm{i}\omega_{*i}=-\frac{T_0}{eBn_0}\nabla n_0\cdot\mathbf{b}_0\times\nabla,\ \mathrm{i}\omega_d=\frac{T_0}{eB^2}\mathbf{b}_0\times\nabla B\cdot\nabla,\ \hat{\nabla}_{\perp}^2\Phi=2b\frac{\partial\Gamma_0^{1/2}}{\partial b}\phi,\ \hat{\nabla}_{\perp}^2\Phi=2b\frac{\partial\Gamma_0^{1/2}}{\partial b}\phi,\ \hat{\hat{\nabla}}_{\perp}^2\Phi=b\frac{\partial^2(b\Gamma_0^{1/2})}{\partial b^2}\phi,\ \eta_i=\frac{L_n}{L_T}$$

- For simplicity, we utilize the gyrofluid model developed by P. Snyder and G. Hammett, but for ETG simulations, the roles of electron and ion are interchanged

P. B. Snyder and G. W. Hammett, Phys. Plasmas 8, 3199 (2001).

3+1 gyrofluid model (Coun.)

- Landau closure

$$\tilde{q}_{\parallel} = -n_0\sqrt{\frac{8}{\pi}}v_{t\parallel}\frac{ik_{\parallel}\tilde{T}_{\parallel}}{|k_{\parallel}|}$$

$$\tilde{q}_{\perp} = -n_0\sqrt{\frac{2}{\pi}}v_{t\parallel}\frac{ik_{\parallel}\tilde{T}_{\perp}}{|k_{\parallel}|}$$

Landau-fluid operators is implemented based on a non-Fourier methods developed by A.Dimits, M.Umansky and I.Joseph

(GP8.00114)

- Toroidal closure

$$i\omega_d(\tilde{r}_{\parallel,\perp} + \tilde{r}_{\perp,\perp}) = i\omega_d\frac{T_0}{m}(7\tilde{p}_{\parallel} + \tilde{p}_{\perp} - 4T_0\tilde{n} - 2i\frac{|\omega_d|}{\omega_d}(\nu_1\tilde{T}_{\parallel} + v_2\tilde{T}_{\perp}))$$

$$i\omega_d(\tilde{r}_{\parallel,\parallel} + \tilde{r}_{\parallel,\perp}) = i\omega_d\frac{T_0}{m}(\tilde{p}_{\parallel} + 5\tilde{p}_{\perp} - 3T_0\tilde{n} - 2i\frac{|\omega_d|}{\omega_d}(\nu_3\tilde{T}_{\parallel} + v_4\tilde{T}_{\perp}))$$

- Quasi-neutrality

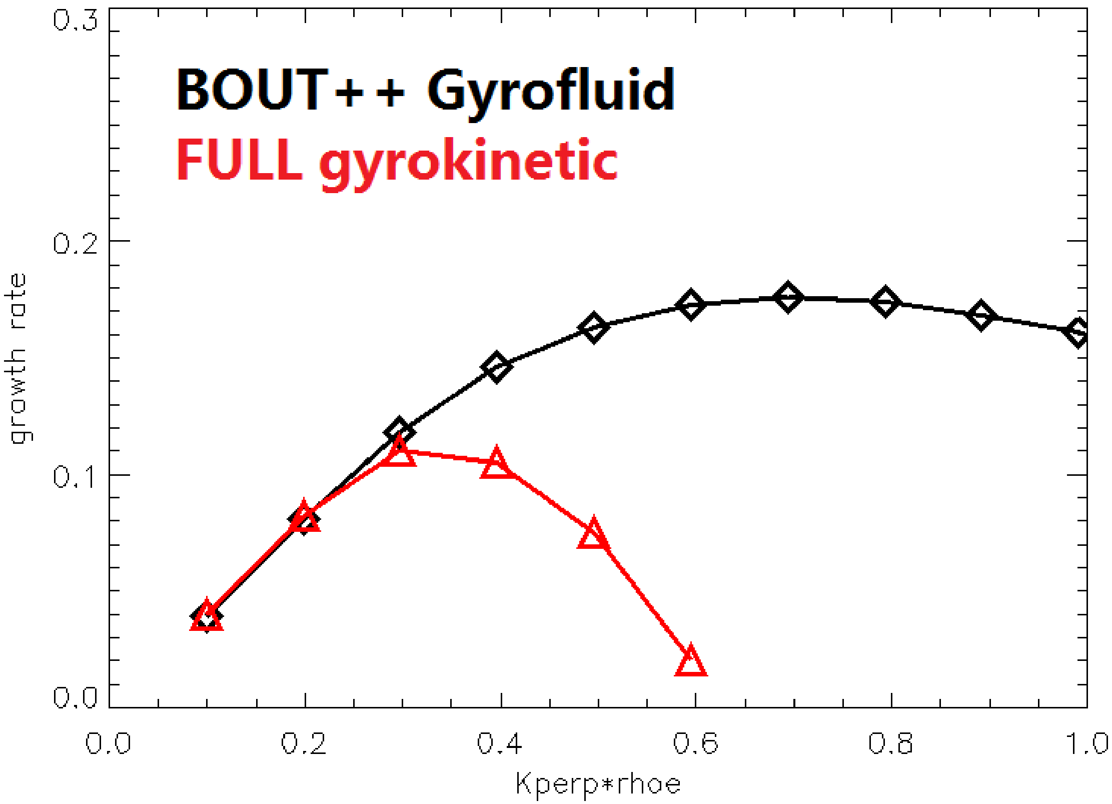
$$\tilde{n}_{\beta} = n_0\frac{e\phi}{T_0}$$

$$\bar{n}_{\alpha} - n_0(1-\Gamma_0)\frac{e\phi}{T_0} = \tilde{n}_{\beta}$$

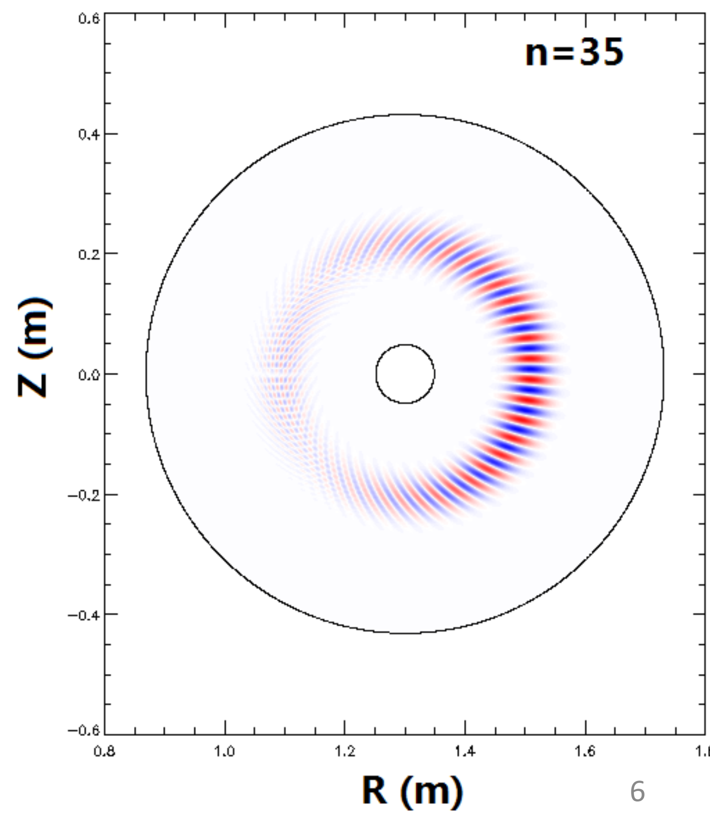
$$\bar{n}_{\alpha} = \frac{1}{1+b/2}\tilde{n}_{\alpha} - \frac{n_0 2b}{T_0(2+b)^2}\tilde{T}_{\perp}$$

where $b = k_{\perp}^2\rho_{\alpha}^2 = -\rho_{\alpha}^2\nabla_{\perp}^2$ and $k_{\perp} = -i\nabla_{\perp}$.

Benchmark: Cyclone ITG case



- To be included
  - Landau damping
  - Toroidal closure



Gyro-fluid Simulations of Edge-Localized-Modes

BOUT++
Boundary Plasma Turbulence Code

P.W.Xi[1,2], X.Q.Xu[2] , T.Y.Xia[2,3], S.S.Kim[4]

[1]Fusion Simulation Center, School of Physics, Peking University, Beijing, China

[2]Lawrence Livermore National Laboratory, Livermore, CA 94550, USA

[3]Institute of Plasma Physics, Chinese Academy of Sciences, Hefei, China

[4]WCI Center for Fusion Theory, NFRI, Korea

54th Annual Meeting of the APS-DPP

29/Oct-2/Nov. 2012, Providence, Rhode Island

This work was performed under the auspices of the U.S. DoE by LLNL under Contract DE-AC52-07NA27344 and is supported by the China NSF under Contract No.10721505, the National Magnetic Confinement Fusion Science Program of China under Contracts No. 2011GB107001, also supported by the China Scholarship Committee under contract N0.2011601099.

An isothermal electromagnetic 3-field gyro-fluid model

- We derived an isothermal electromagnetic 3-field gyro-fluid model with vorticity formulation generalized from Snyder-Hammett gyro-fluid model [1] for edge plasmas.

- Utilizing the Padé approximation for the modified Bessel functions, this set of gyro-fluid equations is implemented in the BOUT++ framework with full ion FLR effects,

- In long-wavelength limit, this set of gyro-fluid equations is reduced to previous 3-field two-fluid model with additional gyro-viscous terms resulting from the incomplete “gyro-viscous cancellation” in two-fluid model given by Xu et al [2].

- At nonlinear phase, gyrofluid sustains a strong n=0 **E**X**B** flow for much longer time than twofluid;

- Higher temperature leads to smaller ELM crash

[1] P. B. Snyder and G. W. Hammett, Phys. Plasmas 8, 3199 (2001).
[2] X. Q. Xu, R. H. Cohen, T. D. Rognlien, et.al., Phys. Plasma 7, 1951 (2000).

3-field isothermal gyrofluid model* for ELM simulation: consider the large density gradient at H-mode pedestal

$$\frac{d\varpi_G}{dt}+\mathbf{V}_E\cdot\nabla\varpi_{G0}\boxed{-eB(\mathbf{V}_{\Phi T}-\mathbf{V}_{ET})\cdot\nabla n_{iG}}=B\nabla_\parallel J_\parallel+2\mathbf{b}_0\times\boldsymbol{\kappa}\cdot\nabla\tilde{P}_G+\mu_{i,\parallel}\partial_\parallel^2\varpi_G$$

$$\frac{d\tilde{P}_G}{dt}+\mathbf{V}_E\cdot\nabla P_{G0}\boxed{+T_0(\mathbf{V}_{\Phi T}-\mathbf{V}_{ET})\cdot\nabla n_{iG}}=0$$

$$\frac{\partial A_\parallel}{\partial t}+\partial_\parallel\phi_T=\frac{\eta}{\mu_0}\nabla_\perp^2A_\parallel-\frac{\eta_H}{\mu_0}\nabla_\perp^4A_\parallel$$

$$\varpi_G=eB\left(\Gamma_0^{1/2}\tilde{n}_{iG}-n_0(1-\Gamma_0)\frac{e\phi}{T_0}+\boxed{\frac{e\rho_i^2}{T_0}\nabla n_0\cdot\nabla(\Gamma_0-\Gamma_1)\phi}-\tilde{n}_{iG}\right)$$

Padé approximation

$$\begin{cases}\Gamma_0^{1/2}\approx\dfrac{1}{1+b/2}\\ \Gamma_0\approx\dfrac{1}{1+b}\qquad,\quad b=-\rho_i^2\nabla_\perp^2\\ \Gamma_0-\Gamma_1\approx1\end{cases}$$

$$d/dt=\partial/\partial t+\mathbf{V}_{ET}\cdot\nabla,\ \mathbf{V}_{ET}=\frac{1}{B}\mathbf{b}_0\times\nabla\phi_T,\ \phi_T=\phi_0+\phi,\ \nabla_\parallel f=B\partial_\parallel\frac{f}{B},\ \partial_\parallel=\partial_\parallel^0+\delta\mathbf{b}\cdot\nabla,\ \delta\mathbf{b}=\frac{1}{B}\nabla A_\parallel\times\mathbf{b}_0,\ J_\parallel=J_{\parallel0}+\tilde{J}_\parallel,\ \tilde{J}_\parallel=-\nabla_\perp^2A_\parallel/\mu_0$$

Twofluid equations

$$\frac{d\varpi}{dt}=B\nabla_\parallel J_\parallel+2\mathbf{b}_0\times\boldsymbol{\kappa}\cdot\nabla\tilde{P}+\mu_{i,\parallel}\partial_\parallel^2\varpi$$

$$\frac{d\tilde{P}}{dt}+\mathbf{V}_E\cdot\nabla P_0=0$$

$$\frac{\partial A_\parallel}{\partial t}+\partial_\parallel\phi_T=\frac{\eta}{\mu_0}\nabla_\perp^2A_\parallel-\frac{\eta_H}{\mu_0}\nabla_\perp^4A_\parallel$$

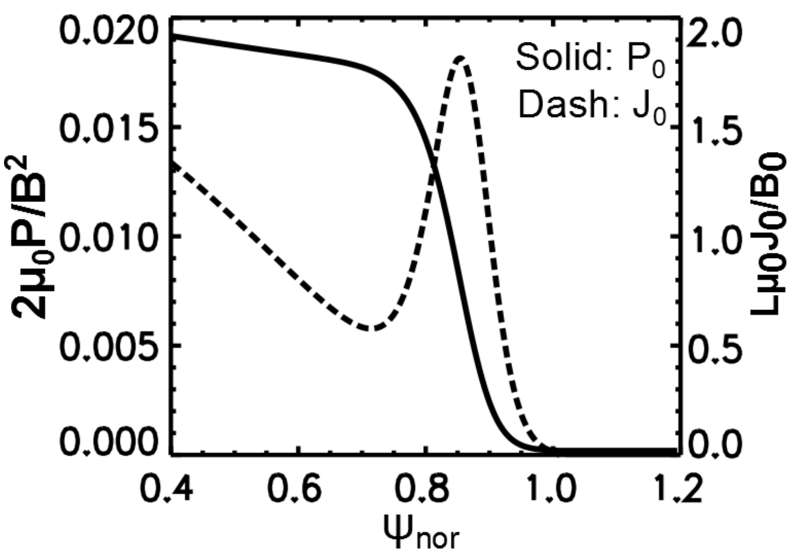
$$\varpi=\frac{m_in_0}{B}\left(\nabla_\perp^2\phi+\frac{1}{en_0}\nabla_\perp^2\tilde{P}_i+\frac{1}{n_0}\nabla n_0\cdot\nabla\phi\right)$$

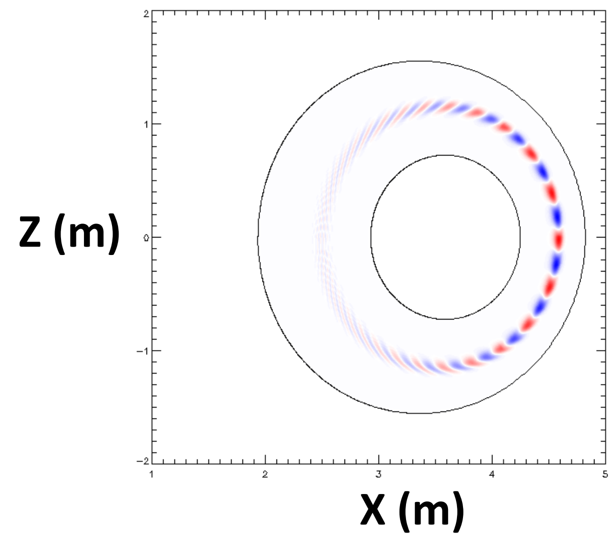
Relation between twofluid vorticity and gyrokinetic vorticity

$$\varpi\approx\varpi_G+\frac{1}{en_0}\nabla_\perp^2\tilde{P}_{iG},\ \ \rho_i^2k_\perp^2<<1$$

*) P. B. Snyder and G. W. Hammett, *Phys. Plasmas* **8**, 3199 (2001)

In the presence of large density gradient, gyro-fluid and two-fluid model show qualitative difference when $k_\perp \rho_i$ is large

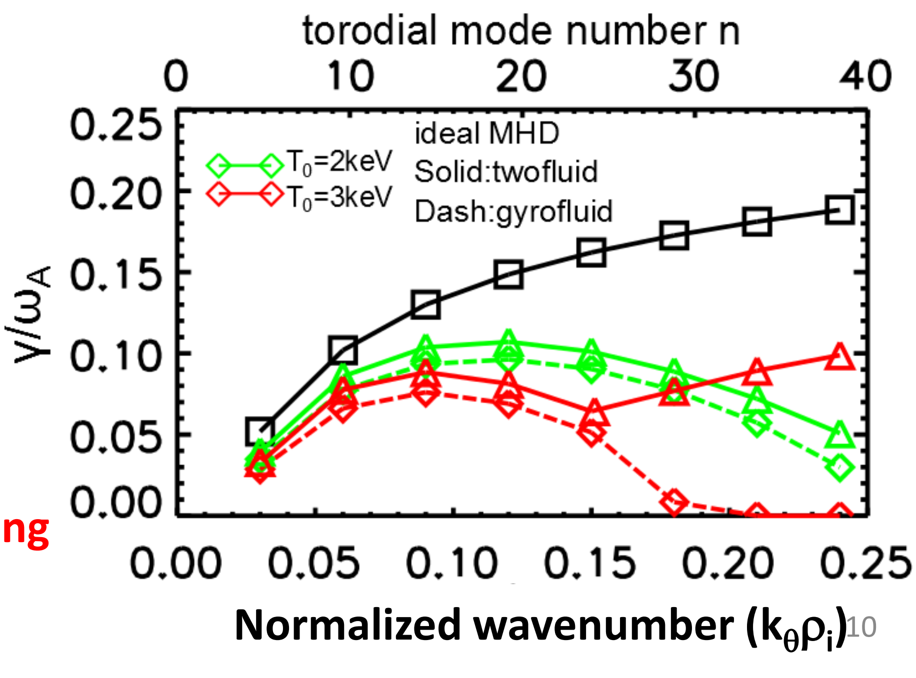




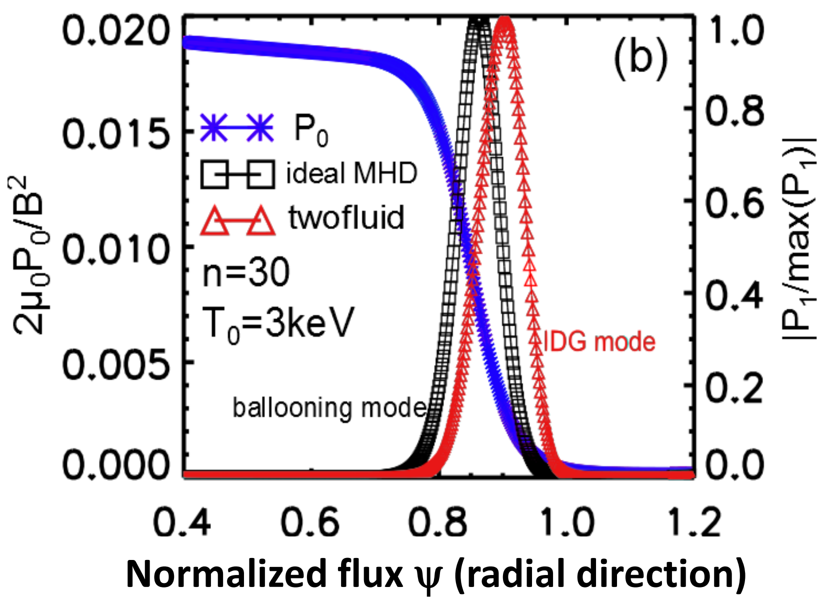
Consider the **large density gradient** at H-mode pedestal:

- Two-fluid model: **no stabilizing** of high-n modes,
- Gyro-fluid model: **strong FLR stabilizing** of high-n modes.

What causes the disappearance of stabilizing in twofluid model?



Ion-Density-Gradient mode in twofluid model



→ The instability does not localize at peak pressure gradient region

→ Not pressure gradient driven ballooning mode, but other instability

→ Lowest order ballooning equation changes

$$\frac{1}{J}\frac{\partial}{\partial\chi}\left[\frac{k_\perp^2}{B^2 J}\frac{\partial}{\partial\chi}\hat{\phi}\right] - \frac{\omega_J}{\rho_i^2 V_A}\left[\frac{i}{BJ}\frac{\partial}{\partial\chi}\hat{\phi}\right]$$

$$= -\left[\frac{k_\perp^2}{V_A^2}\omega(\omega+\omega_{*i}+\boxed{i\frac{\omega}{k_\perp^2 n_0}\frac{dn_0}{d\psi}n\chi q' g^{\psi\psi}}) + 2\frac{\omega_\kappa\omega_{*i}}{V_A^2\rho_i^2}\right]\hat{\phi}$$

Twofluid local dispersion relation

ion diamagnetic stabilizing on ballooning modes

drift instability due to ion density gradient

$$\gamma = \frac{1}{\sqrt{C}}\left(\sqrt{\gamma_I^2 - \frac{\omega_{*i}^2}{4}}\cos\frac{\alpha}{2} + \frac{\omega_{*i}}{2}\sin\frac{\alpha}{2}\right)$$

$$C^2 = 1 + \frac{k_x^2}{k_\perp^4 L_n^2},\quad \cos\alpha = \frac{1}{C},\quad \sin\alpha = \frac{1}{C}\frac{k_x}{k_\perp^2 L_n}$$

When ω_{*i} increases:

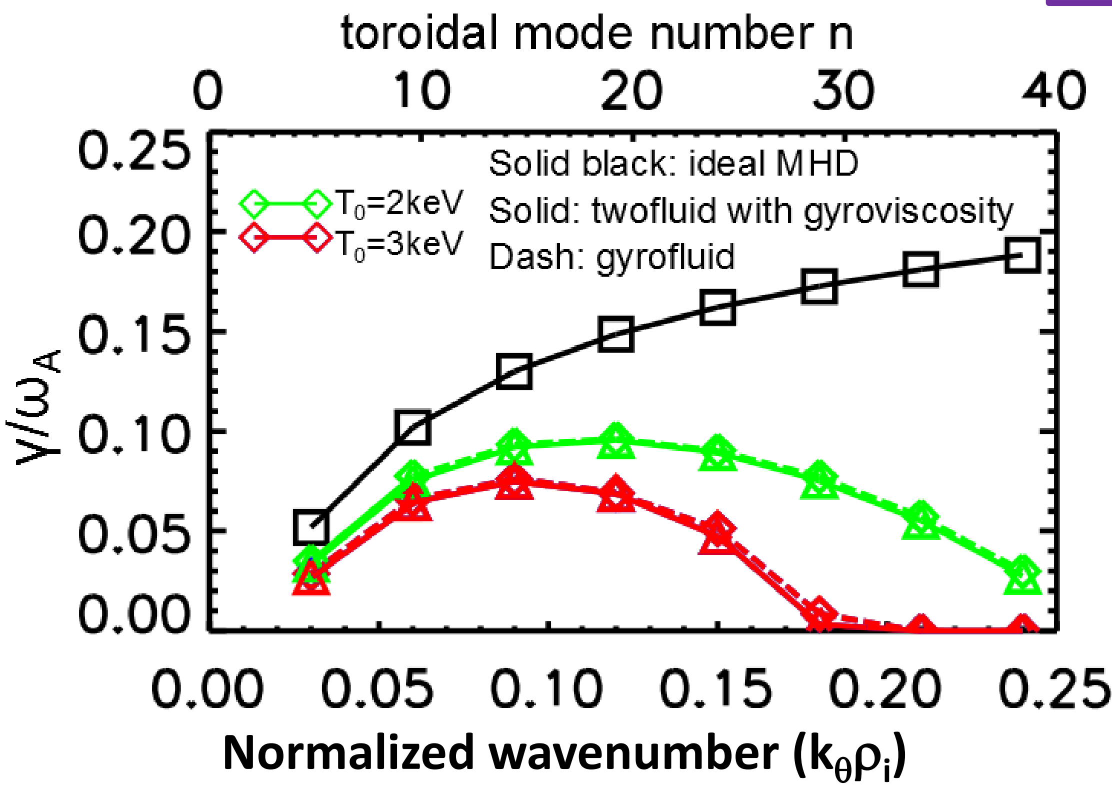
- ❑ Ion diamagnetic effect stabilizes ballooning modes → first term decreases
- ❑ Ion density gradient introduces **Ion-Density-Gradient mode**: second term become dominant

Gyroviscous terms are necessary to stabilize Ion-Density-Gradient modes and should be kept in twofluid model

Only ion diamagnetic effect in two-fluid model is not sufficient to represent FLR stabilizing if density gradient is large!

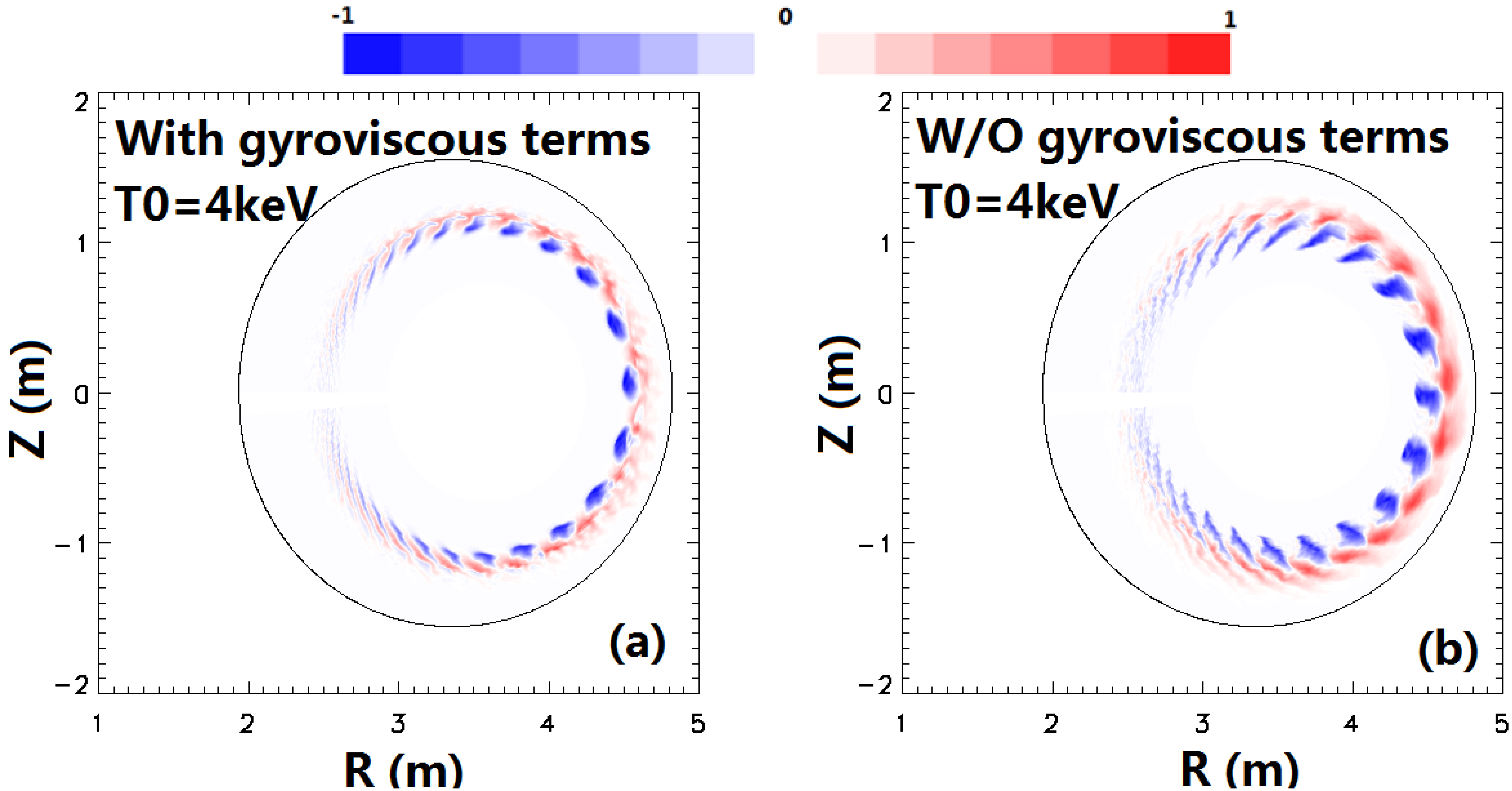
- **At long wavelength limit, gyro-fluid goes back to two-fluid but with additional gyroviscous terms**

$$\frac{d\varpi_G}{dt} + \mathbf{V}\cdot\nabla\varpi_{G0} - eB(\mathbf{V}_{\Phi T} - \mathbf{V}_{ET})\cdot\nabla n_{iG} = \frac{d\varpi}{dt} + \boxed{\frac{1}{2\omega_{ci}}\left\{\nabla_\perp^2[\phi, P_i] - [\nabla_\perp^2\phi, P_i] - [\phi, \nabla_\perp^2 P_i]\right\}}$$



- **Gyroviscous terms represent necessary FLR effect to stabilize IDG modes and should be kept in twofluid model**
- **If without gyroviscous terms, IDG mode will lead to much larger ELM crash in nonlinear phase**

Without gyroviscous terms, IDG mode leads to larger ELM crash and more energy loss at nonlinear phase

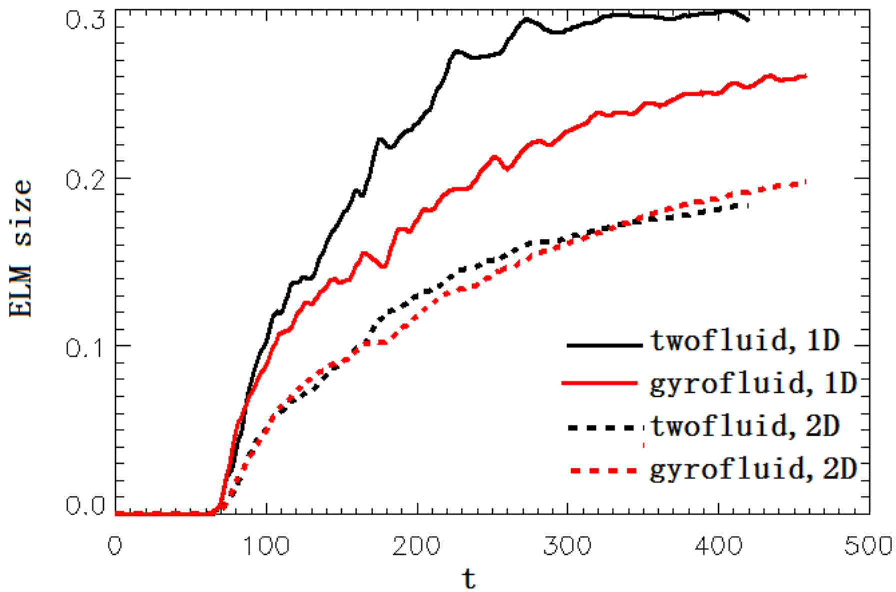


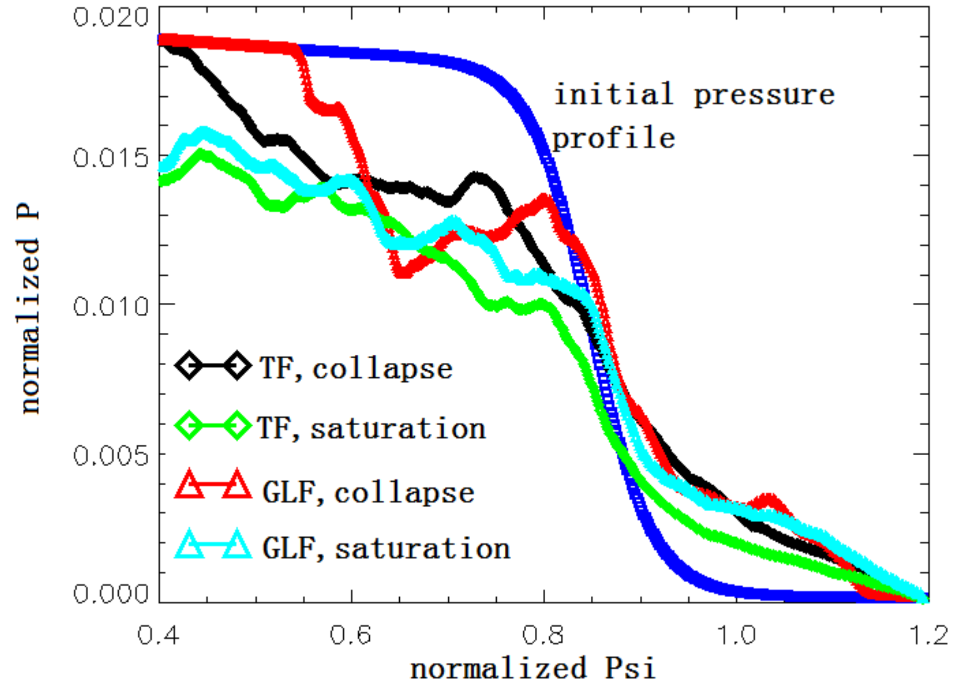
Pressure perturbation at ELM crash time

ELM size: 0.06

ELM size: 0.13

Twofluid and gyrofluid simulations show similar energy loss, but different profile evolution during ELM crash



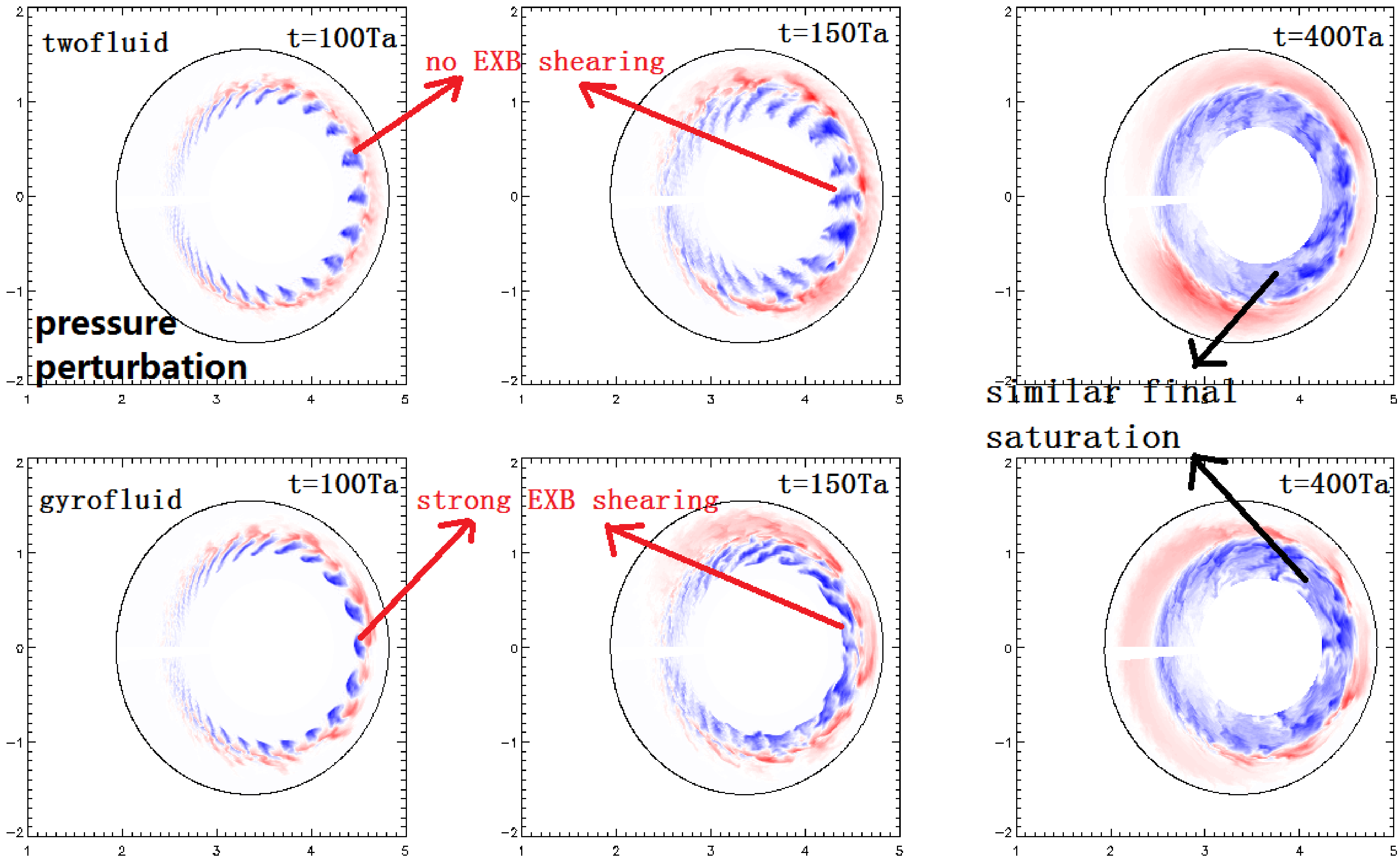


- Definition of ELM size
  - 1D: energy loss at outer mid-plane
  - 2D: energy loss from whole cross-section

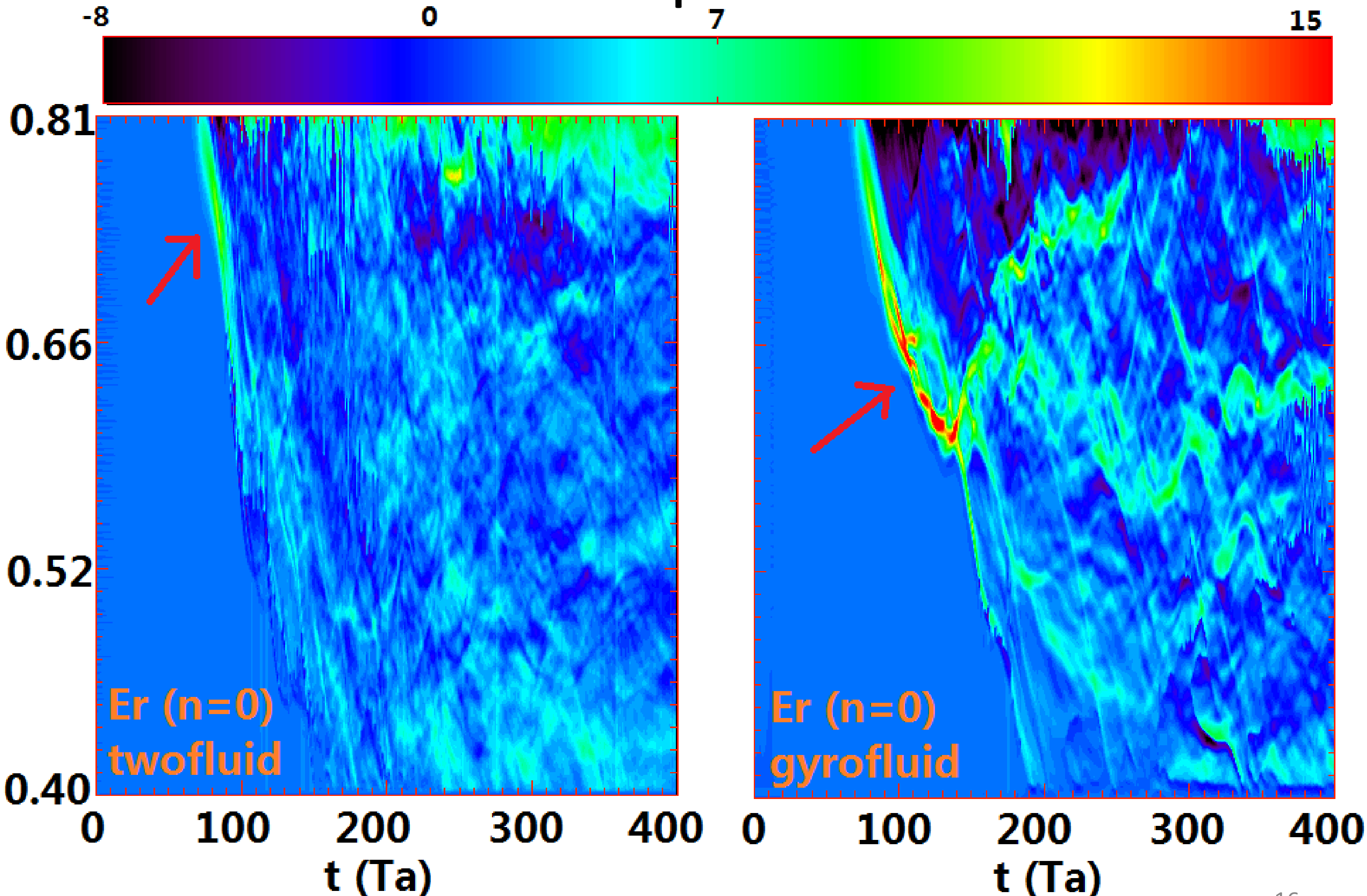
$$\Delta_{ELM}^{th} = \frac{\Delta W_{ped}}{W_{ped}} = \frac{\int_{R_{in}}^{R_{out}} \oint dR d\theta (P_0 - \langle P \rangle_\zeta)}{\int_{R_{in}}^{R_{out}} \oint dR d\theta P_0}$$

- **Gyrofluid and twofluid has similar 2D ELM size but different 1D ELM size**
  →different poloidal perturbation structure
- **Gyrofluid sustain different profile after crash, but finally also get relaxation.**

Twofluid shows no mode twist in poloidal direction, but gyrofluid shows the perturbation is strongly twisted in poloidal direction

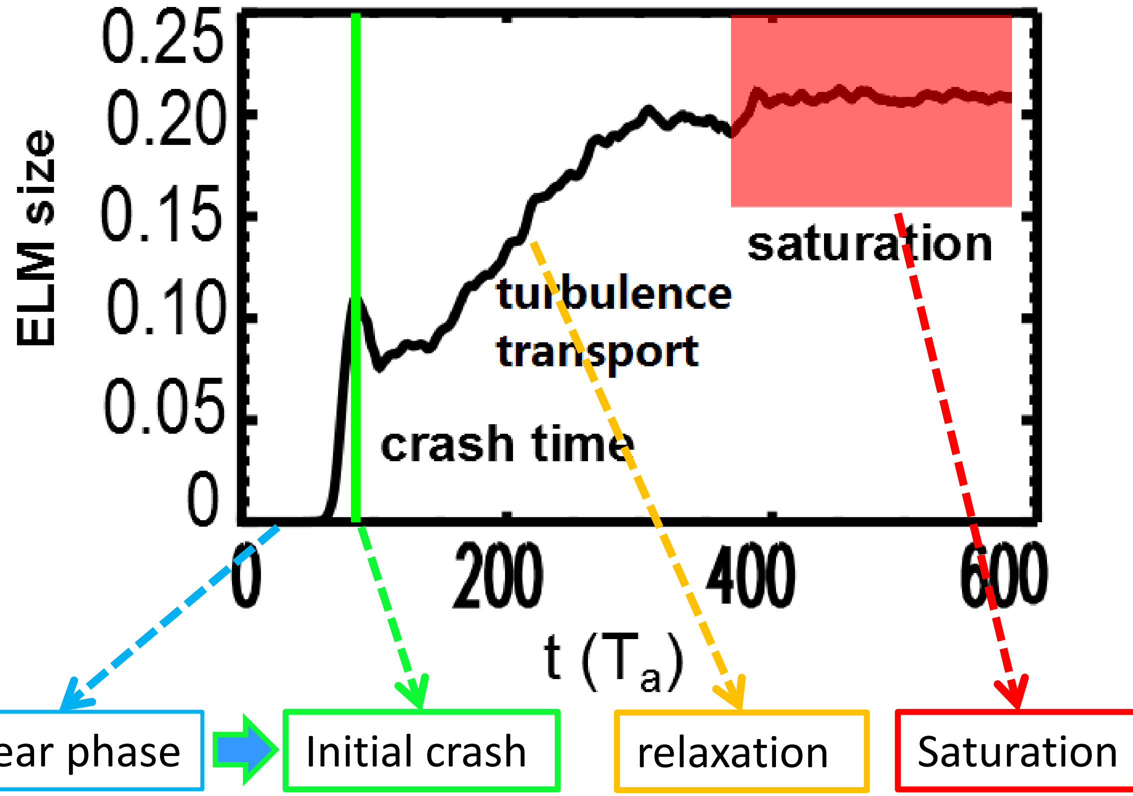


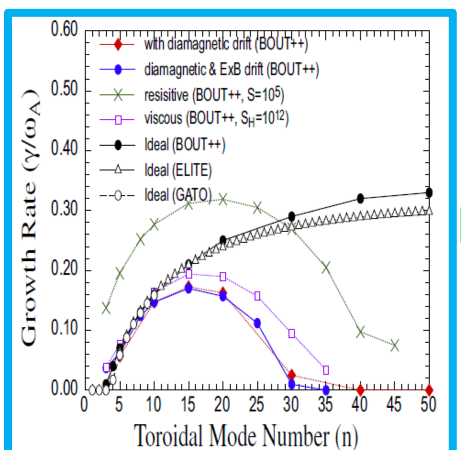
Gyrofluid sustains a strong n=0 EXB shear flow which leads to the perturbation twist, but this flow is destroyed during the further profile relaxation process

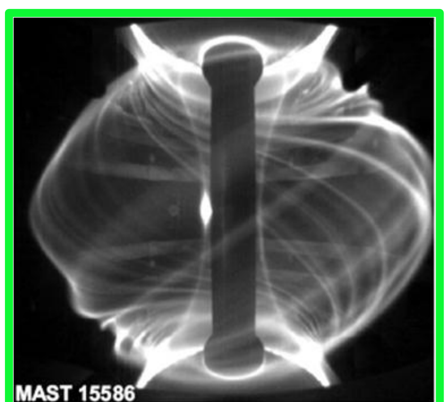


Time evolution of n=0 Er at outer mid-plane

Energy losing process during one ELM event



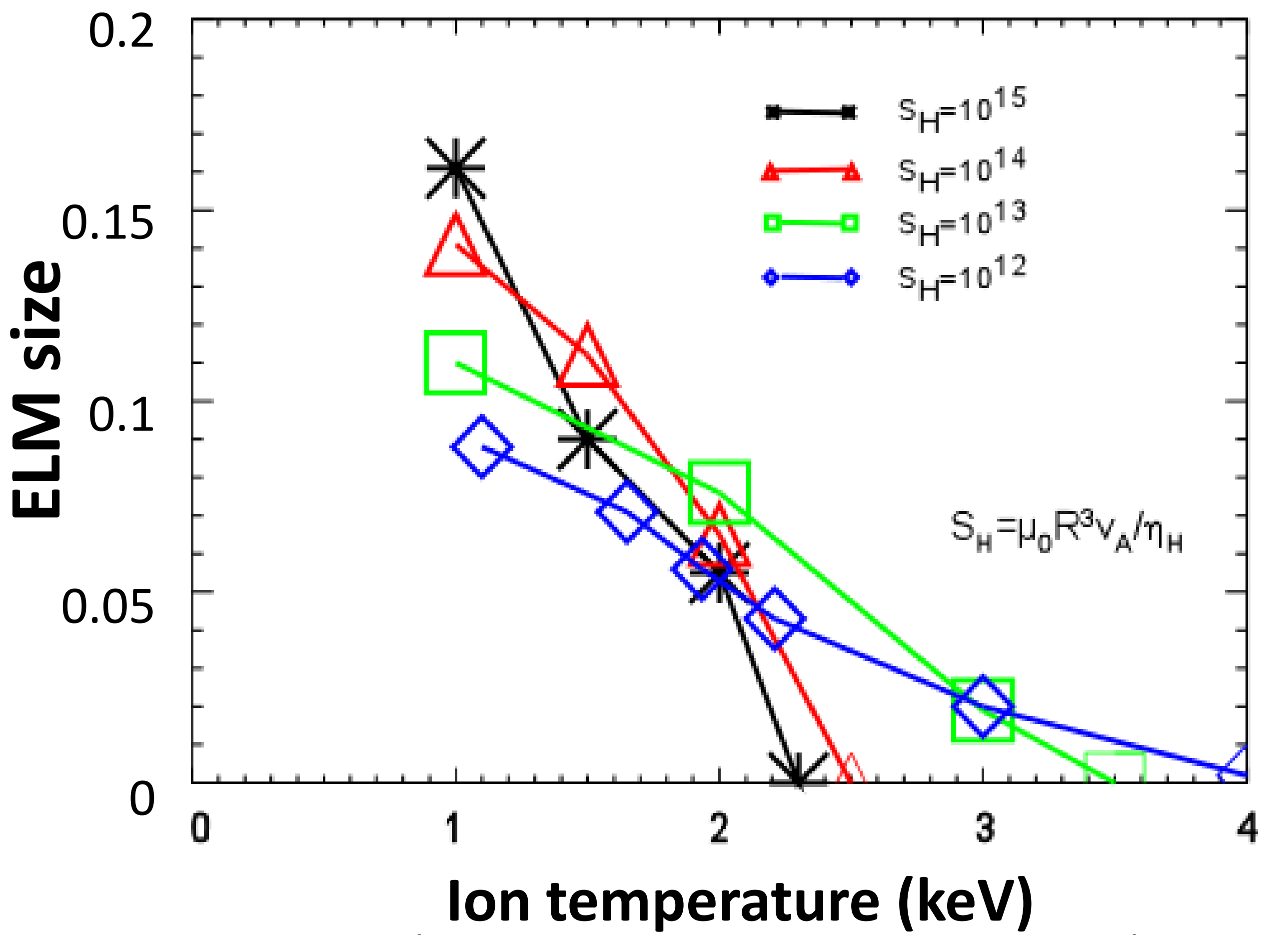




phase	characteristic
Linear phase	Growing of various linear instabilities
Initial crash	• Reconnection • Formation of filaments • Energy loss carried by filaments(used in BOUT++)
relaxation	Turbulence transport, mainly EXB convection
Saturation	Total energy loss(measured in experiments)

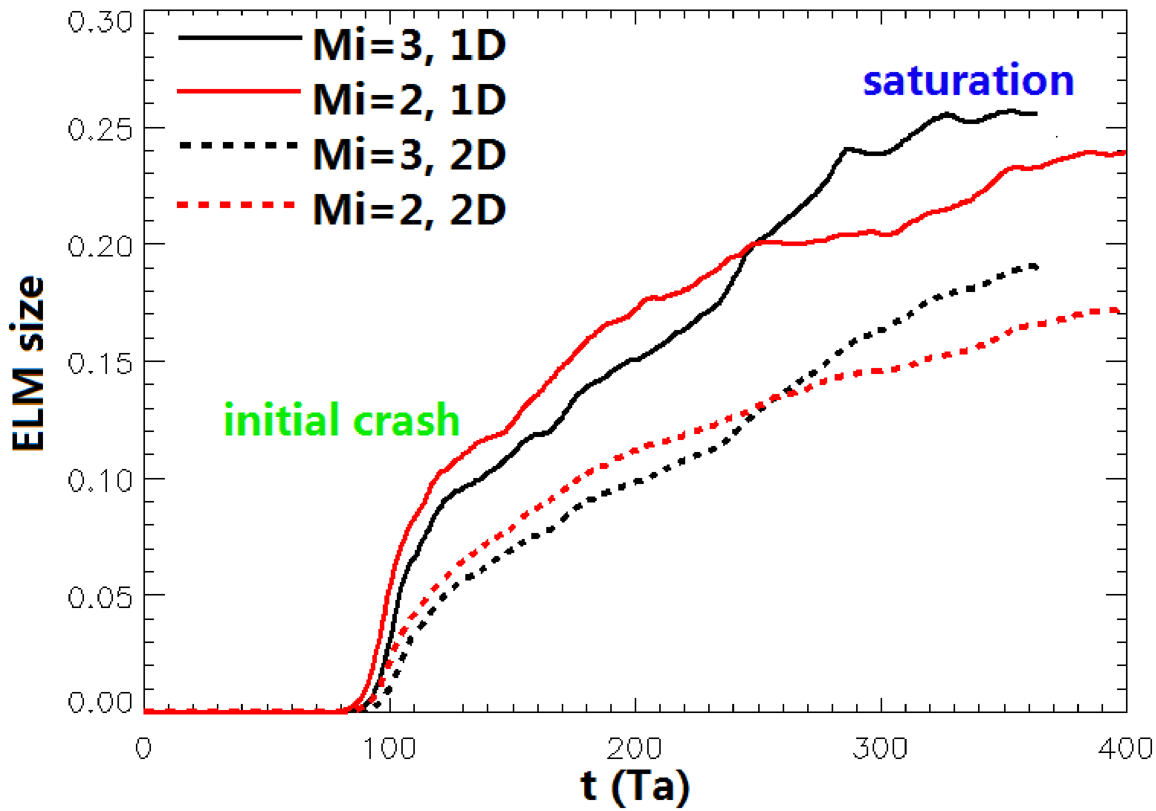
Higher ion temperature introduces more FLR stabilizing effects, thus reduces ELM size (filament part)

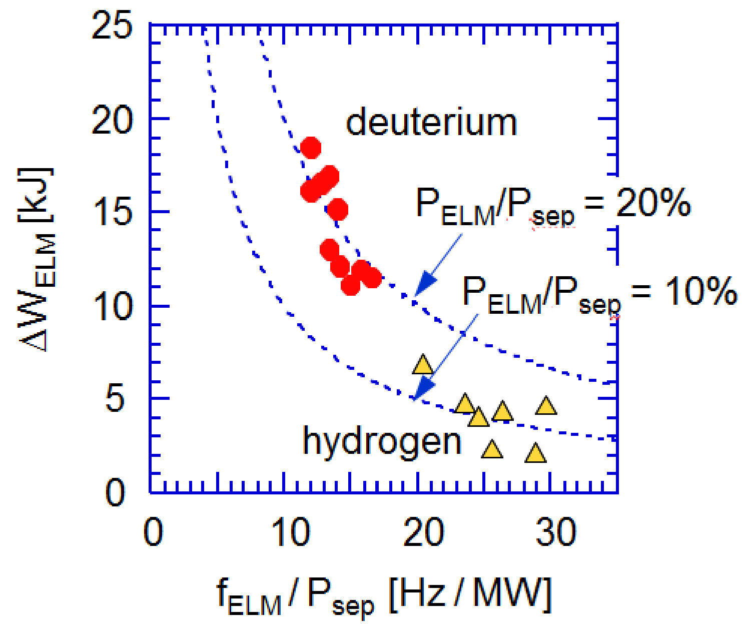
- Hyper-resistivity is necessary to ELM crash, but ELM size is weakly sensitive to hyper-resistivity;
- With fixed pressure profile, high ion temperature introduce stronger FLR effect and thus leads to smaller ELM size



(Without density gradient in vorticity)

A primary comparison about the influence of isotopic on ELM size between BOUT++ simulation and JT-60U experiment





H.Urano, et.al. 24th IAET (San Diego) talk

BOUT++ simulation → **Qualitative agreement** ← **JT-60U experiment**

- With larger isotopic mass
  - initial crash is smaller (cannot be measured in experiments)
  - Total energy loss is larger (consistent with JT-60U measurement)

Conclusion

- In the presence of large density gradient, gyroviscosity is necessary to stabilize IDG modes in twofluid model;
- With gyroviscosity, gyrofluid and twofluid show good consistence on linear instability;
- Gyrofluid generates stronger n=0 EXB flow;
- Higher temperature leads to smaller ELM crash;