TI.00003

Gyro-Landau-Fluid Theory and Simulations of Edge-Localized-Modes

BOUT++
Boundary Plasma Turbulence Code

X.Q.Xu

Lawrence Livermore National Laboratory

in collaboration with

P.W.Xi[1,2], A.Dimits[1], I.Joseph[1], M.V.Umansky[1], T.Y.Xia[1,3], B.Gui[1,3], S.S.Kim[4], G.Y.Park[4], T.Rhee[4], H.Jhang[4], P.H.Diamond[4,5], B.Dudson[6], P.B.Snyder[7], M. F. Fenstermacher[1], W. H. Meyer[1], and B. Tobias[8]

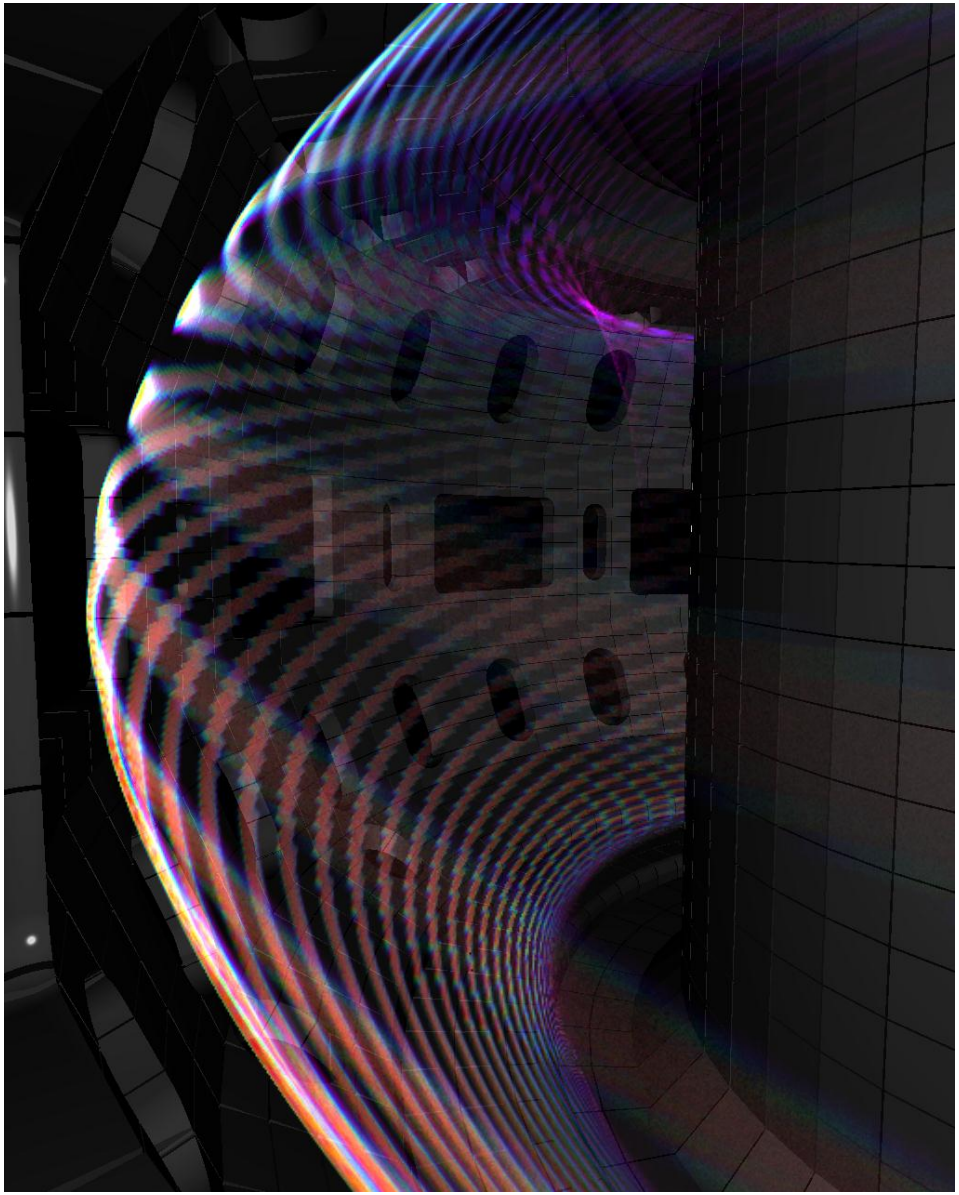
[1]LLNL,[2]PKU,[3]ASIPP,[4]NFRI,[5]UCSD,[6]U.York,[7]GA, [8]PPPL

54th Annual Meeting of the APS Division of Plasma Physics

October 29–November 2 2012; Providence, Rhode Island

This work was performed under the auspices of the U.S. DoE by LLNL under Contract DE-AC52-07NA27344 and is supported by the China NSF under Contract No.10721505, the National Magnetic Confinement Fusion Science Program of China under Contracts No. 2011GB107001. LLNL-PRES-587357.

Principal Results



(1) First order FLR corrections from "gyro-viscous cancellation" in two-fluid model are necessary to agree with gyro-fluid results for high ion temperature case with large density gradient.

(2) Higher ion temperature introduces more FLR stabilizing effects, thus reduces ELM size.

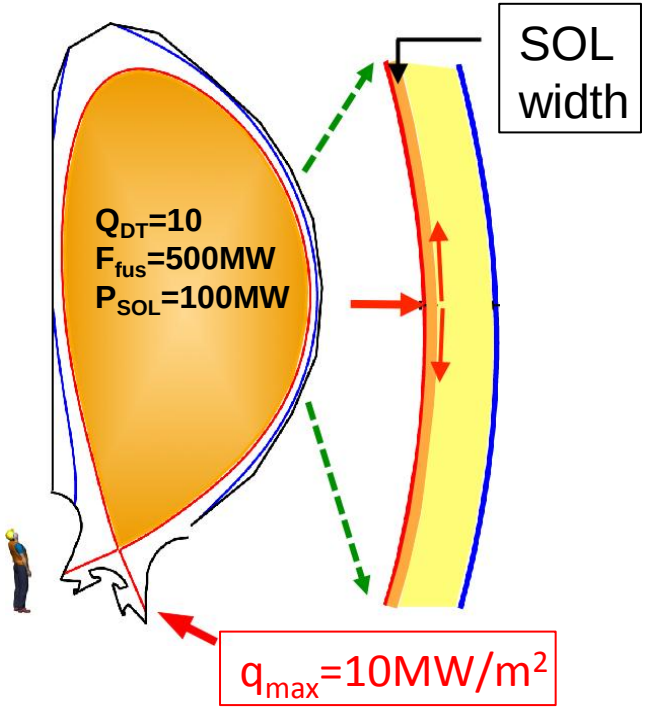
(3) Fundamental model: 3-field 2-fluid model captures the essential physics for P-B instability and early phase of ELM crashes.

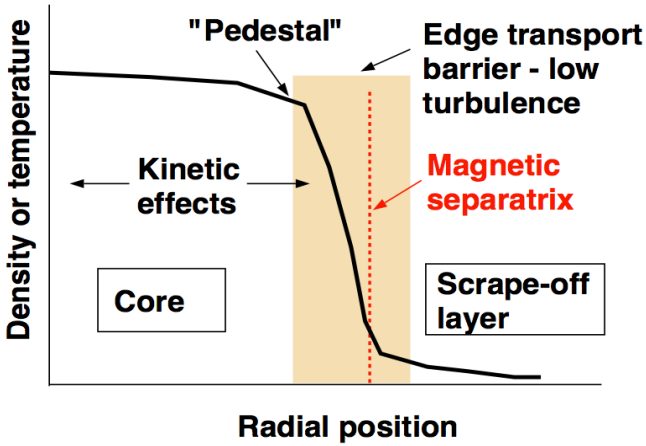
(4) Six-field simulations show that most energy lost via ion channel during an ELM event.

(5) Developed accurate non-Fourier methods for Landau-fluid operators

Global EM gyrofluid simulation can bridge the kinetic and fluid regimes across separatrix

- **Edge Localized Modes (ELMs) are potential damaging to ITER divertor plates, and first walls**
- **ELMs in tokamaks are sudden releases of particle and energy into the SOL, resulting in the eruption of filamentary structures from the plasma edge.**
- **ELMs are believed to be triggered by the peeling-ballooning (P-B) modes**
- **P-B modes are ideal MHD modes which are destabilized by a combination of pressure gradients (ballooning) and currents in the plasma edge (Peeling)**
  - **pedestal height**
- **Global EM simulations across separatrix is a must:**
  - **micro-turbulence in hot H-mode pedestal**
    - ➢ **pedestal width**
  - **power deposition on PFCs in the cold SOL**





The successful KBM physics model in EPED* motivates electromagnetic gyro-kinetic calculations in real geometry

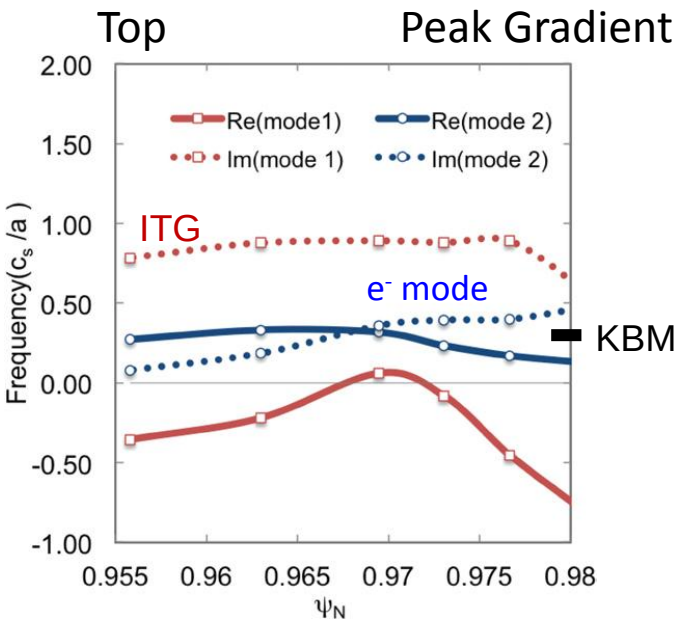
- The EPED model predicts the H-mode pedestal height and width based upon two constraints:
  1) onset of non-local P-B modes at low to intermediate n,
  2) onset of nearly local KBM at high n.

- Performed GYRO linear local gyrokinetic analysis of pressure scan of DIII-D discharge used in EPED study

- Results for top of pedestal
  - ✓ ITG found to be dominant for $k_\perp \rho_i < 1$. At higher $k_\perp \rho_i$ a micro-tearing mode is dominant

- Results for the steep gradient region of the pedestal:
  - ✓ Two modes compete for dominance for $k_\perp \rho_i < 1$. One is in electron drift direction, other is ion.
  - ✓ KBM found in steep gradient region, roughly appearing where ideal ballooning theory predicts an onset.

Frequency vs radius, $k_\theta \rho_s = 0.25$



E. Wang, Nucl. Fusion 52 (2012)

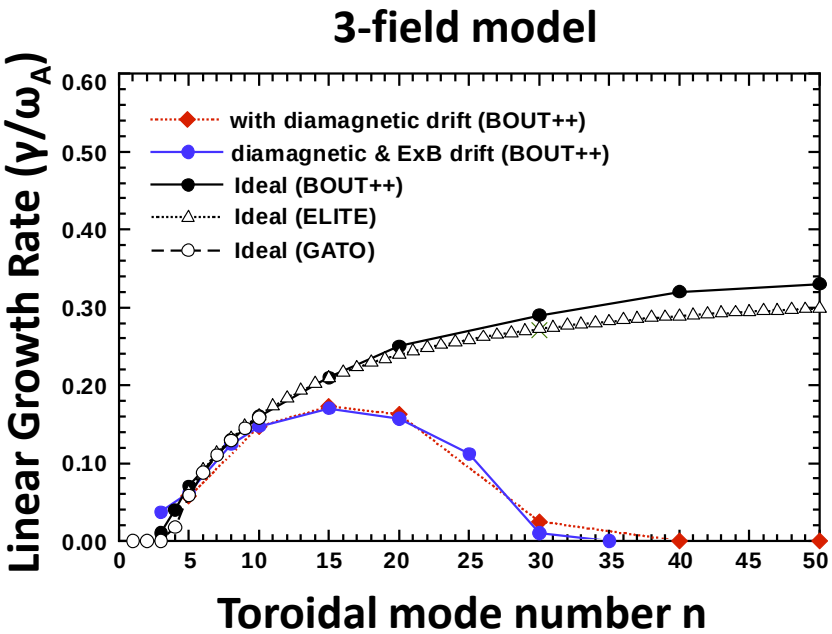
*Snyder, Groebner, Leonard, Osborne, & Wilson, PoP16, 056118 (2009).
Jerry Hughes, TI3.00003 , next talk

The difficulty to run nonlinear global EM gyrokinetic codes across separatrix further motivates the global gyrofluid development.

- BP8.00151: E Wang, et al, Gyrokinetic analysis of H-mode pedestals
- JP8.00119: R. Bravenec, et al, Benchmarking of the Gyrokinetic Microstability Codes GYRO, GS2, and GEM

BOUT++ code is an ideal framework for gyrofluid extention for modeling tokamak edge ELMs and turbulence*

- Framework for writing fluid / plasma simulations in complex tokamak geometry
  - Proximity of open+closed flux surface
  - Presence of X-point
- Written in C++, started from BOUT code, jointly developed by LLNL, Univ. York, other U.S. and international partners
- Well benchmarked with ELITE, GATO and other fluid codes
- Extensions: new formulation for edge GLF models and GLF closures



X.Q. Xu, B.D. Dudson, P.B. Snyder, M.V. Umansky, H.R.Wilson and T. Casper, Nucl. Fusion 51 (2011) 103040

YI3.00003 : Bruce Cohen, BOUT Simulations of Drift Resistive Ballooning L-mode Turbulence in the Edge of the DIII-D Tokamak

---

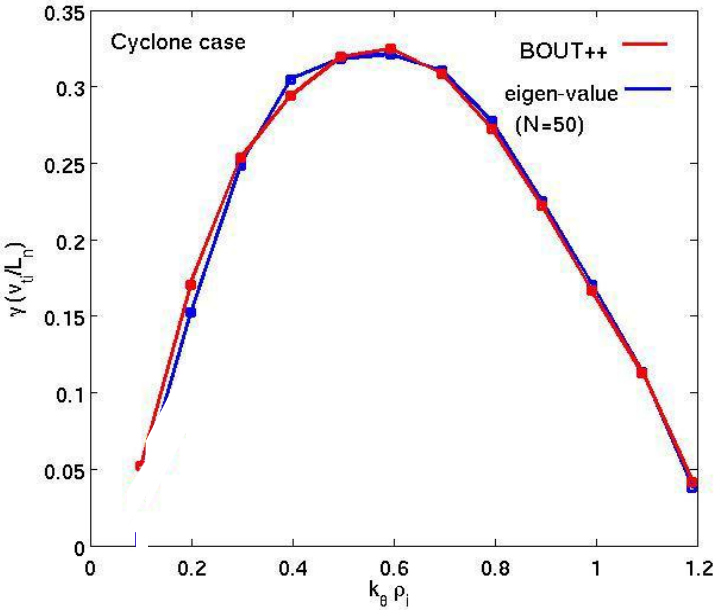
** X.Q. Xu and R.H. Cohen, Contrib. Plasma Phys. 38, 158 (1998)*

- Umansky, Xu, Dudson, et al., , Comp. Phys. Comm. V. *180 , 887-903 (2008).*
- Dudson, Umansky, Xu et al., Comp. Phys. Comm. V.180 (2009) 1467.

An isothermal electromagnetic 3-field gyro-fluid model

- **We derived an isothermal electromagnetic 3-field gyro-fluid model with vorticity formulation generalized from Snyder-Hammett gyro-fluid model [1] for edge plasmas.**
- **Utilizing the Padé approximation for the modified Bessel functions, this set of gyro-fluid equations is implemented in the BOUT++ framework with full ion FLR effects.**
- **In long-wavelength limit, this set of gyro-fluid equations is reduced to previous 3-field two-fluid model with additional gyro-viscous terms resulting from the incomplete "gyro-viscous cancellation" in two-fluid model given by Xu [2].**
- **Only simple ion diamagnetic effect in two-fluid model is not sufficient to represent FLR stabilizing if density gradient is large!**

Excellent agreement of global ITG mode between BOUT++ and Eigenvalue Solver



GP8.00117: S.S. Kim, et al, Core Gyrofluid Simulations of Ion Temperature Gradient Turbulence Using BOUT++.

[1] P. B. Snyder and G. W. Hammett, Phys. Plasmas 8, 3199 (2001).
[2] X. Q. Xu, R. H. Cohen, T. D. Rognlien, et.al., Phys. Plasma 7, 1951 (2000).

3-field isothermal gyrofluid model* for ELM simulation generalized for the large density gradient at H-mode pedestal

$$\frac{d\varpi_G}{dt} + \mathbf{V}_E \cdot \nabla \varpi_{G0} \boxed{- eB(\mathbf{V}_{\Phi T} - \mathbf{V}_{ET}) \cdot \nabla n_{iG}} = B\nabla_{\parallel} J_{\parallel} + 2\mathbf{b}_0 \times \boldsymbol{\kappa} \cdot \nabla \tilde{P}_G + \mu_{i,\parallel} \partial_{\parallel}^2 \varpi_G$$

$$\frac{d\tilde{P}_G}{dt} + \mathbf{V}_E \cdot \nabla P_{G0} \boxed{+ T_0(\mathbf{V}_{\Phi T} - \mathbf{V}_{ET}) \cdot \nabla n_{iG}} = 0$$

$$\frac{\partial A_{\parallel}}{\partial t} + \partial_{\parallel} \phi_T = \frac{\eta}{\mu_0} \nabla_{\perp}^2 A_{\parallel} - \frac{\eta_H}{\mu_0} \nabla_{\perp}^4 A_{\parallel}$$

$$\varpi_G = eB\left(\Gamma_0^{1/2} \tilde{n}_{iG} - n_0(1-\Gamma_0)\frac{e\phi}{T_0} + \boxed{\frac{e\rho_i^2}{T_0} \nabla n_0 \cdot \nabla(\Gamma_0 - \Gamma_1)\phi} - \tilde{n}_{iG} \right)$$

Padé approximation

$$\begin{cases} \Gamma_0^{1/2} \approx \dfrac{1}{1+b/2} \\ \Gamma_0 \approx \dfrac{1}{1+b} \\ \Gamma_0 - \Gamma_1 \approx 1 \end{cases}, \quad b = -\rho_i^2 \nabla_{\perp}^2$$

$$d/dt = \partial/\partial t + \mathbf{V}_{ET} \cdot \nabla, \mathbf{V}_{ET} = \frac{1}{B}\mathbf{b}_0 \times \nabla \phi_T, \phi_T = \phi_0 + \phi, \nabla_{\parallel} f = B\partial_{\parallel} \frac{f}{B}, \partial_{\parallel} = \partial_{\parallel}^0 + \delta\mathbf{b} \cdot \nabla, \delta\mathbf{b} = \frac{1}{B}\nabla A_{\parallel} \times \mathbf{b}_0, J_{\parallel} = J_{\parallel 0} + \tilde{J}_{\parallel}, \tilde{J}_{\parallel} = -\nabla_{\perp}^2 A_{\parallel}/\mu_0$$

$$\frac{d\varpi}{dt} = B\nabla_{\parallel} J_{\parallel} + 2\mathbf{b}_0 \times \boldsymbol{\kappa} \cdot \nabla \tilde{P} + \mu_{i,\parallel} \partial_{\parallel}^2 \varpi$$

$$\frac{d\tilde{P}}{dt} + \mathbf{V}_E \cdot \nabla P_0 = 0$$

$$\frac{\partial A_{\parallel}}{\partial t} + \partial_{\parallel} \phi_T = \frac{\eta}{\mu_0} \nabla_{\perp}^2 A_{\parallel} - \frac{\eta_H}{\mu_0} \nabla_{\perp}^4 A_{\parallel}$$

$$\varpi = \frac{m_i n_0}{B}\left(\nabla_{\perp}^2 \phi + \frac{1}{en_0} \nabla_{\perp}^2 \tilde{P}_i + \frac{1}{n_0} \nabla n_0 \cdot \nabla \phi \right)$$

two-fluid equations

Relation between two-fluid vorticity and gyrokinetic vorticity

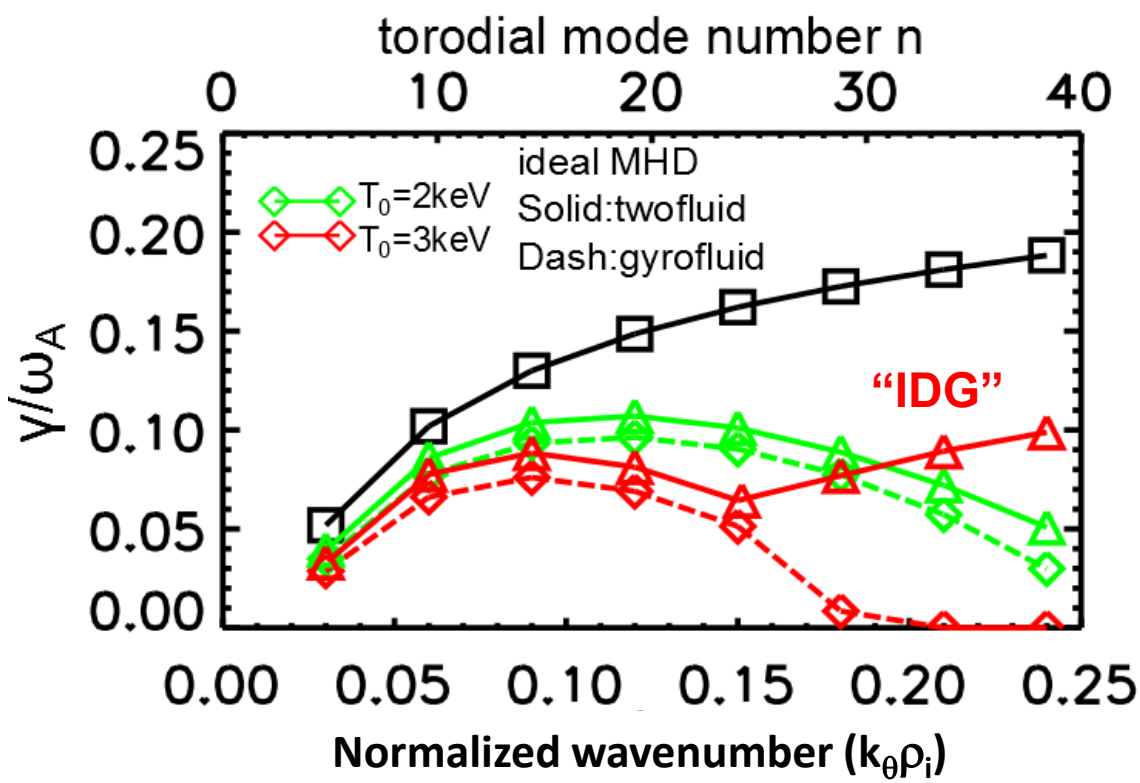
$$\varpi \approx \varpi_G + \frac{1}{2en_0} \nabla_{\perp}^2 \tilde{P}_{iG}, \qquad \rho_i^2 \nabla_{\perp}^2 << 1$$

* P. B. Snyder and G. W. Hammett, *Phys. Plasmas* **8**, 3199 (2001)

In the presence of large density gradient, gyro-fluid and two-fluid model show qualitative difference when $k_\perp \rho_i$ is large

Consider the **large density gradient** at H-mode pedestal, when ion temperature ↑ :

- Two-fluid model: **no stabilizing** of high-n modes,
- Gyro-fluid model: **strong FLR stabilizing** of high-n modes.
- **What causes the disappearance of stabilizing in two-fluid model?**

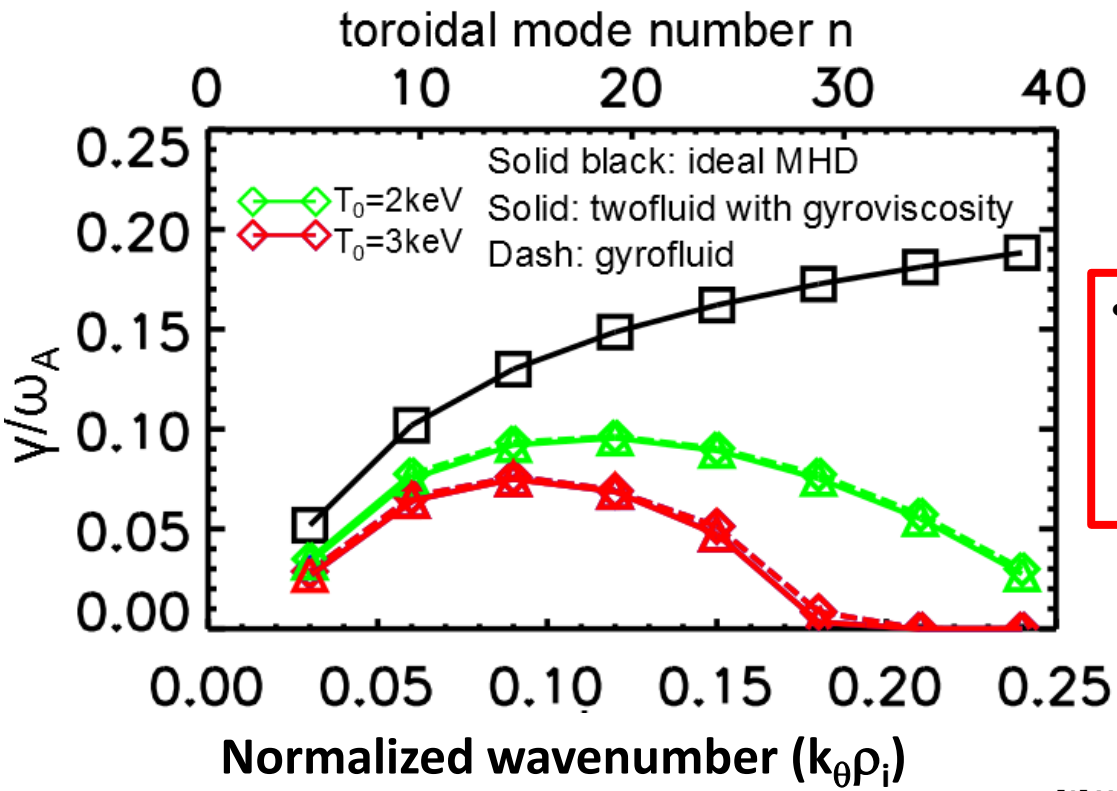


GP8.00116: P.W. Xi, et al, Gyro-fluid Simulations of Edge-Localized-Modes

Gyroviscous terms are necessary to stabilize "Ion-Density-Gradient modes" and should be kept in two-fluid model

- **At long wavelength limit, gyro-fluid goes back to two-fluid but with additional gyroviscous terms[1,2]**

$$\frac{d\varpi_G}{dt} + \mathbf{V}\cdot\nabla\varpi_{G0} - eB(\mathbf{V}_{\Phi T} - \mathbf{V}_{ET})\cdot\nabla n_{iG} = \frac{d\varpi}{dt} + \boxed{\frac{1}{2\omega_{ci}}\left\{\nabla_\perp^2[\phi, P_i] - [\nabla_\perp^2\phi, P_i] - [\phi, \nabla_\perp^2 P_i]\right\}}$$



- **Only ion diamagnetic effect in two-fluid model is not sufficient to represent FLR stabilizing if density gradient is large!**

[1] X.Q.Xu, et al, TH/5-2Rb, 24th IAEA FEC, 2012, San Diego, CA.
[2] X.Q.Xu, et.al., Phys. Plasma 7, 1951 (2000).

Hyper-resistivity η_H is anomalous electron viscosity μ_e
Hyper-resistivity describes the anomalous radial transport of current

- **ELM dynamics is a multi-scale problem**
  - **meso-scale MHD** ⟷ **electron gyro-radius scale dissipation**

- **Hyper-resistivity can be used to set the finest resolved radial scale in simulations**
  - **Ion FLR effect cannot replace hyper-resistivity**
  - **Dissipation in Ohm's law does exist on e^- scale for reconnection**
    - ✓ **classical e^- viscosity is too small, $\mu_e^{cl} \approx \nu_{ei}\rho_e^2 < 10^{-5} m^2/s$**
    - ✓ **anomalous e^- viscosity works, assuming $\mu_e \sim \chi_e \approx 0.1\text{-}1\ m^2/s$ and $\nu_{ei} \approx 10^5$, $S_H = 10^{12\text{-}14}$**
      - → **turbulent mixing**
      - → **stochastic fields**

$$\frac{\partial A_\parallel}{\partial t} = -\nabla_\parallel \phi_T + \frac{\eta}{\mu_0}\nabla_\perp^2 A_\parallel \boxed{-\frac{\eta_H}{\mu_0}\nabla_\perp^4 A_\parallel}, \qquad \eta_H = \eta\frac{\mu_e}{\nu_{ei}}$$

$$\frac{\partial \hat{A}_\parallel}{\partial \hat{t}} = -\hat{\nabla}_\parallel \hat{\phi}_T + \left(\frac{1}{S}\right)\hat{\nabla}_\perp^2 \hat{A}_\parallel - \left(\frac{1}{S_H}\right)\hat{\nabla}_\perp^4 \hat{A}_\parallel, \qquad S = \mu_0 R v_A/\eta, \qquad S_H = \mu_0 R^3 v_A/\eta_H = S/\alpha_H$$

- **In our present model, the frozen-in flux condition of ideal MHD theory is broken by**
  - **resistivity**
  - **hyper-resistivity.**

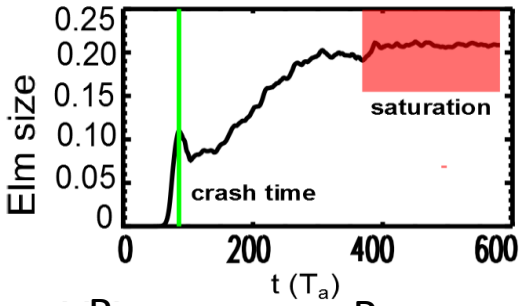
- **The self-consistent ELM simulation needs the first principle theory of hyper-resistivity***

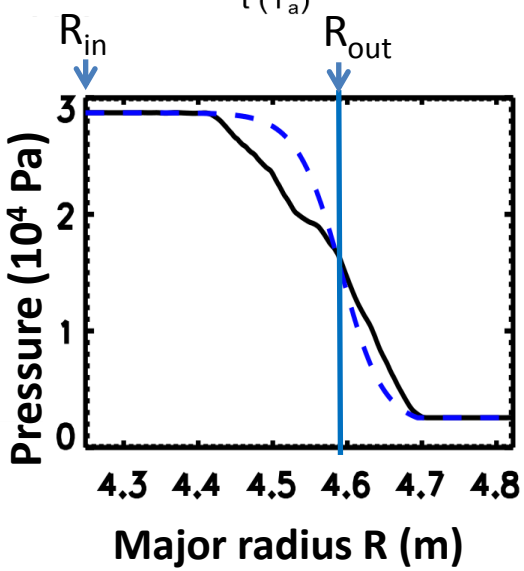
*Z. B. Guo, P. H. Diamond and X. G. Wang, 2012 *ApJ* 757, 173
GP8.00116: P.W. Xi, et al, ETG turbulence simulation of tokamak edge plasmas via 3+1 gyrofluid code

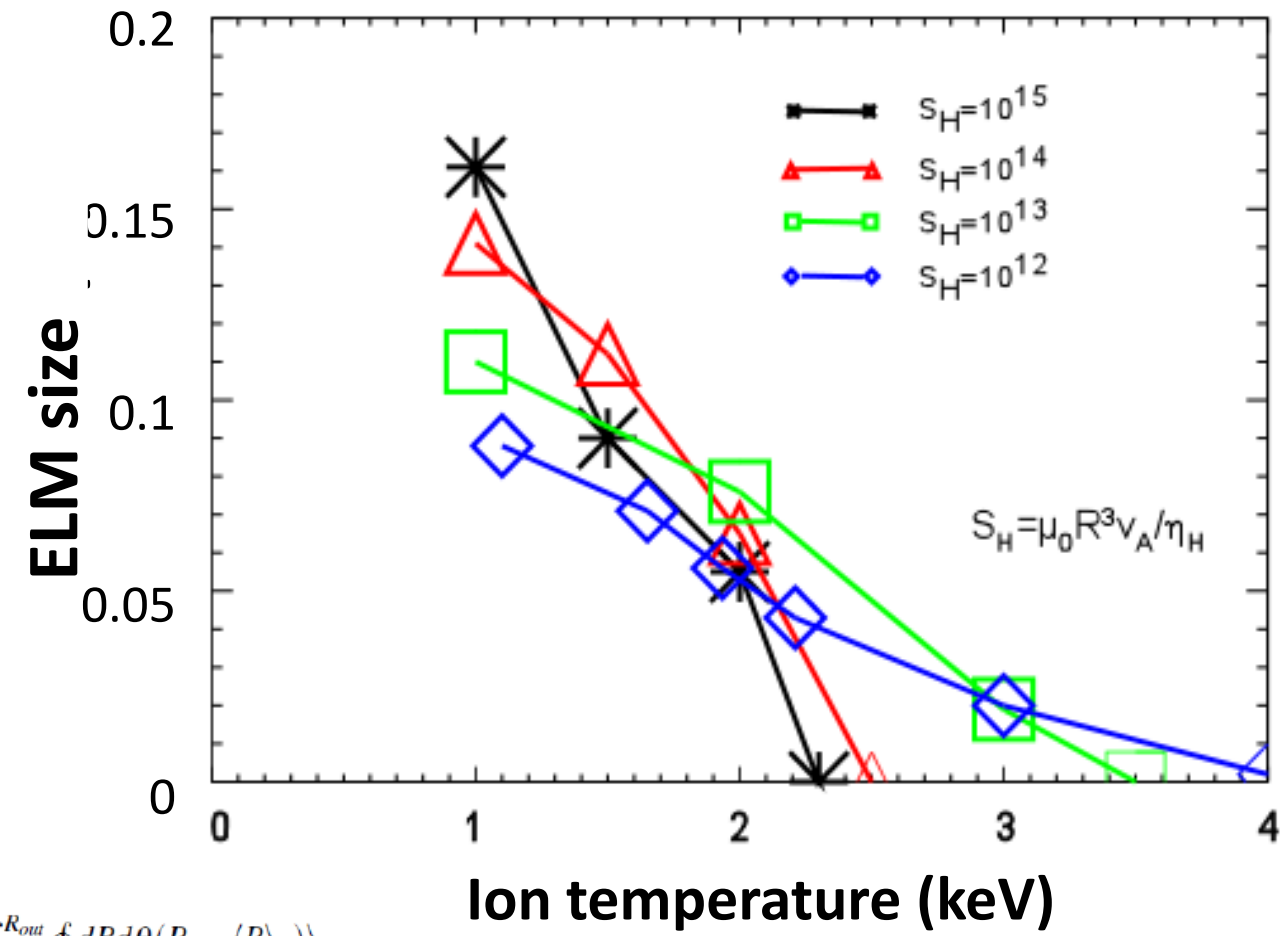
Higher ion temperature introduces more FLR stabilizing effects, thus reduces ELM size

TI3.00002

- Hyper-resistivity is necessary to simulate ELM crash, but ELM size is weakly sensitive to hyper-resistivity;
- With fixed pressure profile, high ion temperature introduces stronger FLR effect and leads to smaller ELM size







$$\Delta_{ELM}^{th} = \frac{\Delta W_{ped}}{W_{ped}} = \frac{\langle \int_{R_{in}}^{R_{out}} \oint dR d\theta (P_0 - \langle P \rangle_\zeta) \rangle_t}{\int_{R_{in}}^{R_{out}} \oint dR d\theta P_0}$$

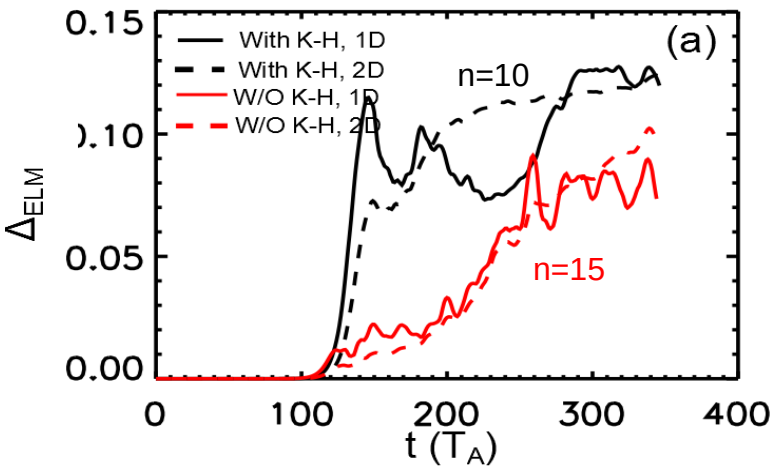
(Without density gradient in vorticity)

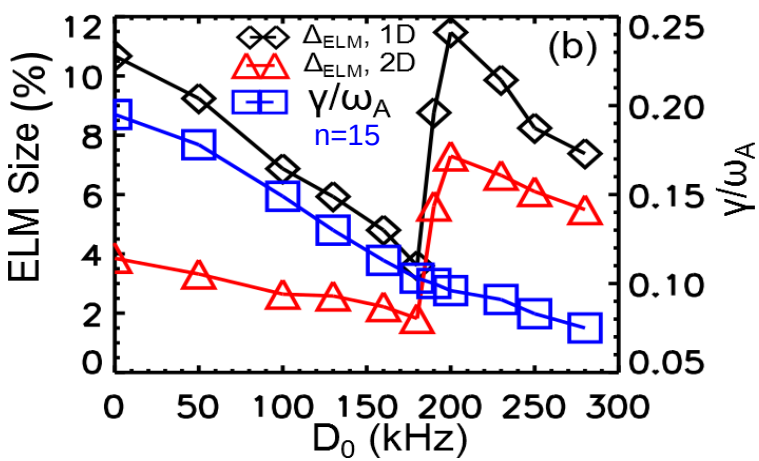
Equilibrium EXB shear flow can stabilize high-n ballooning modes & reduce ELM size, but additional Kelvin-Helmholtz drive can enhance growth rate of low-n modes and leads to larger ELM when flow shear is too large.

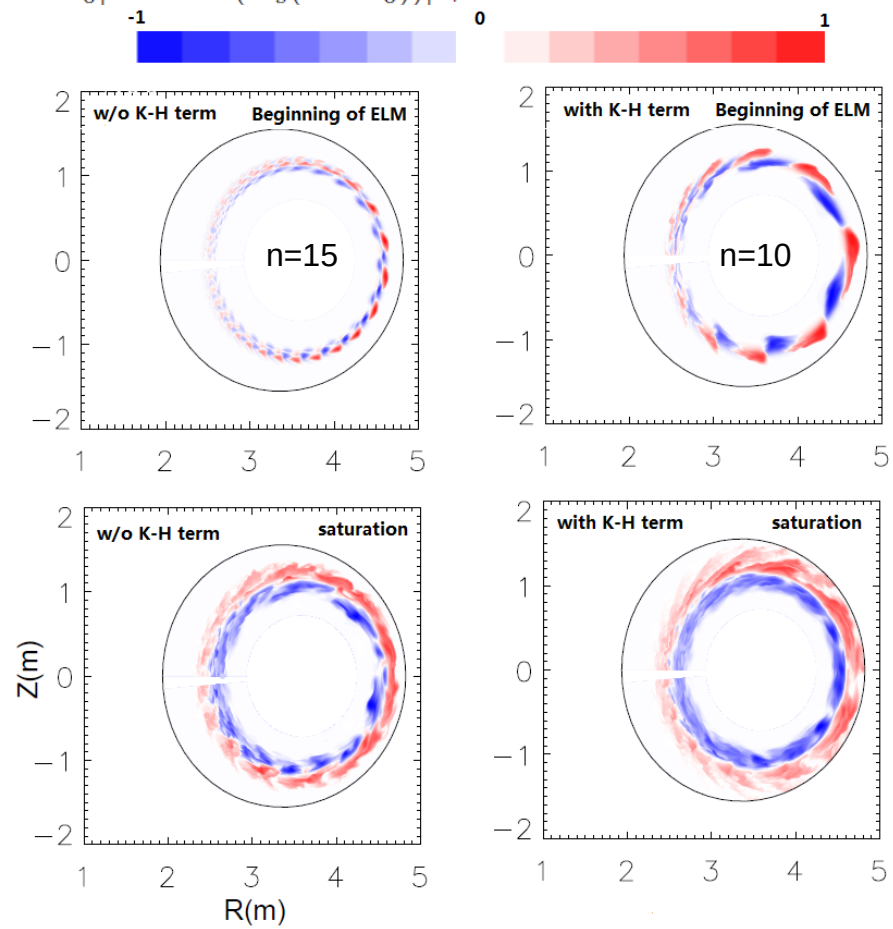
$$\frac{\partial \varpi}{\partial t} + \mathbf{V}_{E\times B}\cdot\nabla\varpi + \boxed{\mathbf{V}_1\cdot\nabla\varpi_0} = B_0\nabla_{\parallel}J_{\parallel} + 2\mathbf{b}_0\times\kappa\cdot\nabla P$$

$$\mathbf{V}_{E\times B} = \frac{\mathbf{b}\times\nabla\Phi_{dia0}}{B} + \frac{\mathbf{b}\times\nabla\Phi_{V0}}{B} + \frac{\mathbf{b}\times\nabla\phi}{B} \qquad \varpi_0 \approx \frac{n_0 m_i}{B_0}\nabla_{\perp}^2\Phi_{V0}$$

$$\Phi_{dia0} = -\frac{P_{i0}}{Zen_{i0}}, \qquad \frac{d\Phi_{V0}(\psi)}{d\psi} = D_0[1-\tanh(D_s(x-x_0))] + C$$





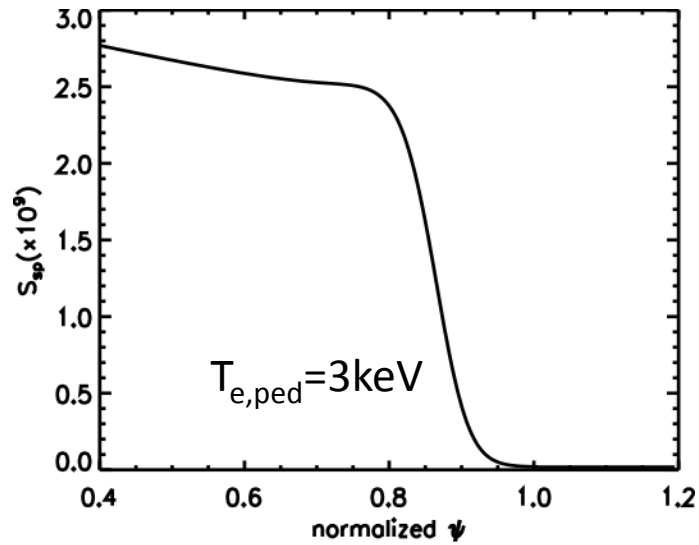


P. W. Xi, X. Q. Xu, X. G. Wang, and T. Y. Xia, *Phys. Plasmas* **19**, 092503 (2012)

Multi-field two-fluid models in BOUT++ Guide GLF development

- **Three-field (ϖ, P, $A_{||}$): peeling-ballooning model.**
- **Four-field (ϖ, P, $A_{||}$, $V_{||}$): include sound waves.**
- **Five-field (ϖ, n_i, T_i, T_e, $A_{||}$): parallel thermal diffusivities**
- **Six-field (ϖ, n_i, T_i, T_e, $A_{||}$, $V_{||}$): based on Braginskii equations, the density, momentum and energy of ions and electrons are described in drift ordering*.**
  - ✓ **nonlinear || thermal diffusivities**
  - ✓ **nonlinear resistivity**
  - ✓ **additional drift wave instabilities**

Lundquist number profile



- **|| thermal diffusivities with flux limited expressions reduce ELM size:**

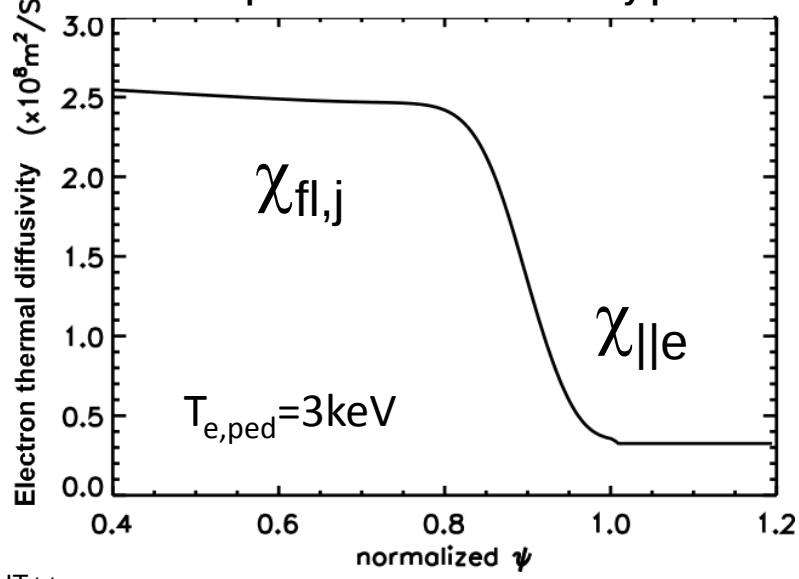
$$\chi_{||i} = 3.9\frac{v_{th,i}^2}{\nu_i} \quad \chi_{||e} = 3.2\frac{v_{th,e}^2}{\nu_e} \quad \chi_{fl,j} = v_{th,j} q_{95} R_0$$

Flux limited expression:

$$\chi_{||j}^{e} = \left(\frac{1}{\chi_{||j}} + \frac{1}{\chi_{flj}}\right)^{-1}$$

- **GLF models for $\chi_{||j}$ are under development**

Electron parallel thermal diffusivity profile



*X. Q. Xu et al., Commun. Comput. Phys. 4, 949 (2008).
PO7.00005: T. Y. Xia, et al, Six-field two-fluid simulations on edge localized modes with BOUT++

The 3-field 2-fluid model is good enough to simulate P-B stability and early phase of ELM crashes, additional physics from multi-field contributes less than 25% corrections

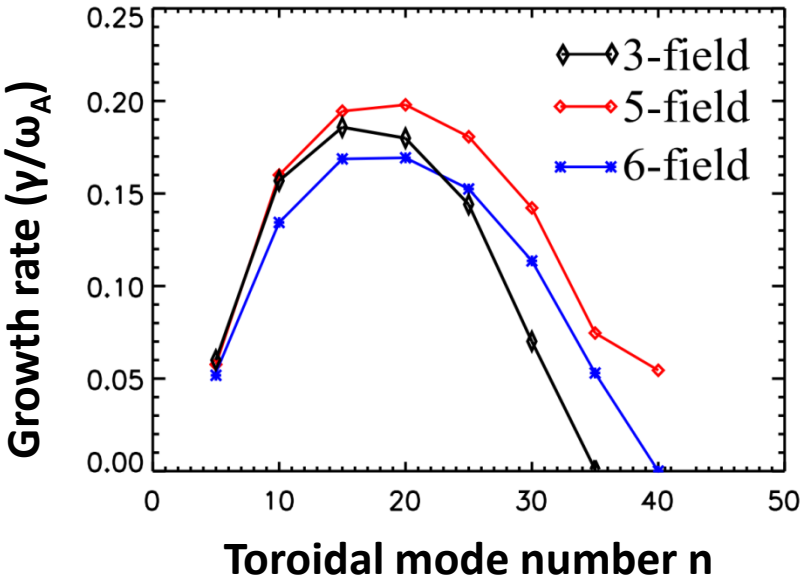
- **Fundamental physics in ELMs:**
  - ✓ Peeling-Ballooning instability
  - ✓ Ion diamagnetic stabilization
    - → kinetic effect
  - ✓ Resistivity and hyper-resistivity
    - → reconnection

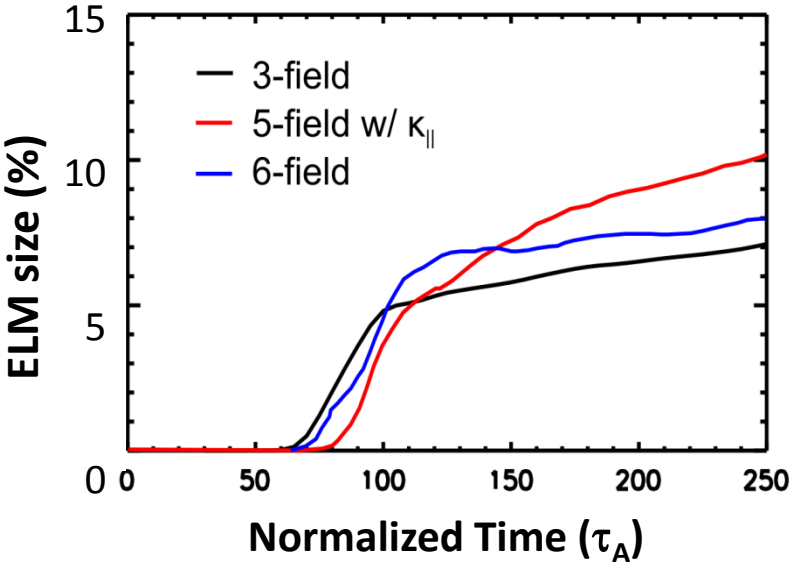
- **Additional physics:**
  - Ion acoustic waves
  - || thermal conductivities
  - Hall effect
  - Compressibility
  - Electron-ion friction

BUT

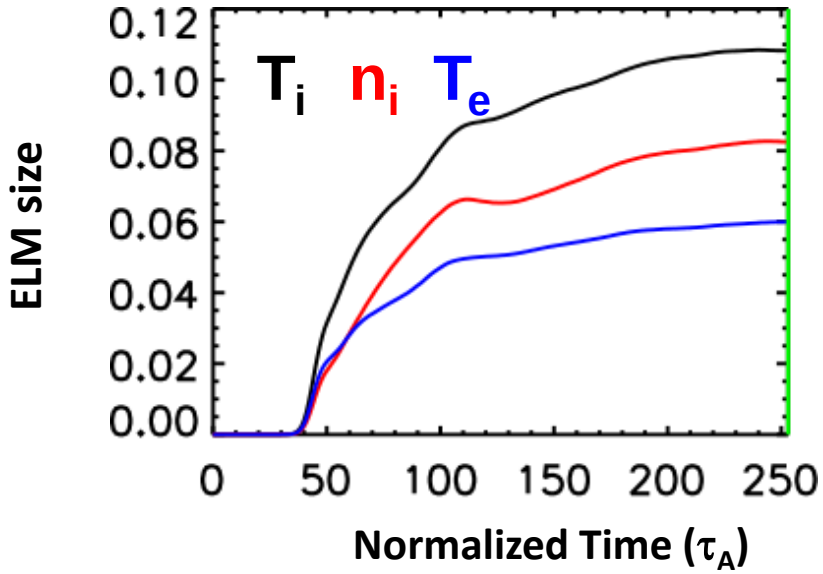
change the peak linear growth rate less than 25%

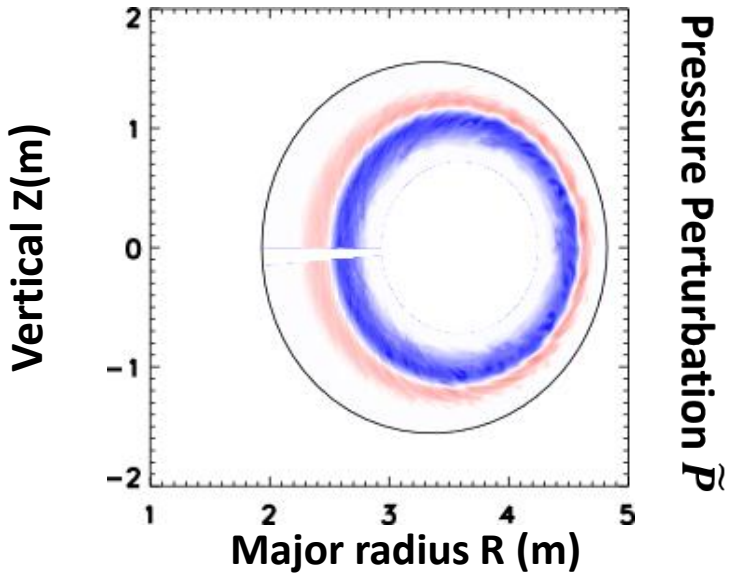
- ✓ **Power loss via separate ion & electron channels**
- ✓ **Power depositions on PFCs.**
- ✓ **Turbulence and transport**

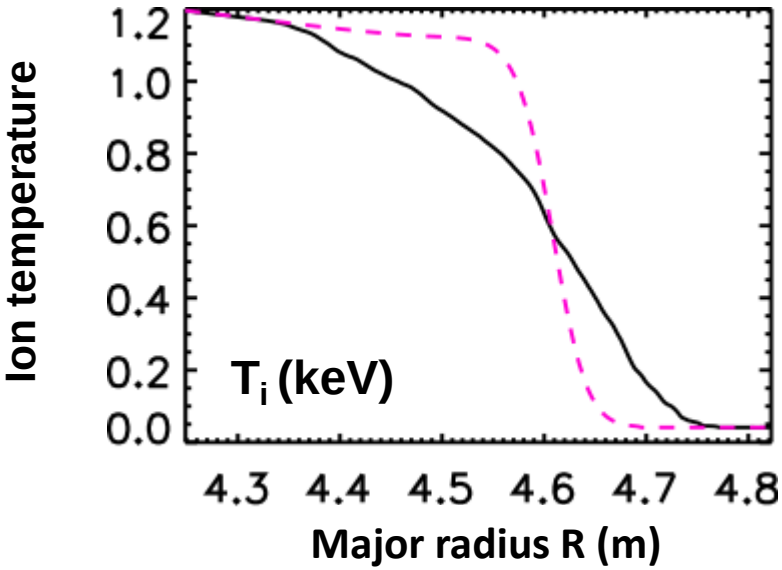


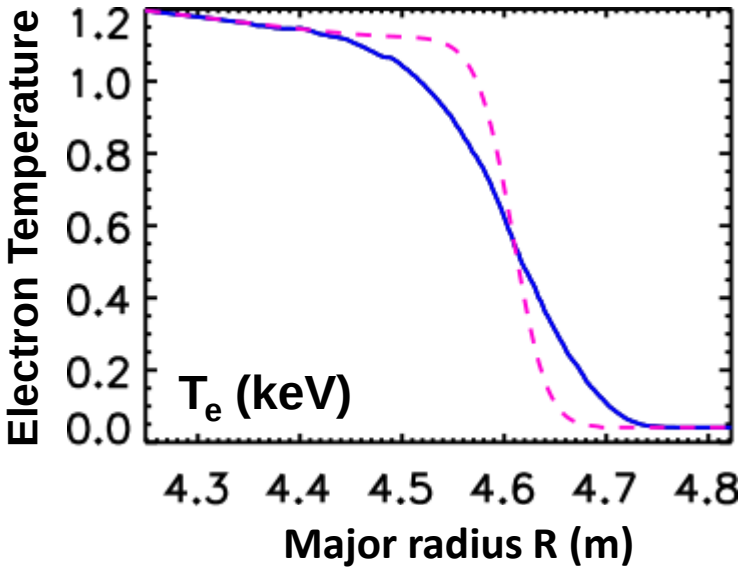


Six-field simulations show that most energy lost via ion channel during an ELM event



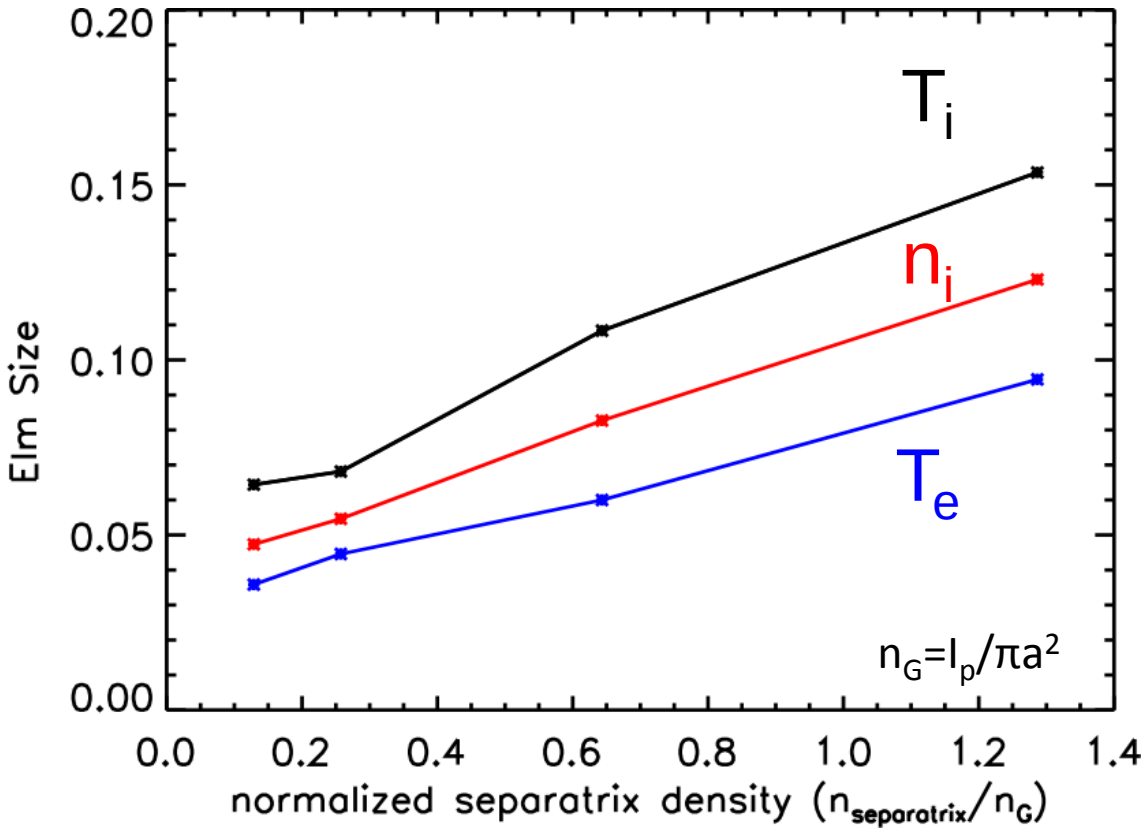


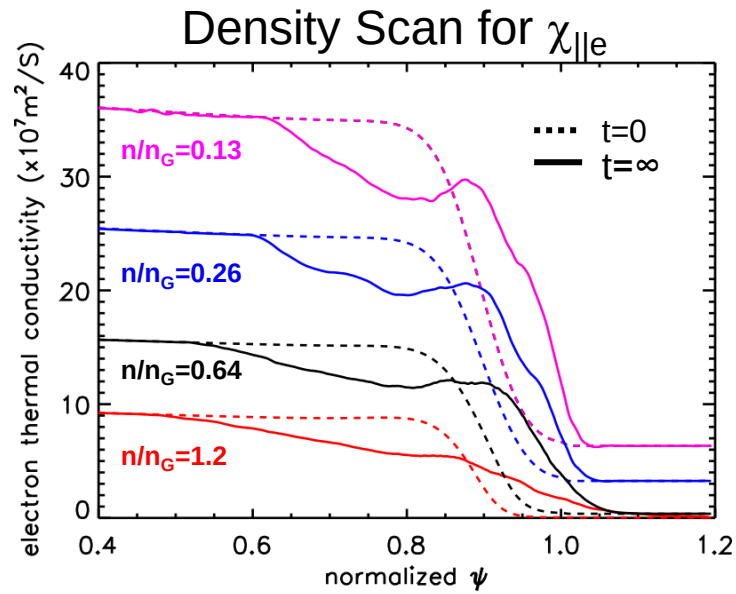




Higher density leads to large ELM size during an ELM event because of reduced parallel thermal conduction from lower temperature

(Fixed pressure profile)





During density scan

- ✓ **fixed profiles of the scale lengths: L_n, L_{Ti}, L_{te}**
- ✓ **$\chi_{||e}$↓ due to T_e↓ , not ν_{ei}↑**

When density n ⬆ increases & temperature (T_e,T_i⬇) decreases ➡ thermal diffusivity ($\chi_{||i}$, $\chi_{||e}$) ⬇ decreases & parallel damping decreases, the ELM encroaches further into core plasmas, which leads to larger ELM size ⬆.

Developed accurate non-Fourier methods for Landau-fluid operators and nonlocal parallel heat transport based on an approximation by a sum of Lorentzians

Tokamak edge:

- **kinetic effects important → need**
  - **Landau-fluid (LF) operators**

$$\gamma \propto -v_{\text{char}} \left| k \right|$$

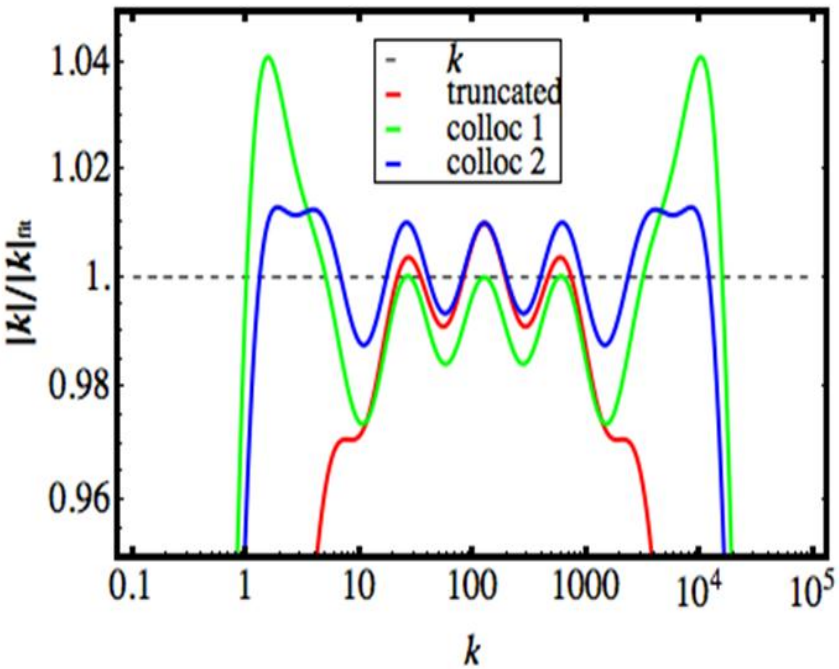
  - **nonlocal || thermal transport $q_{||j}$**

- **Large spatial inhomogeneities & complicated boundary**
  - **need non-Fourier implementation**
  - **Useful accurate approximation:**

$$\frac{1}{\left|k\right|} \approx \sum_{n=0}^{N} \frac{\alpha^n k_0}{k^2 + \left(\alpha^n k_0\right)^2}$$

- **The new method has Fourier-like computational scaling**

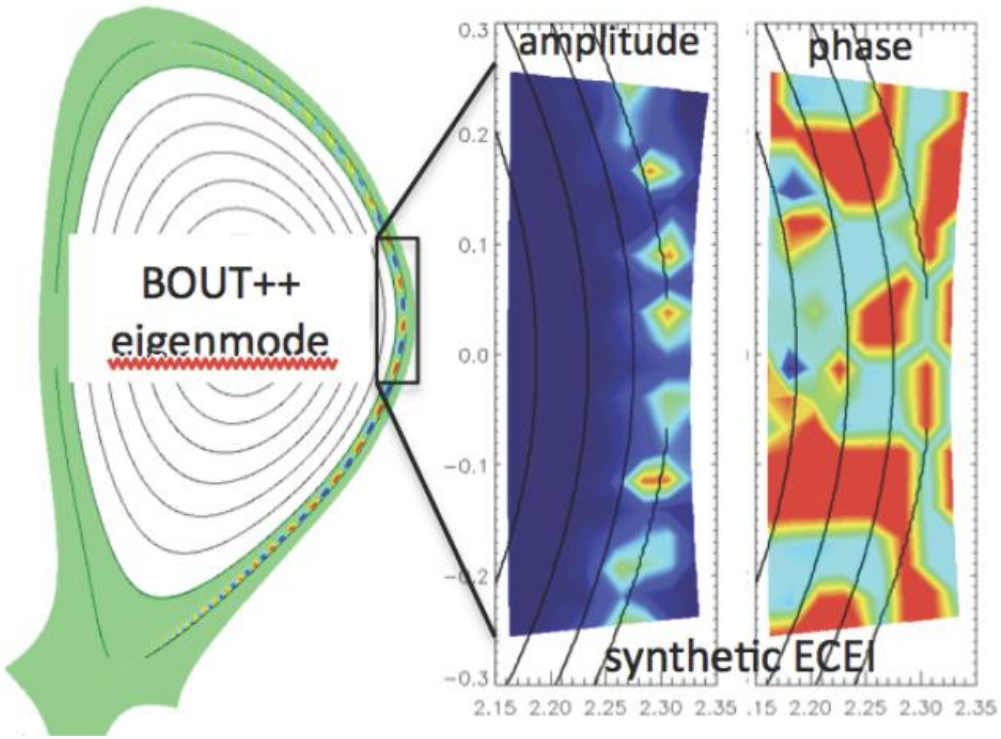
Ratio of fit to analytical operator vs. wavenumber



✓ **The error is less than 1.5%.**

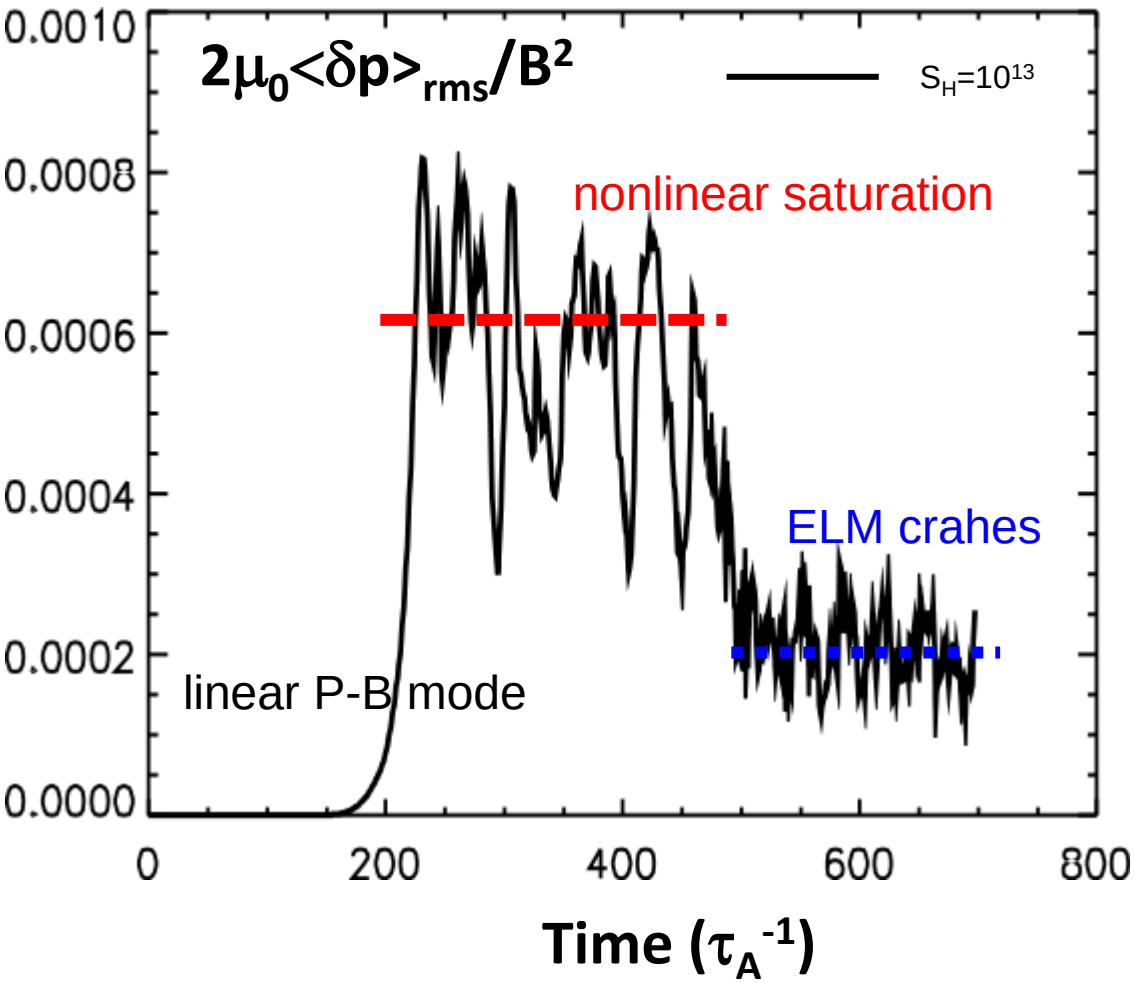
GP8.00114: A.M. Dimits, et al, Efficient Non-Fourier Implementation of Landau-Fluid Operators in the BOUT++ Code.

Nonlinear ELM simulations to be validated with fast pedestal, SOL, and divertor measurements from DIII-D using fundamental three-field two-fluid Model

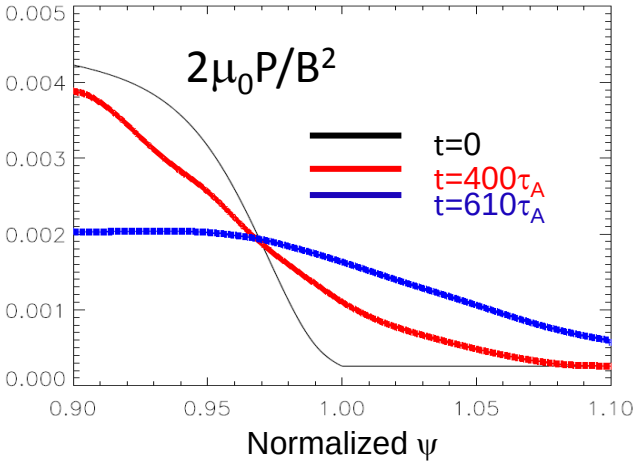


GP8.00085: M.E. Fenstermacher, et al, Validation of BOUT++ Nonlinear ELM Simulations Using Fast Measurements from DIII-D.

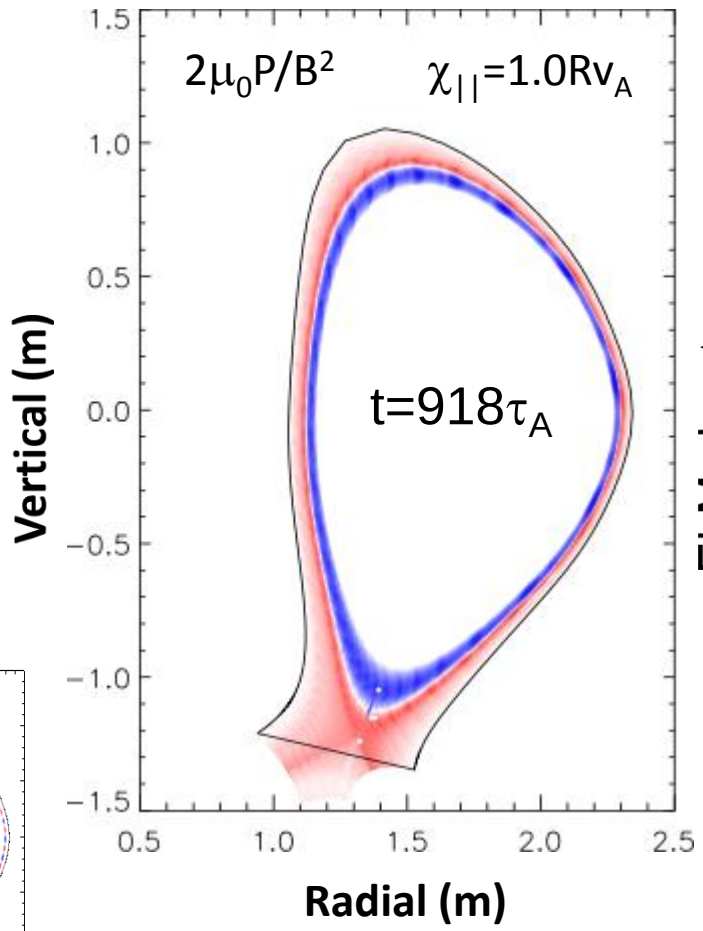
Four stages in ELM simulations: Linear P-B growth, nonlinear saturation, ELM crash & power deposition on PFCs



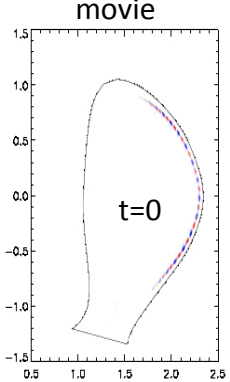
- **Simulated Stationary H-mode with Type-I ELMs of DIII-D Discharge 146394 in divertor geometry**
- **Will compare 1D 2D and 3D snapshot data plus coherent profile reconstruction**



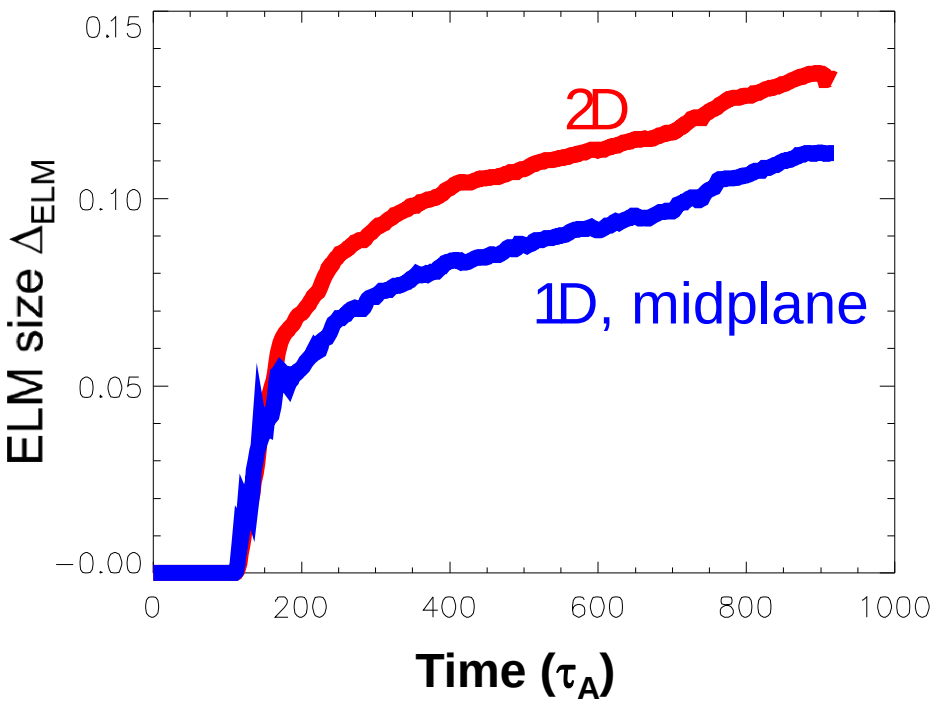
Nonlinear simulations show the linear P-B mode growth, nonlinear saturation, ELM crash and power deposition on PFCs



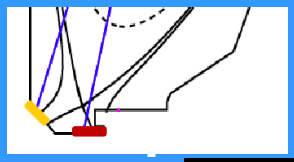
movie



Calculated ELM size from BOUT++ simulations



$$\Delta_{ELM} = \frac{\Delta W_{ped}}{W_{ped}} = \frac{\int_{0.9}^{1.0} d\psi \oint d\theta \left(P_0 - \langle P \rangle_\zeta\right)}{\int_{0.9}^{1.0} d\psi \oint d\theta \, P_0}$$



Inner divertor plate

$2\mu_0\delta p/B^2$

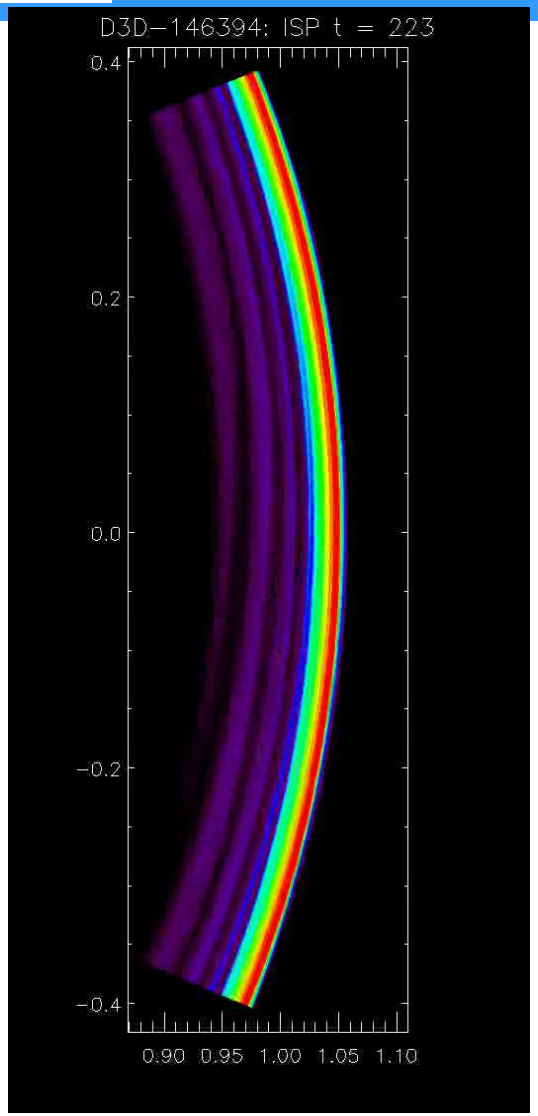
Outer midplane

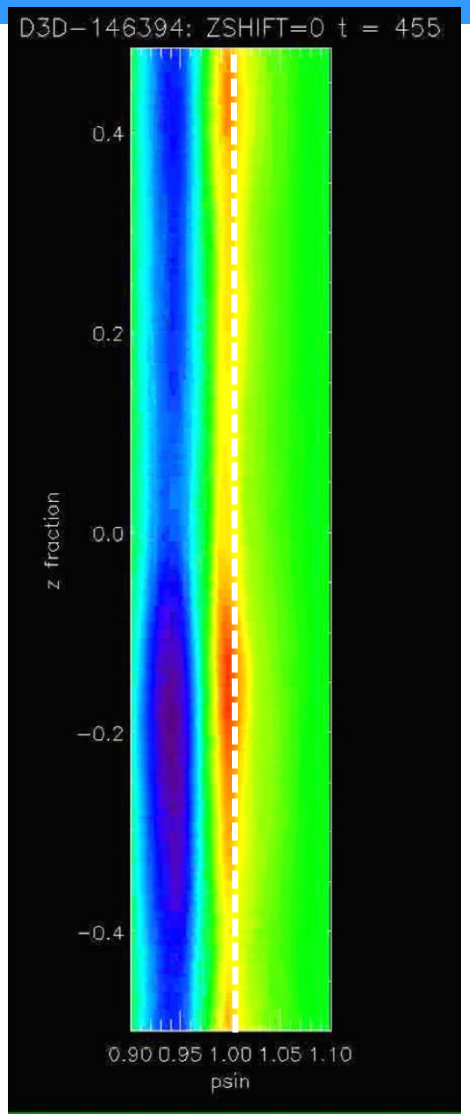
Radial index

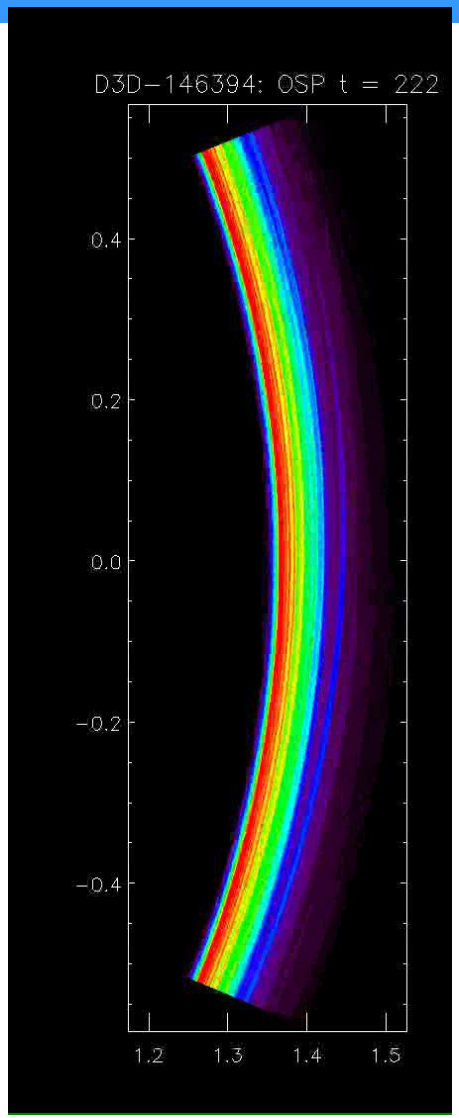
Outer divertor plate

Radial index

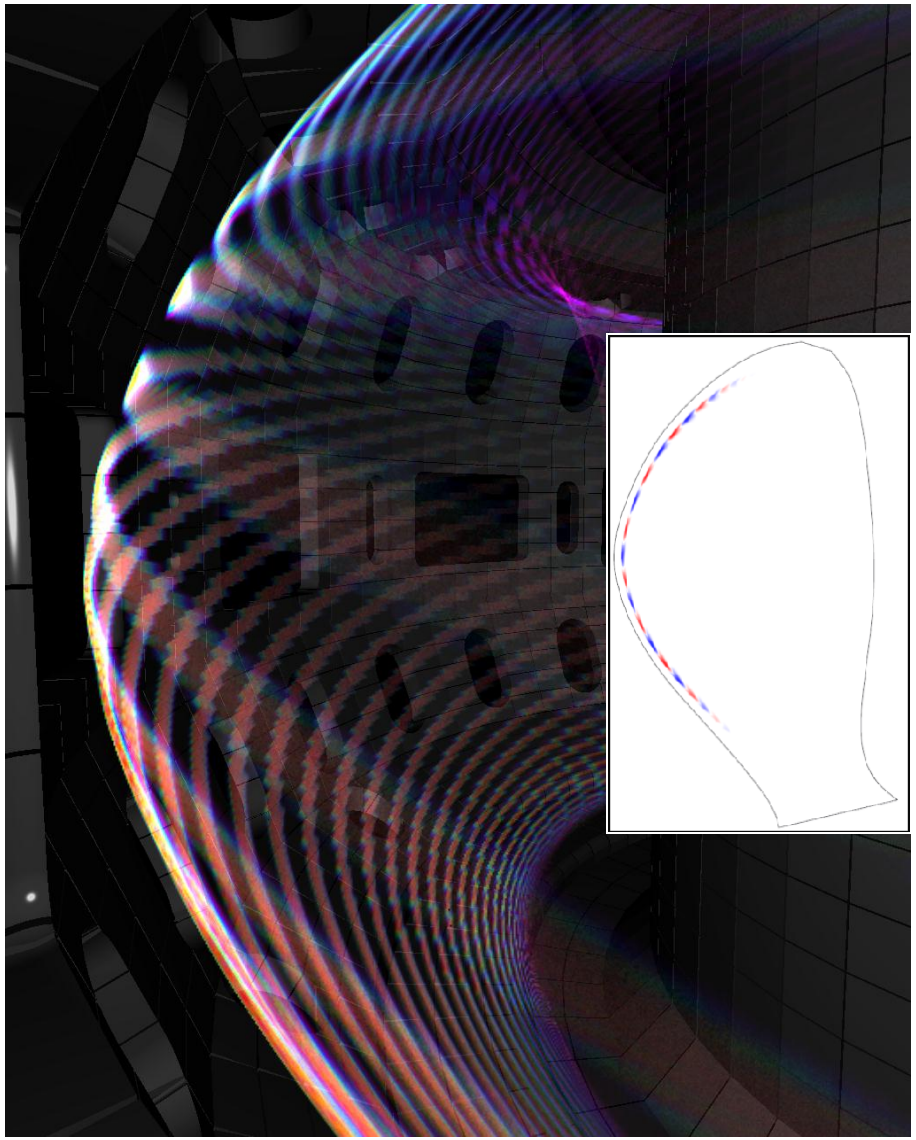
Toroidal index





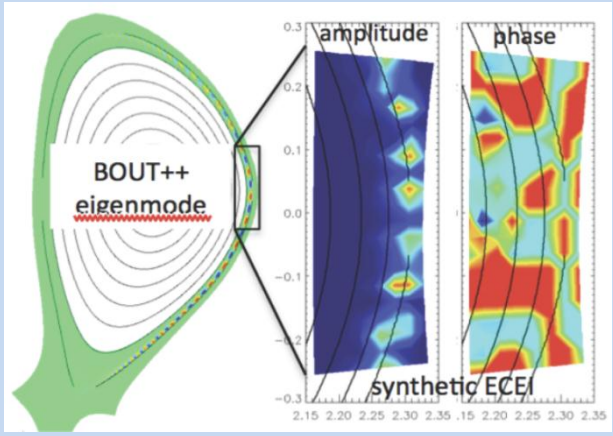


Future Work – Ongoing Validation of BOUT++ Non-Linear ELM Simulations Including ECEI and NEW Periscope Data

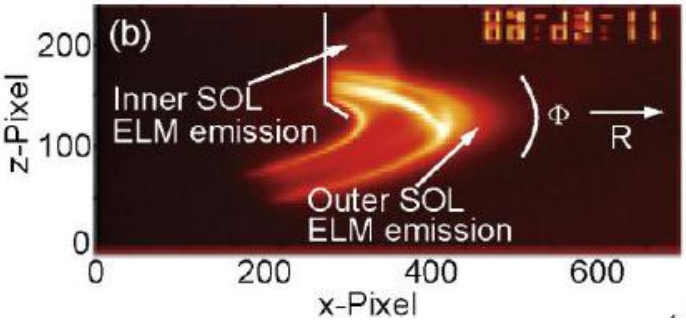


- **Projections from BOUT++ n=25 linear mode**
  - **Periscope view will measure CIII (Te=10 eV emission)**

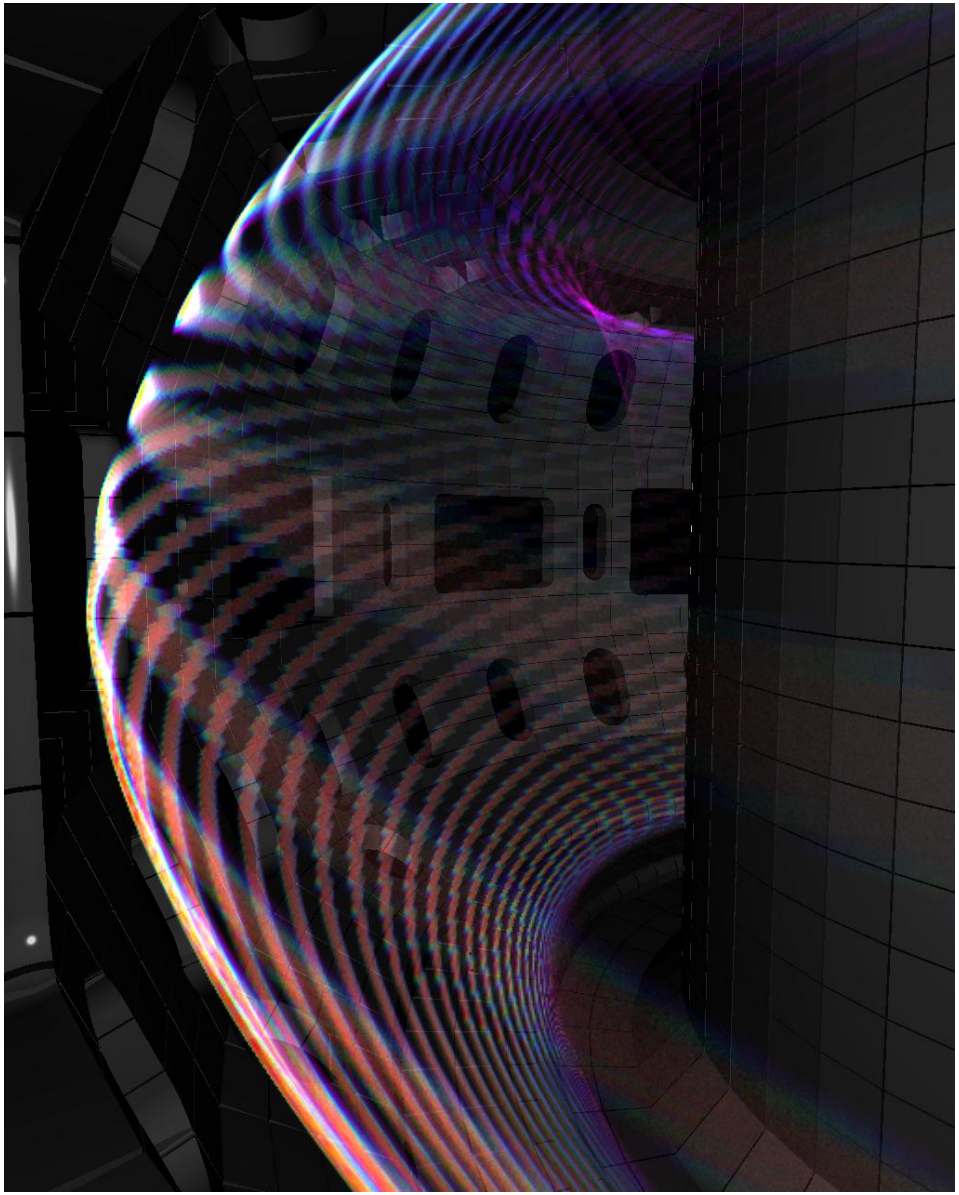
Synthetic ECEI data from BOUT++ solution



IRTV image on the divertor target floor



Principal Results



(1) First order FLR corrections from "gyro-viscous cancellation" in two-fluid model are necessary to agree with gyro-fluid results for high ion temperature case with large density gradient.

(2) Higher ion temperature introduces more FLR stabilizing effects, thus reduces ELM size.

(3) Fundamental model: 3-field 2-fluid model captures the essential physics for P-B instability and early phase of ELM crashes.

(4) Six-field simulations show that most energy lost via ion channel during an ELM event.

(5) Developed accurate non-Fourier methods for Landau-fluid operators

Additional related presentations at this APS DPP meeting

Another Invited talk

Session YI3: Edge Turbulence: 9:30 AM–12:30 PM, Friday, November 2, 2012. Room: Ballroom BC.
YI3.00003 : Bruce Cohen, BOUT Simulations of Drift Resistive Ballooning L-mode Turbulence in the Edge of the DIII-D Tokamak

Posters:

Session BP8: Poster Session I: 9:30 AM–12:30 AM, Monday, October 29, 2012. Room: Hall BC
BP8.00036:Troy Carter, et al, Studies of Turbulence, Transport and Flow in the Large Plasma Device

Session GP8: Poster Session III: 9:30 AM–9:30 AM, Tuesday, October 30, 2012. Room: Hall BC
GP8.00085: M.E. Fenstermacher, et al, Validation of BOUT++ Nonlinear ELM Simulations Using Fast Measurements from DIII-D.
GP8.00114: A.M. Dimits, et al, Efficient Non-Fourier Implementation of Landau-Fluid Operators in the BOUT++ Code.
GP8.00116: P.W. Xi, et al, ETG turbulence simulation of tokamak edge plasmas via 3+1 gyrofluid code
GP8.00117: S.S. Kim, et al, Core Gyrofluid Simulations of Ion Temperature Gradient Turbulence Using BOUT++.
GP8.00120: T. Rhee, et al, BOUT++ Simulations of ELMs with Four-Field Model
GP8.00121: Z. H. Wang, et al, Simulations of Plasma Profile Evolution during SMBI using BOUT++ Code
GP8.00115: C. H. Ma, et al., A upwind PPM with limiter for tokamak edge plasmas simulation under BOUT++ framework.
GP8.00118: Minwoo Kim, et al, Comparison study between the observed ELM dynamics in the KSTAR H-mode and simulation results from BOUT++
GP8.00122: Bin Gui, et al., Simulations of plasma responses due to RMP and external antenna with BOUT++ code
GP8.00123: I Joseph, et al, Flute-reduced drift-MHD model for external magnetic perturbations using the BOUT++ code

Session JP8: Poster Session IV: 2:00 PM–5:30 PM, Tuesday, October 30, 2012. Room: Hall BC
JP8.00086: Evan Davis, et al, BOUT++ Simulations of Edge Turbulence in Alcator C-Mod's EDA H-Mode.
JP8.00087: T. Golfinopoulos, et al, Alcator C-Mod's Quasi-Coherent Mode Antenna: Experimental Results and Interpretation.

Session PO7: 2:00 PM–4:24 PM, Wednesday, October 31, 2012. Room: 556AB
PO7.00005: T. Y. Xia, et al, Six-field two-fluid simulations on edge localized modes with BOUT++

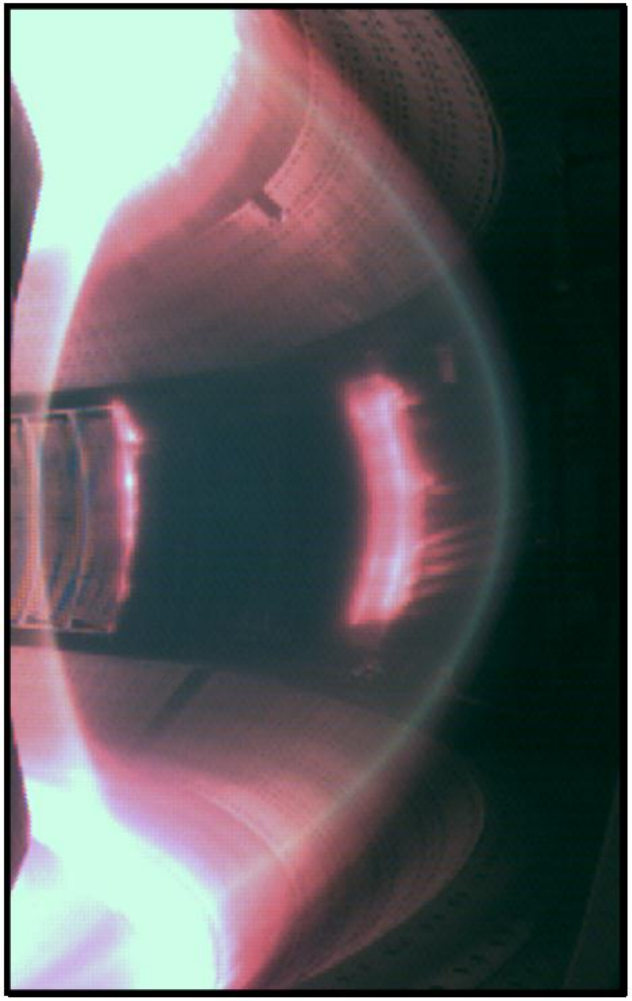
Session YP8: 9:30 AM–12:30 AM, Friday, November 2, 2012. Room: Hall BC
YP8.00068 : Winston Frias, et al, Simulation of gradient drift instabilities in Hall thruster plasmas with the BOUT++ code

Backup slides

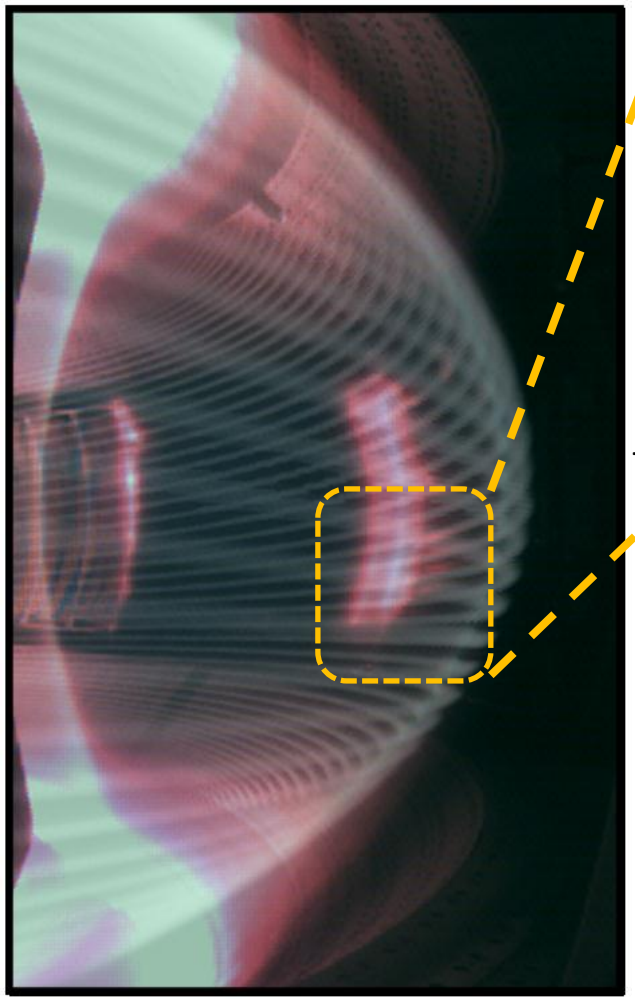
BOUT++ simulations show that the stripes from visible camera match ELM filamentary structures

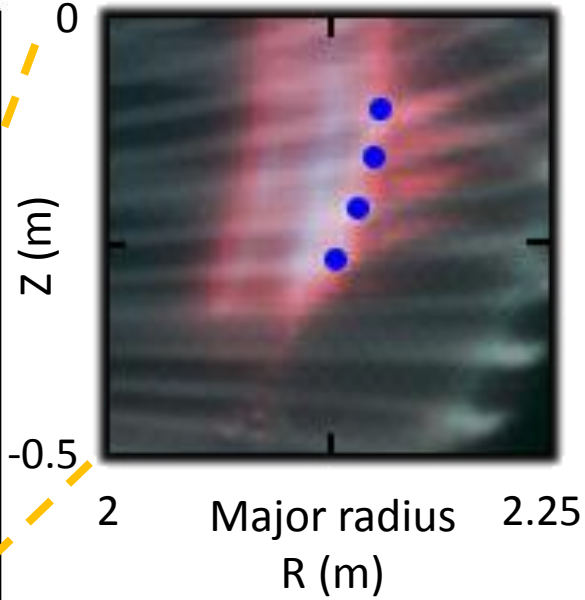
TI3.00002

EAST#41019@3034ms
Visible camera shows bright ELM structure$



BOUT++ simulation shows that the ELM stripe are filamentary structures*





- Pitch angle match!
- Mode number match!

T. Y. Xia, X.Q. Xu, Z. X. Liu, et al, TH/5-2Ra, 24th IEAE FEC, San Diego, CA, USA, 2012

$Photo by J. H. Yang
*Figure by W.H. Meyer