H-mode pedestal turbulence in DIII-D and NSTX using BOUT++ code*  

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Abstract

In this work, we will report BOUT++ simulations for H-mode pedestal instabilities and turbulent transport. For DIII-D H-mode discharges, the BOUT++ peeling-ballooning ELM model including electron inertia was used to analyze the ideal linear stability and ELM dynamics. The beta scan is carried out from a series of self-consistent MHD equilibria generated from EFIT by varying pressure and/or current. For typical tokamak pedestal plasmas with high temperature and low collisionality, we found that the collisionless ballooning modes driven by electron inertia are unstable in the H-mode pedestal and have a lower beta threshold than ideal peeling-ballooning modes, which are the triggers for Edge Localized Modes. Thus, collisionless (electron inertia) ballooning modes might be responsible for H-mode turbulence transport when the pedestal is stable to peeling-ballooning modes. BOUT++ calculations also show that NSTX ELM stability boundaries are sensitive to flow shear profile. Attempts are underway to calculate nonlinear turbulence and transport in H-mode discharges due to the non-ideal effects.
The Nonlinear System of Equations for Simulating Non-Ideal MHD Peeling-Ballooning Modes

\[ \frac{\partial \tilde{\omega}}{\partial t} + \mathbf{v}_E \cdot \nabla \tilde{\omega} = B_0 \nabla_{\|} \tilde{J}_\| + 2b_0 \times \kappa_0 \cdot \nabla \tilde{p} + \mu_{i,\|} \partial^2_{\|0} \tilde{\omega} + \mu_{i,\perp} \nabla^2_{\perp} \tilde{\omega}, \]

Here \( \nabla_{\|} F = B \nabla_{\|} (F/B) \) for any \( F, \) \( \partial_t = \partial_{\|0} + \tilde{b} \cdot \nabla, \) \( \tilde{B}/B = \nabla \tilde{A}_\| \times b_0/B, \) \( \partial_{\|0} = b_0 \cdot \nabla, \) \( \kappa_0 = b_0 \cdot \nabla b_0, \) ion pressure \( P_i \) and total pressure \( P = P_i + P_e. \) (1)

\[ \frac{\partial P}{\partial t} + \mathbf{v}_E \cdot \nabla P = \chi_{\|} \partial^2_{\|0} P, \] (2)

\[ \frac{\partial \tilde{A}_\|}{\partial t} = -\nabla_{\|} \Phi + \frac{\eta}{\mu_0} \nabla^2_{\perp} \tilde{A}_\| - \frac{\eta_H}{\mu_0} \nabla^4_{\perp} \tilde{A}_\|, \] (3)

\[ \tilde{\omega} = \frac{n_0 M_i}{B_0} \left( \nabla^2_{\perp} \tilde{\phi} + \frac{1}{n_0 Z_i e} \nabla^2_{\perp} \tilde{p}_i \right), \quad \Phi = \tilde{\phi} + \Phi_0, \]

\[ P = \tilde{p} + P_0, \]

\[ J_{\|} = J_{\|0} - \frac{1}{\mu_0} \nabla^2_{\perp} \tilde{A}_\|, \]

\[ \mathbf{v}_E = \frac{1}{B_0} \left( b_0 \times \nabla_{\perp} \Phi \right). \] (5)

This simple set of reduced two-fluid equations effectively bypasses the issue of the gyroviscous cancellations in simulations while the important diamagnetic effect is retained in the second term of the generalized vorticity expression.
The effect of transport coefficients on linear P-B instabilities.

The growth rate of the \( n = 15 \) eigenmode versus various transport coefficients with the \( E \times B \) drift and diamagnetic drift for \( S = 10^8 \) and \( \alpha_H = 10^{-4} \): (a) the parallel diffusivity \( \chi \parallel \), (b) the ion perpendicular viscosity \( \mu_{i,\perp} \) and (c) the ion parallel viscosity \( \mu_{i,\parallel} \).

For ITER pedestal parameters \( T_{e,\text{ped}} \simeq 4.5 \text{ keV}, n_{e,\text{ped}} \simeq 5 \times 10^{-10} \text{ m}^{-3}, \chi_{e,\parallel}^{SH} \simeq 2.62 \times 10^{11} \text{ m}^2 \text{ s}^{-1} \) and \( \chi_{e,\parallel}^{SH}/D_A \simeq 794 \), while \( \chi_{e,\perp}^{PL} \simeq n_{e,\text{eq}} R \simeq 1.16D_A \). Similarly, for typical pedestal plasma parameters, \( \mu_{i,\perp} \simeq (0.1 - 1) \text{ m}^2 \text{ s}^{-1} \) as radial thermal diffusivity with the assumption that the turbulent Prandtl numbers are close to unity. Namely, \( \mu_{e,\perp}/\chi_{e,\perp} \simeq \mu_{i,\perp}/\chi_{i,\perp} \) and \( \chi_{e,\perp} \simeq \chi_{i,\perp} \), which yields \( \mu_{i,\perp}/D_A \simeq (0.3 - 3) \times 10^{-8} \), the impact of the perpendicular ion viscosity on the growth rate is negligibly small.

Equations (1)–(5) are solved using a field-aligned (flux) coordinate system \((x, y, z)\) with shifted radial derivatives. Different methods used are fourth-order central differencing and third-order WENO advection scheme. The resulting difference equations are solved with a fully implicit Newton–Krylov solver: Sundials CVODE package. Radial boundary conditions used are \( \frac{\partial}{\partial r} \tilde{A}_1 = 0, \frac{\partial}{\partial r} \tilde{A}_2 = 0, \frac{\partial \tilde{\rho}}{\partial r} = 0 \) and \( \frac{\partial}{\partial r} \tilde{\psi} = 0 \) on the inner radial boundary; \( \tilde{\sigma} = 0, \frac{\partial^2}{\partial r^2} \tilde{A}_1 = 0, \tilde{\rho} = 0 \) and \( \tilde{\phi} = 0 \) on outer radial boundary. The domain is periodic in the parallel coordinates \( y \) (with a twist-shift condition) and in \( z \) (toroidal angle).
Lundquist Number (S) is a dimensionless ratio of the resistive diffusion time to the Alfvén time

\[ S = \frac{\mu_0 R v_A}{\eta} \]

- \( S \sim 10^7 \) in C-Mod EDA pedestal

Davis, et al, UP9.00008
BOUT++ calculations show that Diamagnetic Effects Damp Higher Mode Numbers, yielding the growth rate peaks at n=25, consistent with measurements.

Preliminary Nonlinear Simulations have begun --- Mode Saturation and Turbulent Steady-State have been Observed. Comparisons with experimental measurements will begin.

Davis, et al, UP9.00008
BOUT++ simulations for DIII-D ELMy H-mode shot #131997 at reduced $J_{\parallel}$

- Ideal MHD stability boundary is consistent with infinite-$n$ BALLOO code
- Inclusion of $e^-$ inertial eliminates the stability boundary
BOUT++ simulations for DIII-D ELMy H-mode shot #131997 at reduced J$_{||}$

Varyped: $P_{0,v17} = 0.6 P_{0,exp}$, $P_{0,v69} = P_{0,exp}$

Growth rate ($\gamma \tau_A$)

Inclusion of $e^-$ inertial eliminates or reduces the ion diamagnetic stabilization
BOUT++ nonlinear simulations for DIII-D H-mode shot #132016 at $t=3034\text{ms}$ & $I_p=1.49\text{MA}$

$q_{\text{MHD}}$

$P_0(\text{Pa})$, pressure
$J_{\parallel}/100(\text{A/m}^2)$, parallel current
BOUT++ nonlinear simulations for DIII-D H-mode shot #132016 at t=3034ms & I_p=1.49MA

Varyped: $P_{0,v5} = P_{0,\text{exp}}$, $P_{0,v10} = 1.5P_{0,\text{exp}}$

- Inclusion of $e^{-}$ inertial eliminates the stability boundary
BOUT++ nonlinear simulations for DIII-D H-mode shot #132016 at t=3034ms & I_p=1.49MA

Keeping ballooning mode eigen-function from linear to nonlinear phase
BOUT++ nonlinear simulations for DIII-D H-mode shot #132016 at \( t=3034 \text{ms} \) & \( I_p=1.49 \text{MA} \)

\[ 2\mu_0 \delta p_{\text{rms}, v_{10}/B^2} \]

\( t=280\tau_{\text{Alfvén}} \)

✓ X-point magnetic shear limits the poloidal extent of perturbation on low field side.
BOUT++ simulations for one of the latest designs of the ITER 15 MA inductive ELMy H-mode scenario (under the burning condition)

- Simulations starting from equilibrium generated by the CORSICA code.

- Marginal unstable pedestal case, $T_{\text{ped}} = 5.5\text{keV}$, $n_{\text{max}} = 15$
- The calculations impact previous ITER ELMy H-mode scenario design as it was based on the pedestal height $T_{\text{ped}} = 4.5\text{keV}$

X.Q.Xu, B.D.Dudson, P.B.Snyder, M.V.Umansky, H.R.Wilson and T.Casper, Nucl. Fusion 51 (2011)
BOUT++ simulations for one of the latest designs of the ITER 15 MA inductive ELMy H-mode scenario

It is numerical challenge to simulation ITER divertor geometry, requiring high resolutions $n_x > 1000$, $n_y > 100$, even for linear mode.

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X.Q.Xu, B.D.Dudson, P.B.Snyder, M.V.Umansky, H.R.Wilson and T.Casper, Nucl. Fusion 51 (2011)
BOUT++ simulations show radial and poloidal mode structures and for the ITER 15 MA inductive ELMy H-mode scenario

X.Q.Xu, B.D.Dudson, P.B.Snyder, M.V.Umansky, H.R.Wilson and T.Casper, Nucl. Fusion 51 (2011)
BOUT++ Calculations Show NSTX discharge 129032 Resistively Unstable

With assumption that $V_{\text{ExB}}=V_{\text{diam}}$, all modes are stabilized.

The detailed flow profile does matter for this discharge.
Magnetic Reconnection and Pedestal Collapse during ELMs
Equilibrium current and pressure profiles used as BOUT++ input
Flux-surface-averaged pressure profile $2m_0 \frac{<P>}{B^2}$ vs $S$ with $S_H=10^{12}$

low $S$ -> large ELM size, ELM size is insensitive when $S>10^7$

(1) a sudden collapse: P-B modes -> magnetic reconnection -> bursting process
(2) a slow backfill as a turbulence transport process

For $S=10^8$, $S_H = 10^{12}$, the reconnection region is small and the collapse is limited.

Role of the hyper-resistivity on nonlinear ELM simulations.

Figure 6. ELM sizes versus Lundquist number $S$ with $S_H = 10^{12}$.

\[
\frac{\partial \hat{A}_\parallel}{\partial \hat{t}} = \hat{\nabla}_\parallel \hat{\Phi} + \frac{1}{S} \hat{\nabla}_\perp^2 \hat{A}_\parallel + \frac{1}{S_H} \hat{\nabla}_\perp^4 \hat{A}_\parallel
\]

ideal MHD term contains $\nabla_{\parallel 0} = k_\parallel q R_0 = m - nq$
Equilibrium flow shear model

\[ \frac{\partial \omega}{\partial t} + \left( \mathbf{V}_{EP0} + \mathbf{V}_{EV0} \right) \cdot \nabla \omega + \mathbf{V}_1 \cdot \nabla \omega_0 = B_0^2 \nabla \|J\| + 2b_0 \times \kappa \cdot \nabla P_1 \]

Diamagnetic convection flow \hspace{1cm} \text{Net flow} \hspace{1cm} \text{Kelvin-Helmholtz term}

\[ \bar{\omega} = \frac{\rho_0}{B_0} \left( \nabla_\perp^2 \phi + \frac{1}{n_0 Z_i e} \nabla_\perp^2 P \right) \]

- **Diamagnetic effects:**
  - **Diamagnetic convection flow:** EXB flow that balances diamagnetic flow, is determined by pressure profile, introduces negative electric field \(\Phi_{\text{dia0}}\);
  - **Diamagnetic drift:** inversely depends on density;

- **Net flow:** perpendicular component of toroidal rotation, modeled by a simple function via \(\Phi_{V0}\), flexible;
- **Kelvin-Helmholtz term:** curl of net flow, can be switched off;
- **Total convection flow:** flow shear effects come from this total convection flow rather than the net flow.

Xi, et al, JP9.00103
Equilibrium flow shear can be a double-edged sword on P-B modes

- The flow shear plays the same role as diamagnetic stabilization for ideal MHD case without diamagnetic term.
- Kelvin-Helmholtz drive mainly destabilize intermediate n modes: n=10~30.

Xi, et al, JP9.00103
5-field Peeling-Ballooning model

- In order to investigate the separate effects of density and temperature effect, we extend the 3-field simple P-B model into 5-field model by separating the total pressure into density electron and temperature.

\[
\frac{\partial n_i}{\partial t} + \mathbf{V}_E \cdot \nabla n_i = 0, \quad \nabla \cdot \mathbf{E} = 0,
\]

\[
\frac{\partial T_j}{\partial t} + \mathbf{V}_E \cdot \nabla T_j = 0,
\]

\[
\frac{\partial \bar{\sigma}}{\partial t} + \nabla \cdot \mathbf{V}_E = B_0^2 \mathbf{b} \cdot \nabla \frac{J_\parallel}{B_0} + 2 \mathbf{b} \times \kappa \cdot \nabla P,
\]

\[
\frac{\partial \psi}{\partial t} = -\frac{1}{B_0} \mathbf{b} \cdot \nabla \Phi + \frac{\eta}{\mu_0} \nabla^2 \psi - \frac{\eta_H}{\mu_0} \nabla^4 \psi,
\]

\[
\sigma = n_0 \frac{m_i}{B_0} \left[ \nabla_\perp^2 \phi + \frac{1}{n_0} \nabla_\perp \phi \cdot \nabla_\perp n_0 + \frac{1}{n_0 Z_i e} \nabla_\perp^2 p \right],
\]

\[
J_\parallel = J_{\parallel 0} - \frac{1}{\mu_0} \nabla_\perp^2 (B_0 \psi),
\]

\[
\mathbf{V}_E = \frac{1}{B_0} (\mathbf{b}_0 \times \nabla_\perp \Phi),
\]

\[
P = k_B n(T_i + T_e) = P_0 + p,
\]

\[
\Phi = \Phi_0 + \phi.
\]

The strong stabilizing density effect on P-B modes is due to ion diamagnetic drift

\[ n_0 = \text{constant in } x, \ T_{e0} \text{ and } T_{i0} \text{ vary in } x \]

- For ideal MHD, \( n_0 \) does not affects the normalized linear growth rate.
- With diamagnetic effects,
  - low density results in more stable high-\( n \) modes.

\[ n_{i0}(x) = \left( \frac{n_{0\text{height}} \times n_{ped}}{2} \right) \left[ 1 - \tanh \left( \frac{x - x_{ped}}{\Delta x_{ped}} \right) \right] + n_{0\text{ave}} \times n_{ped}, \]

\text{Xia, et al, JP9.00102}